4.3a 2nd order, linear, const coeff,

Ex Solve the IVP

$$y'' - 9y = 0 \qquad y(0) = 2 \text{ and } y'(0) = -1$$

(i) characteristic eqn: $r^2 - 9 = 0$, $\boxed{r = -3, 3}$

(ii) gen soln $\boxed{y(t) = c_1 e^{-3t} + c_2 e^{3t}}$

$\Rightarrow$ derivative: $y' = -3c_1 e^{-3t} + 3c_2 e^{3t}$

(iii) Apply the I.C.

$y(0) = 2$: $\quad y(0) = c_1 e^{-3\cdot 0} + c_2 e^{3\cdot 0}$

$$\boxed{2 = c_1 + c_2}$$

2 eqns
↑ 2 unknown

$y'(0) = -1$: $\quad y'(0) = -3c_1 e^{-3\cdot 0} + 3c_2 e^{3\cdot 0}$

$$\boxed{-1 = -3c_1 + 3c_2}$$

(iv) Solve for the constants:

- graphical
- substitution
- elimination
- Gauss-Jordan
- Cramer's Rule
- matrix inversion
  - "armani" method
  - cofactor method
  - 2x2: method

$$c_1 + c_2 = 2$$
$$-3c_1 + 3c_2 = -1$$

$$\underbrace{\begin{pmatrix} 1 & 1 \\ -3 & 3 \end{pmatrix}}_{A}\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$$

$$[A \,|\, I] \xrightarrow{\text{E.R. ops}} [I \,|\, A^{-1}]$$

$$\left[\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ -3 & 3 & 0 & 1 \end{array}\right] {*3 \atop \downarrow} \rightarrow \left[\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 0 & 6 & 3 & 1 \end{array}\right] *-6$$

$$\rightarrow \left[\begin{array}{cc|cc} -6 & -6 & -6 & 0 \\ 0 & 6 & 3 & 1 \end{array}\right] \big\}+ \quad \left[\begin{array}{cc|cc} -6 & 0 & -3 & 1 \\ 0 & 6 & 3 & 1 \end{array}\right] {\div -6 \atop \div 6}$$

$$A^{-1} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{6} \\ \frac{1}{2} & \frac{1}{6} \end{bmatrix}$$

If $\mathbb{A}\vec{c} = \vec{b}$ then $\vec{c} = \mathbb{A}^{-1}\vec{b}$

$$\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & -\frac{1}{6} \\ \frac{1}{2} & 1/6 \end{pmatrix} \begin{pmatrix} 2 \\ -1 \end{pmatrix}$$

$$\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} \frac{2}{2} + \left(\frac{-1}{6}\right)(-1) \\ \frac{2}{2} + \left(\frac{1}{6}\right)(-1) \end{pmatrix} = \begin{pmatrix} 7/6 \\ 5/6 \end{pmatrix}$$

(v) specific solution:

$$\boxed{y(t) = \frac{7}{6}e^{-3t} + \frac{5}{6}e^{3t}}$$

This is the solution to $\boxed{y'' - 9y = 0,\ y(0) = 2,\ y'(0) = -1}$

EX Solve $y'' + 11y' + 24y = 0$ with $y(0) = 0$
$y'(0) = -7$

(i) characteristic $r^2 + 11r + 24 = 0$

$(r+8)(r+3) = 0 \Rightarrow \boxed{r_1 = -8, r_2 = -3}$

(ii) Gen Soln: $\boxed{y(t) = c_1 e^{-8t} + c_2 e^{-3t}}$

$y'(t) = -8c_1 e^{-8t} - 3c_2 e^{-3t}$

(iii) IVP: $y(0) = 0 \Rightarrow c_1 + c_2 = 0$

$y'(0) = -7 \Rightarrow -8c_1 - 3c_2 = -7$

2 eqns and 2 unknowns

(iv) Solve: $c_1 + c_2 = 0 \rightarrow c_1 = -c_2 \rightarrow \underline{\underline{c_1 = \frac{7}{5}}}$

$8c_1 + 3c_2 = 7$

$-8c_2 + 3c_2 = 7 \Rightarrow \underline{\underline{c_2 = -\frac{7}{5}}}$

(v) Specific Soln: $\boxed{y(t) = \frac{7}{5} e^{-8t} - \frac{7}{5} e^{-3t}}$

⊛ Be ready for some messy real roots:

EX $y'' - 6y' - 2y = 0 \Rightarrow r^2 - 6r - 2 = 0$

$\rightarrow r_{1,2} = 3 \pm \sqrt{11}$

$$\boxed{y(t) = c_1 e^{(3+\sqrt{11})t} + c_2 e^{(3-\sqrt{11})t}}$$