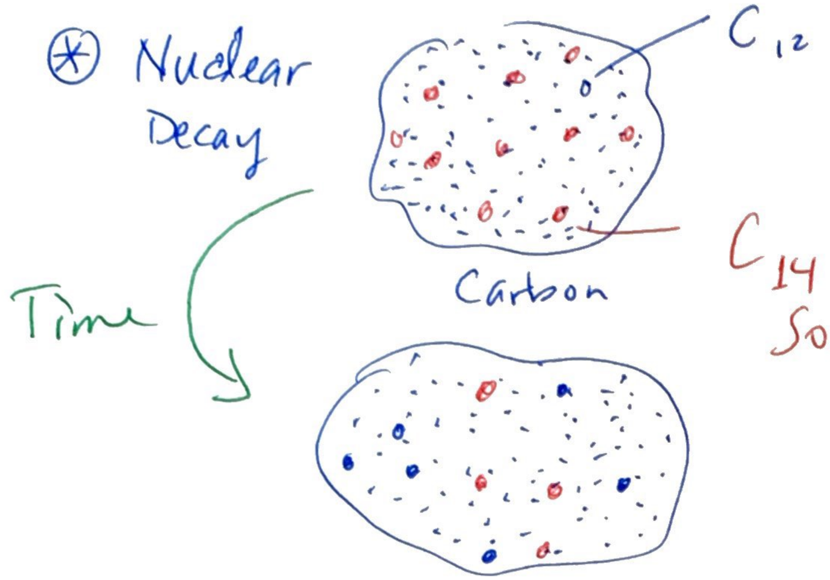
$\boxed{1.3}$ <u>Models for 1^{st} order ODEs</u>

⊛ Nuclear Decay



C_{14} ← unstable and at some point decays to C_{12}

N_{14}

Change (decay) of the number of Carbon 14 atoms is proportional to the number of C_{14} atom present

build in the decay

$$\frac{\Delta N_{14}}{\Delta t} \propto N_{14} \rightarrow \boxed{\frac{dN_{14}}{dt} = -k N_{14}}$$

- Solve via separation of variables

$$\frac{dN_{14}}{N_{14}} = -k\,dt$$

$$\int \frac{dN_{14}}{N_{14}} = -\int k\,dt$$

$$\ln N_{14} = -kt + C$$

↖ always (+)

$$\exp(\ln N_{14}) = \exp(-kt + C)$$

$$N_{14} = e^{-kt+C}$$

$$N_{14} = e^{-kt} e^{C}$$

$$\boxed{N_{14}(t) = Ce^{-kt}}$$

gen. soln.

I.C.

$$N_{14}(0) = N_0$$

$$\boxed{N_{14}(t) = N_0 e^{-kt}}$$

spec. soln.

④ Logistitic Equation

- Heat Flow , also population growth with a predator

$$\frac{dP}{dt} = aP - KP^2$$

wolfes

uninhibited growth

$$\left\{\begin{array}{ll} \boxed{P' = aP(1-\alpha P)} & \text{O.D.E.} \\ \text{I.C.} \quad P(0) \equiv P_0 & \end{array}\right\} \text{I.V. Prob.}$$

So "a" and "α" parameters

- Solve by separating variables

$$\frac{dP}{P(1-\alpha P)} = a\,dt$$

- Integrate

$$\int \frac{dP}{P(1-\alpha P)} = a\int dt$$

- Solution

$$\frac{A}{P} + \frac{B}{1-\alpha P} \quad \text{(partial fractions)}$$

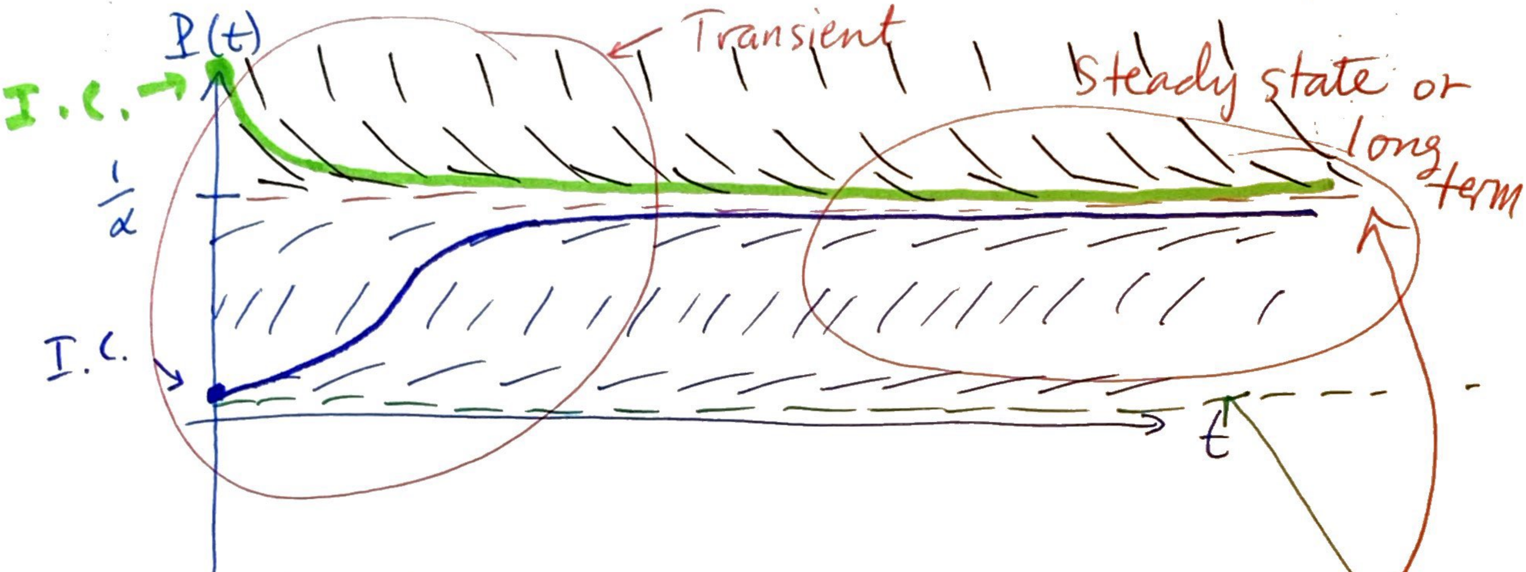
$$\Rightarrow \boxed{P(t) = \frac{P_0 e^{at}}{\alpha P_0 e^{at} + (1-\alpha P_0)}}$$

Solution to the I. Value Prob.

$\boxed{\text{Ex}}$ (Cont.)

Sol: $$\underline{P}(t) = \frac{\underline{P}_0 e^{at}}{\alpha \underline{P}_0 e^{at} + (1-\alpha \underline{P}_0)}$$

ODE: $$\underbrace{\frac{d\underline{P}}{dt}}_{\text{slope in the } \underline{P} \text{ vs. } t \text{ plot.}} = a\underline{P}(1-\alpha\underline{P})$$



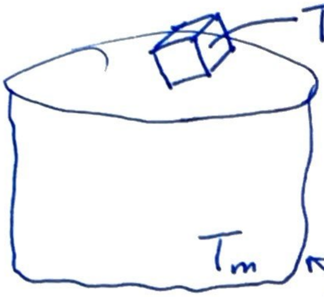
• zero slope $\left\{ \begin{aligned} \underline{P}'\Big|_{\underline{P}=\frac{1}{\alpha}} &= a\left(\frac{1}{\alpha}\right)\left(1-\alpha\left(\frac{1}{\alpha}\right)\right) \\ &= 0 \end{aligned} \right.$

• zero slope $\left\{ \begin{aligned} \underline{P}'\Big|_{\underline{P}=0} &= a\cdot 0\cdot(1-\alpha\cdot 0) \\ &= 0 \end{aligned} \right.$

• If $\underline{P} > \frac{1}{\alpha}$ then $\underline{P}' = a\underline{P}(1-\alpha\underline{P}) < 0$ (−) slope

• If $0 < \underline{P} < \frac{1}{\alpha}$ then $\underline{P}' > 0$ (+) slope

⊛ Newton's Law of Cooling



$T(t) = T_0$ @ $t=0$

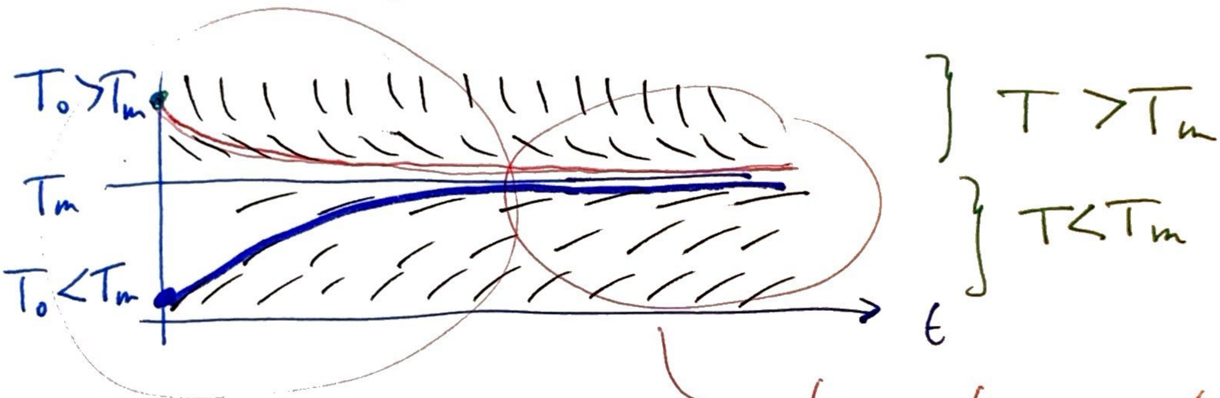
T_m = the temperature of the "medium"

"reservoir" because we assume they are so vast that the sample will not change their temp.

- the equation that manages (models) the heat flow:

$$\boxed{\frac{dT}{dt} = -k\left(T - \overset{\text{const.}}{T_m}\right)}$$

transient



long term steady state

$T = T_m$

1.3 is Finished {skip 1.2 till after Test #1}