Add-On HW: Eigenvalue Problems

The matrix operation $A\vec{x} = \vec{y}$ swallows $\vec{x}$ and spits out $\vec{y}$

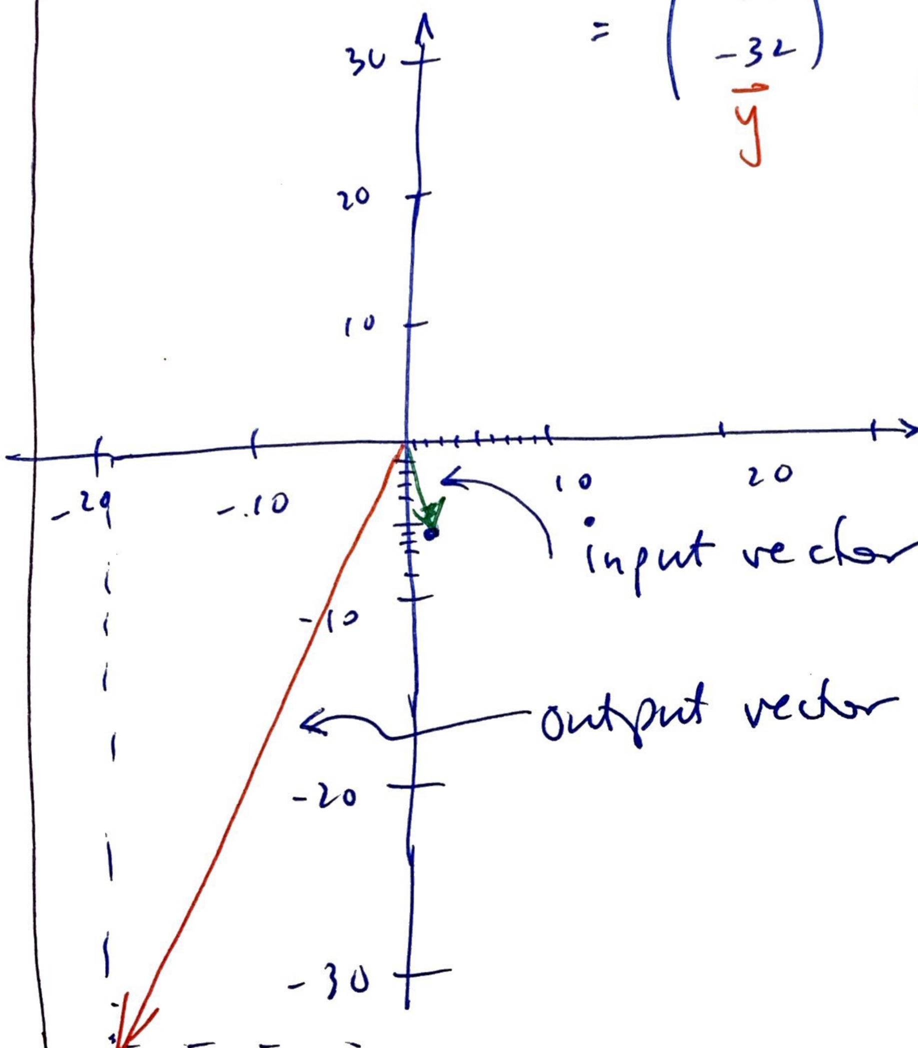
Ex

2-D

$$\overset{A}{\begin{pmatrix} 1 & 3 \\ -2 & 4 \end{pmatrix}} \overset{\vec{x}}{\begin{pmatrix} 2 \\ -7 \end{pmatrix}} = \begin{pmatrix} 1\cdot 2 + 3\cdot(-7) \\ -2\cdot 2 + 4(-7) \end{pmatrix} = \underset{\vec{y}}{\begin{pmatrix} -19 \\ -32 \end{pmatrix}}$$

In: $(x, y) = (2, -7)$

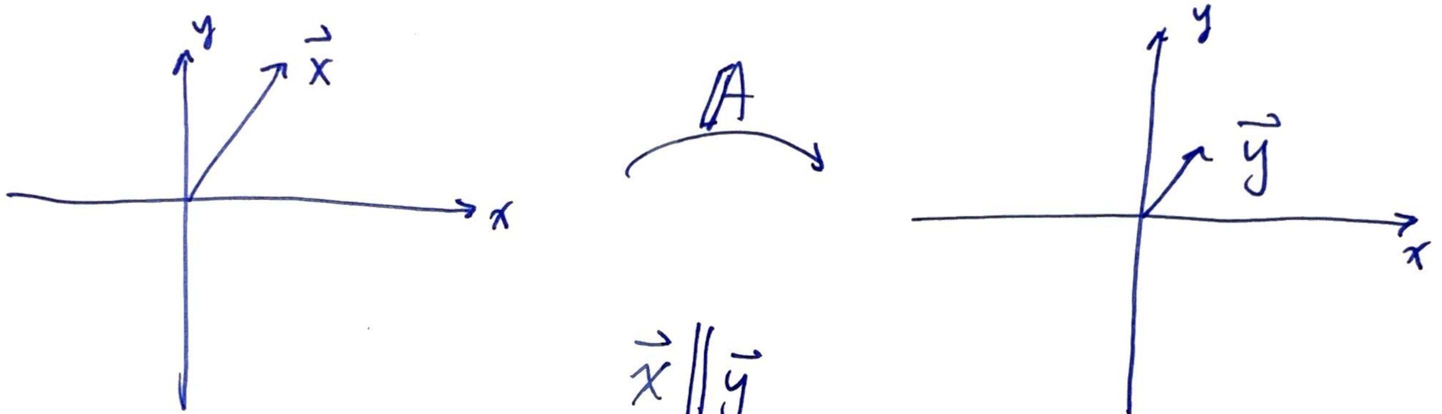
Out: $(x, y) = (-19, -32)$



We are interested in ==input vectors== that produce ==output vectors== that are ==parallel== to the input vector. These are "Feedback" vectors: problems can happen in certian disipilines -or- optimal success can be had in negative -or- positive feed back.

video — Tacoma Narrows w/ practical engr'n

$$\mathbb{A}\vec{x} = \vec{y}$$



$$\vec{x} \parallel \vec{y}$$

- we call these vectors "==eigenvectors=="

$\Longrightarrow$ $$\boxed{\mathbb{A}\vec{x} = \lambda\vec{X}}$$

$\vec{x}$ ↑ in, λ ↑ scaling value, $\vec{X}$ ↓ out, same $\vec{x}$

- the scaling value is called the "==eigenvalue=="

(*) ==each eigen value has its associated eigenvector(s)==

Ex Show the vector $(3,1)$ is an eigenvector of the matrix $\begin{pmatrix} 2 & -12 \\ 1 & -5 \end{pmatrix}$

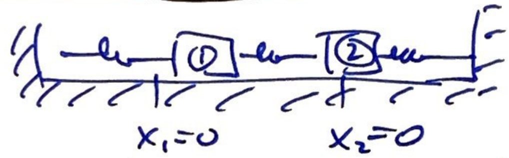
$$\begin{pmatrix} 2 & -12 \\ 1 & -5 \end{pmatrix}\begin{pmatrix} 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 2\cdot 3 - 12\cdot 1 \\ 1\cdot 3 - 5\cdot 1 \end{pmatrix} = \begin{pmatrix} -6 \\ -2 \end{pmatrix} = -2\begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

$\begin{pmatrix} 3 \\ 1 \end{pmatrix}$ is the E. vector

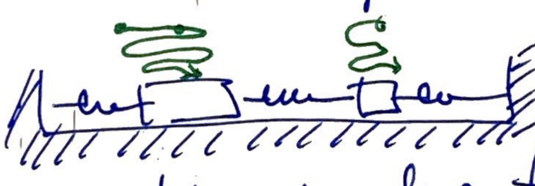
$\lambda = -2$ is the E. value

(*) eigen values and eigen vectors correspond to reasonance modes it physical system.

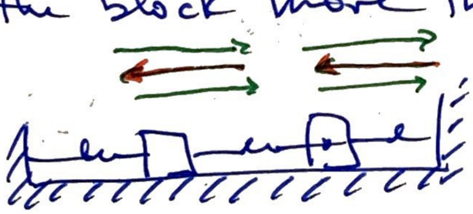
Ex 2-springs

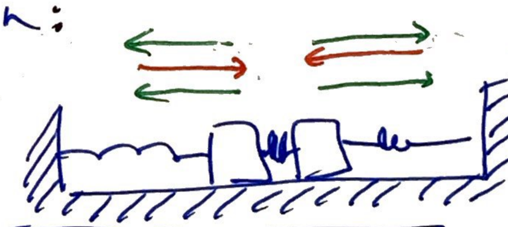


Normally there is a complicated motion



- But there are two modes of oscillation where the block move in syn:



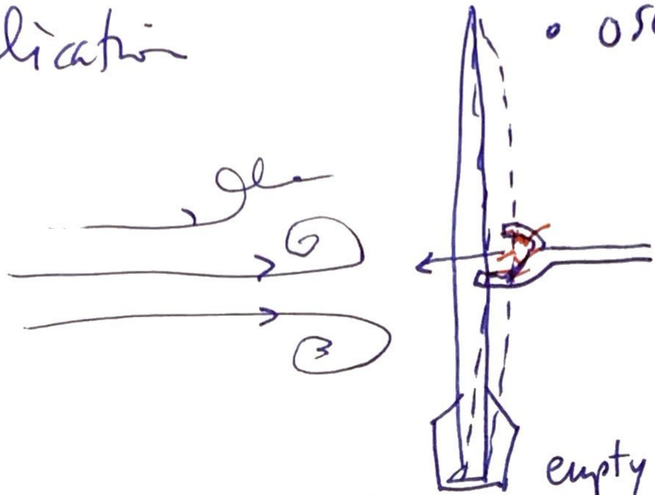


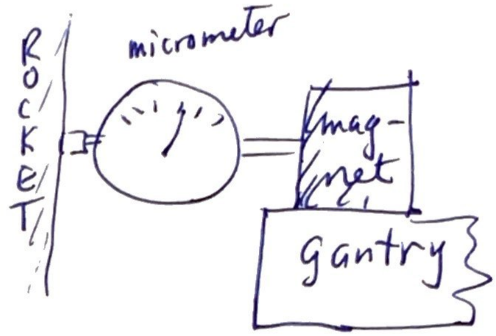
eigen mode one "in sync"

eigen mode two oppositely synchronized

Application

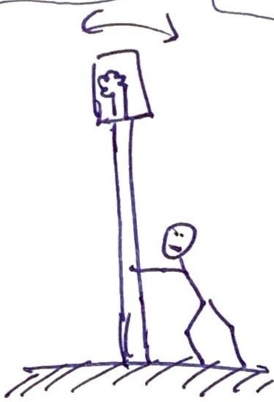
- oscillation of a rocket





empty rocket vibrates in wind.

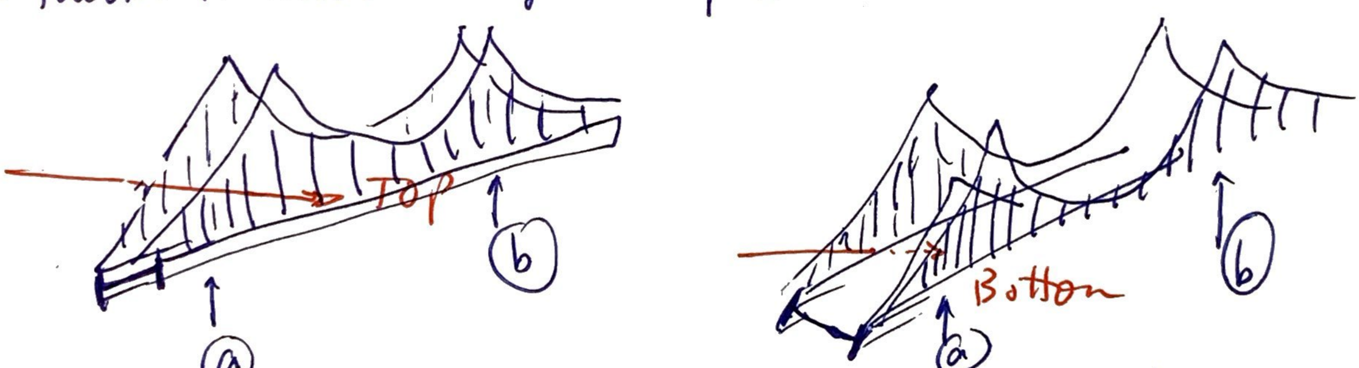
- Light pole



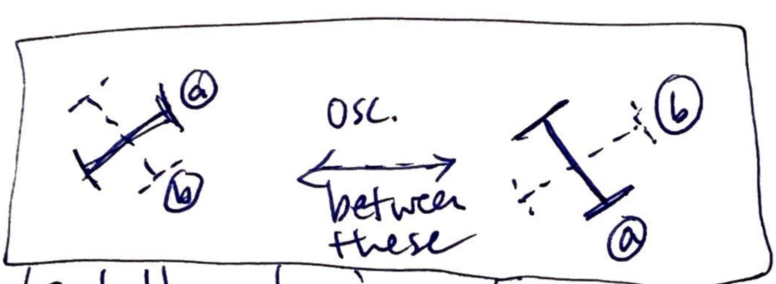
Resonance when drive (push) @ the natural frequency of the structure.

eigen modes are resonance modes.

- Tacoma Narrow's Bridge Collapse



eigenmode (resonance)



cut through views

- Back to the math: we seek an $\vec{x}$ such that

$$\mathbb{A}\vec{x} = \lambda\vec{x}$$

multiply by $\mathbb{I}$, the RHS:

since $\mathbb{I}\vec{x} = \vec{x}$

$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ identity matrix

$$\mathbb{A}\vec{x} = \lambda\mathbb{I}\vec{x}$$

subtract

$$\mathbb{A}\vec{x} - \lambda\mathbb{I}\vec{x} = \vec{0}$$

factor $\boxed{(\mathbb{A} - \lambda\mathbb{I})\vec{x} = \vec{0}}$

- For non-trivial solutions we need the following

$$\det(\mathbb{A} - \lambda\mathbb{I}) = 0$$

, otherwize the only solution is $\vec{x} = \vec{0}$, the trivial solution

- Def: $\boxed{\det(\mathbb{A} - \lambda\mathbb{I}) = 0}$ is called the characteristic polynomial of the matrix $\mathbb{A}$.

Thm 7.2

(*) Solving the eigenproblem has two steps:

1) Solve for λ : $\det(\mathbb{A} - \lambda\mathbb{I}) = 0$

2) insert each λ from 1) and solve for $\vec{x}$

$(\mathbb{A} - \lambda\mathbb{I})\vec{x} = \vec{0}$, call the vectors $\vec{n}_1, \vec{n}_2 \dots$

ODE systems: $\vec{y} = c_1\vec{n}_1 e^{\lambda_1 t} + c_2\vec{n}_2 e^{\lambda_2 t}$

ex Find the eigen values and eigen vectors of $\mathbb{A} = \begin{pmatrix} 2 & 7 \\ -1 & -6 \end{pmatrix}$

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1 eigenvalues

$$\det(\mathbb{A} - \lambda \mathbb{I}) = 0$$

$$\det\left(\begin{pmatrix} 2 & 7 \\ -1 & -6 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right) = 0$$

$$\det\begin{pmatrix} 2-\lambda & 7 \\ -1 & -6-\lambda \end{pmatrix} = 0$$

$$\left\{ \det\begin{pmatrix} a & b \\ c & d \end{pmatrix} = a \cdot d - cb \right\}$$

$$(2-\lambda)(-6-\lambda) - (-1)(7) = 0$$

$$-(2-\lambda)(6+\lambda) + 7 = 0$$

$$(\lambda-2)(\lambda+6) + 7 = 0$$

$$\lambda^2 - 2\lambda + 6\lambda - 12 + 7 = 0$$

$$\boxed{\lambda^2 + 4\lambda - 5 = 0}$$

Characteristic polynomial of the matrix $A = \begin{pmatrix} 2 & 7 \\ -1 & -6 \end{pmatrix}$

$$(\lambda+5)(\lambda-1) = 0$$

$$\Rightarrow \boxed{\lambda_1 = 1 \,, \quad \lambda_2 = -5}$$

eigenvalues of $\mathbb{A}$

EX (cont.)

② Find the eigenvector for each eigenvalue.

$\boxed{\lambda_1 = 1}$ $\quad (\mathbb{A} - \lambda \mathbb{I})\vec{\eta} = \vec{0}$ where $\vec{\eta} = \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix}$

$\vec{\eta}$ ← vector; η_2 ← scalar

or $\mathbb{A}\vec{x} = \vec{b}$

$(\mathbb{A}|\vec{b})$ augmented system $\boxed{\left(\begin{array}{cc|c} 2-\lambda & 7 & 0 \\ -1 & -6-\lambda & 0 \end{array}\right)}$ $\lambda = 1$

$$\left(\begin{array}{cc|c} 2-1 & 7 & 0 \\ -1 & -6-1 & 0 \end{array}\right) \to \left(\begin{array}{cc|c} 1 & 7 & 0 \\ -1 & -7 & 0 \end{array}\right) + \text{add}$$

$$\Rightarrow \left(\begin{array}{cc|c} 1 & 7 & 0 \\ 0 & 0 & 0 \end{array}\right) \to \begin{pmatrix} 1 & 7 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

expected since we require $|\mathbb{A} - \lambda \mathbb{I}| = 0$

back to equations

$$\Rightarrow 1\cdot\eta_1 + 7\cdot\eta_2 = 0$$

$$\eta_2 = -\frac{1}{7}\eta_1$$

- Form the vector $\vec{\eta}_1$:

$$\vec{\eta}_1 = \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} = \begin{pmatrix} \eta_1 \\ -\frac{1}{7}\eta_1 \end{pmatrix} = \boxed{\begin{pmatrix} 7 \\ -1 \end{pmatrix}}\frac{\eta_1}{7}$$

eigenvector for $\lambda_1 = 1$

Ignore $\eta_1/7$ since recall $\vec{y} = c_1\vec{\eta}_1 e^{\lambda_1 t} + c_2\eta_2 e^{\lambda_2 t}$ and we will bury the $\eta_1/7$ into c_1

next eigenvectors . . .

$\boxed{\lambda_2 = -5}$

- Solve $\left(\begin{array}{cc|c} 2-(-5) & 7 & 0 \\ -1 & -6-(-5) & 0 \end{array}\right)$

$$\Rightarrow \left(\begin{array}{cc|c} 7 & 7 & 0 \\ -1 & -1 & 0 \end{array}\right) *\tfrac{1}{7}) +$$

$$\Rightarrow \left(\begin{array}{cc|c} 7 & 7 & 0 \\ 0 & 0 & 0 \end{array}\right)$$

back to eqn space

$$7\eta_1 + 7\eta_2 = 0$$

$$\eta_2 = -\eta_1$$

second eigenvector

- Form vector: $\vec{\eta}_2 = \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} = \begin{pmatrix} \eta_1 \\ -\eta_1 \end{pmatrix} = \boxed{\begin{pmatrix} 1 \\ -1 \end{pmatrix}} \eta_1$

Summary: $\boxed{A = \begin{pmatrix} 2 & 7 \\ -1 & -6 \end{pmatrix}}$

- eigenvalue $\lambda_1 = 1$ has the eigenvector $\begin{pmatrix} 7 \\ -1 \end{pmatrix}$ which is a Basis for eigenspace for λ_1
- eigenvalue $\lambda_2 = -5$ has the eigenvector $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ which is a Basis for λ_2's eigenspace

Test: $\begin{pmatrix} 2 & 7 \\ -1 & -6 \end{pmatrix}\underline{\underline{\begin{pmatrix} 1 \\ -1 \end{pmatrix}}}$

$$= \begin{pmatrix} 2\cdot 1 + 7\cdot(-1) \\ -1(1) + (-6)(-1) \end{pmatrix} = \begin{pmatrix} 2-7 \\ -1+6 \end{pmatrix} = \begin{pmatrix} -5 \\ 5 \end{pmatrix} = -5\underline{\underline{\begin{pmatrix} 1 \\ -1 \end{pmatrix}}}$$

λ_2 $\quad \vec{\eta}_2$

Properties:

- For A an $n \times n$, $\det(A - \lambda I) = 0$ is an nth order polynomial and thus has n solutions.

- If $\lambda_1, \lambda_2, \cdots \lambda_n$ are those eigenvalues (including repeats)

1. If λ occurs only one time it is a simple eigenvalue
2. If λ occurs $k > 1$ times, then it has multiplicity k
3. If $\lambda_1, \lambda_2, \cdots \lambda_n$ are simple e. values then each of their eigenvectors is Lin. Indep.
4. If λ is of multiplicity $k > 1$ then λ will have anywhere from 1 to k Lin. Indep. eigenvectors.

$\boxed{\text{thm } 7.1}$ If $A_{n \times n}$ has an e. value λ then the set of all e. vect. $\vec{x}$ together with $\vec{0}$ form a subspace of R^n, called the eigenspace

Ex Solve the eigenproblem for $A = \begin{pmatrix} 1 & -1 \\ 4/9 & -1/3 \end{pmatrix}$

(1) $\det(A - \lambda I) = \begin{vmatrix} 1-\lambda & -1 \\ 4/9 & -1/3-\lambda \end{vmatrix} = 0$

$\Rightarrow \lambda^2 - \frac{2}{3}\lambda + \frac{1}{9} = 0$

multiplicity 2

$\lambda_1 = \frac{1}{3}$; $\lambda_2 = \frac{1}{3}$

(2) $\boxed{\lambda_1 = \frac{1}{3}}$

$$\left(\begin{array}{cc|c} 1-\frac{1}{3} & -1 & 0 \\ 4/9 & -\frac{1}{3}-\frac{1}{3} & 0 \end{array}\right)$$

$$\Rightarrow \left(\begin{array}{cc|c} 2/3 & -1 & 0 \\ 4/9 & -2/3 & 0 \end{array}\right) * -\frac{2}{3}$$

$$\Rightarrow \left(\begin{array}{cc|c} 2/3 & -1 & 0 \\ 0 & 0 & 0 \end{array}\right)$$

- eqn $\frac{2}{3}n_1 - n_2 = 0$ $\quad n_2 = \frac{2}{3}n_1$

- Build e.vector

$$\vec{n}_1 = \begin{pmatrix} n_1 \\ n_2 \end{pmatrix} = \begin{pmatrix} n_1 \\ \frac{2}{3}n_1 \end{pmatrix} = \frac{n_1}{3}\begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

summary

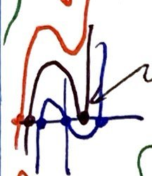
the eigenvector for $\lambda_1 = \lambda_2 = \frac{1}{3}$ is

$$\vec{n}_1 = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

only one e.vector for this matrix!!

Ex

3×3: $\mathbb{A} = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$



Then $\det(\mathbb{A} - \lambda \mathbb{I}) = \begin{pmatrix} -\lambda & 1 & 1 \\ 1 & -\lambda & 1 \\ 1 & 1 & -\lambda \end{pmatrix} = 0$

$\Rightarrow$ $\boxed{\lambda^3 - 3\lambda - 2 = 0}$ charact. poly.

If λ is a rational root then it has to be a combination of factors of p over factors of q

for $q\lambda^n + r\lambda^{n-1} + s\lambda^{n-2} + \cdots + w\lambda + p = 0$

rational roots, if they exist, $\pm \dfrac{\text{factors } p}{\text{factors } q}$

see mymathmantra.com $\rightarrow$ Diff Eqn Reserve "Finding zeros of polynomials"

$\boxed{\lambda_1 = 2, \quad \lambda_2 = -1, \quad \lambda_3 = -1}$

$\boxed{\lambda_1 = 2}$ Solve $\left(\begin{array}{ccc|c} -2 & 1 & 1 & 0 \\ 1 & -2 & 1 & 0 \\ 1 & 1 & -2 & 0 \end{array}\right)$

(to be cont.)

$\boxed{\lambda_1 = 2}$ $\quad (\mathbb{A} - \lambda \mathbb{I})\vec{n} = \vec{0}$

$$\left.\left(\begin{array}{ccc|c} -\lambda & 1 & 1 & 0 \\ 1 & -\lambda & 1 & 0 \\ 1 & 1 & -\lambda & 0 \end{array}\right)\right|_{\lambda=2}$$

$$\left(\begin{array}{ccc|c} -2 & 1 & 1 & 0 \\ 1 & -2 & 1 & 0 \\ 1 & 1 & -2 & 0 \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ -2 & 1 & 1 & 0 \\ 1 & 1 & -2 & 0 \end{array}\right) *2 \; ; \; *-1$$

$$\left(\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & -3 & 3 & 0 \\ 0 & 3 & -3 & 0 \end{array}\right) \xrightarrow{\div -3} \left(\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \rightarrow \begin{array}{l} n_1 - n_2 = 0 \rightarrow \boxed{n_1 = n_2} \\ n_2 - n_3 = 0 \rightarrow \boxed{n_3 = n_2} \end{array}$$

Form the vector $\quad \vec{n}_1 = \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix} = \begin{pmatrix} n_2 \\ n_2 \\ n_2 \end{pmatrix} = \boxed{\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}} n_2$ for $\boxed{\lambda_1 = 2}$

$\boxed{\lambda_2 = -1}$ multiplicity 2

$$(\mathbb{A} - \lambda \mathbb{I})\vec{\eta} = \vec{0}$$

$$\left(\begin{array}{ccc|c} -(-1) & 1 & 1 & 0 \\ 1 & -(-1) & 1 & 0 \\ 1 & 1 & -(-1) & 0 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \end{array}\right)$$

$$\to \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \to \begin{array}{l} \eta_1 + \eta_2 + \eta_3 = 0 \\ \boxed{\eta_1 = -\eta_2 - \eta_3} \end{array}$$

two parameters

Form $\vec{\eta}_2 = \begin{pmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \end{pmatrix} = \begin{pmatrix} -\eta_2 - \eta_3 \\ \eta_2 \\ \eta_3 \end{pmatrix}$

Seperate this vector into two: $\vec{\eta}_2 = \begin{pmatrix} -\eta_2 \\ \eta_2 \\ 0 \end{pmatrix} + \begin{pmatrix} -\eta_3 \\ 0 \\ \eta_3 \end{pmatrix}$

Factor out η_1 & η_2:

$$\vec{\eta}_2 = \boxed{\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}} \eta_2 + \boxed{\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}} \eta_3$$

Two Lin. Indep. e.vectors for $\lambda_{2,3} = -1$

Summary

$$\boxed{\begin{array}{ll} \lambda_1 = 2 & \vec{\eta}_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \\ \lambda_2 = -1 & \left\{ \begin{array}{l} \vec{\eta}_2 = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} \\ \vec{\eta}_3 = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \end{array} \right. \end{array}}$$

Lin. Indep.

EX Find the eigenvalues/vectors for

$$A = \begin{pmatrix} -4 & -17 \\ 2 & 2 \end{pmatrix}$$

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1. $\det(A - \lambda I) = 0$

$$\begin{vmatrix} -4-\lambda & -17 \\ 2 & 2-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (-4-\lambda)(2-\lambda) - (2)(-17) = 0$$

$$\Rightarrow (\lambda+4)(\lambda-2) + 34 = 0$$

$$\boxed{\lambda^2 + 2\lambda + 26 = 0}$$ characteristic Polynom.

Solve $\lambda = \dfrac{-(2) \pm \sqrt{2^2 - 4(1)(26)}}{2 \cdot 1} = \underline{\underline{-1 \pm 5i}}$

$$\boxed{\lambda_1 = -1 + 5i \quad , \quad \lambda_2 = -1 - 5i}$$ e.values.

EX cont.

(2) Find e.vectors

$$\left(\begin{array}{cc|c} -4-\lambda & -17 & 0 \\ 2 & 2-\lambda & 0 \end{array}\right)\Bigg|_{\lambda=-1+5i}$$

$$\Rightarrow \left(\begin{array}{cc|c} -4-(-1+5i) & -17 & 0 \\ 2 & 2-(-1+5i) & 0 \end{array}\right) *-1$$

$$\Rightarrow \left(\begin{array}{cc|c} 3+5i & 17 & 0 \\ 2 & 3-5i & 0 \end{array}\right) * \frac{-2}{3+5i}$$

$$\Rightarrow \left(\begin{array}{cc|c} \cancel{(3+5i)}\left(\frac{-2}{\cancel{3+5i}}\right) & 17\left(\frac{-2}{3+5i}\right) & 0 \\ 2 & 3-5i & 0 \end{array}\right)$$

$$\Rightarrow \left(\begin{array}{cc|c} -2 & \left(\frac{-34}{3+5i}\right) & 0 \\ 2 & 3-5i & 0 \end{array}\right) +$$

side work

$$\left(\frac{-34}{3+5i}\right) + (3+5i)$$

$$= \left(\frac{-34}{3+5i}\right)\left(\frac{3-5i}{3-5i}\right) + (3+5i)$$

$$= \frac{-34(3+5i)}{\underbrace{9+25}_{34}} + (3+5i)$$

$$\Rightarrow -(3+5i) + (3+5i) = \underline{\underline{0}}$$

resulting action $\Rightarrow$

So the e.vector matrix system becomes

$$\left(\begin{array}{cc|c} -2 & -34/3+5i & 0 \\ 0 & 0 & 0 \end{array}\right)$$

* In the future for 2x2 complex e.values pick either the top or the bottom and go into eqn. form.

For us $\left(\begin{array}{cc|c} 3+5i & 17 & 0 \\ 2 & 3-5i & 0 \end{array}\right)$ pick Bottom Row

$$\to \quad 2\eta_1 + (3-5i)\eta_2 = 0 \to \eta_1 = -\frac{3-5i}{2}\eta_2$$

Build vector:

$$\vec{\eta}_1 = \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} = \begin{pmatrix} -\frac{3-5i}{2}\eta_2 \\ \eta_2 \end{pmatrix}$$

$$\vec{\eta}_1 = \boxed{\begin{pmatrix} -3+5i \\ 2 \end{pmatrix}} \frac{\eta_2}{2}$$

conjugate

Property: If $\vec{\eta}_1$ is complex, then $\vec{\eta}_2 = \vec{\eta}_1^{\,*}$

$$\boxed{\left.\begin{array}{l} \lambda_1 = -1+5i, \quad \vec{\eta}_1 = \begin{pmatrix} -3+5i \\ 2 \end{pmatrix} \\ \lambda_2 = -1-5i, \quad \vec{\eta}_2 = \begin{pmatrix} -3-5i \\ 2 \end{pmatrix} \end{array}\right\} \; A = \begin{pmatrix} -4 & -17 \\ 2 & 2 \end{pmatrix}}$$

$(\lambda_{2,3} = -1+5i)$ cont. *(Alternative) Pick Top Row*

$$\Rightarrow \left(\begin{array}{cc|c} 3+5i & 17 & 0 \\ 0 & 0 & 0 \end{array}\right) \leftarrow \text{original row 1}$$

$$\Rightarrow \text{then } (3+5i)\eta_1 + 17\eta_2 = 0$$

$$\boxed{\eta_2 = -\frac{1}{17}(3+5i)\eta_1}$$

Form $\vec{\eta}_1 = \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} = \begin{pmatrix} \eta_1 \\ \left(-\frac{3}{17} - \frac{5}{17}i\right)\eta_1 \end{pmatrix}$

$$\vec{\eta}_1^{\,(\text{top row})} = \boxed{\begin{pmatrix} -17 \\ 3+5i \end{pmatrix}} \left(-\frac{\eta_1}{17}\right) \quad \text{vs} \quad \vec{\eta}_1 = \begin{pmatrix} -3+5i \\ 2 \end{pmatrix} \text{ (Bottom row)}$$

$//$

Q: Is $\vec{\eta}_1^{\,\text{top row}}$ here "parallel" to $\vec{\eta}_1^{\,\text{bot. row}}$ using the bottom row?

A: they should be scalar multiples of each other:

i.e. $\boxed{\vec{\eta}_1^{\,\text{bot. row}}\, k = \vec{\eta}_1^{\,\text{top row}}}$, Both components must scale.

$$\begin{pmatrix} -3+5i \\ 2 \end{pmatrix} k \stackrel{?}{=} \begin{pmatrix} -17 \\ 3+5i \end{pmatrix} \xrightarrow{\text{Top Row}} (-3+5i)k = -17 \Rightarrow k = \frac{-17}{-3+5i}$$

Now does this work on Bottom?

$$\Rightarrow \text{Is } 2 \cdot \overbrace{\left(\frac{-17}{-3+5i}\right)}^{k} \stackrel{?}{=} 3+5i$$

$$\Rightarrow \frac{-34}{-3+5i} \cdot \left(\frac{-3-5i}{-3-5i}\right) \stackrel{?}{=} 3+5i$$

$$\frac{-34(-3-5i)}{9+25} \stackrel{?}{=} 3+5i \Rightarrow \frac{+\cancel{34}(3+5i)}{\cancel{34}} \stackrel{?}{=} 3+5i$$

yes!

either row will yeild the eigenvector!