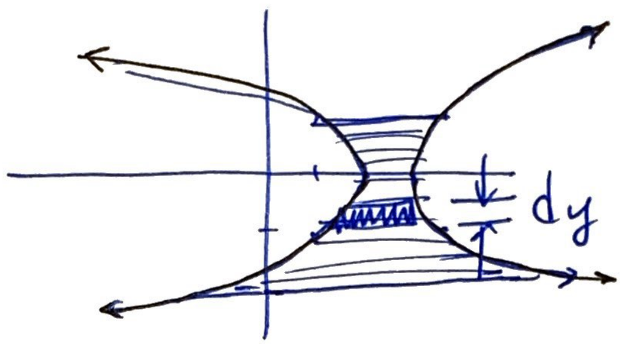
Show all work for FULL credit.
Box in the set-up before proceeding to perform the integration.

1. [5.1 Area] (10 pts) Determine the area of the region bounded by:

$$x=3+y^2, \quad x=2-y^2, \quad y=1 \text{ and } y=-2$$

{HINT: These are sideways parabolas so use dy instead of dx, in other words, the whole integral will be in y }



set-up

$$A = \int_{y=-2}^{1} (x_R - x_L)\,dy = \boxed{\int_{-2}^{1} \left[(3+y^2)-(2-y^2)\right] dy}$$

10

$$= \int_{-2}^{1} \left[1+2y^2\right] dy$$

$$= \left(y + \frac{2y^3}{3}\right)\Bigg|_{-2}^{1}$$

$$= \left(1+\frac{2}{3}\cdot 1^3\right) - \left(-2+\frac{2}{3}(-2)^3\right)$$

$$= \frac{5}{3} - \left(-\frac{6}{3} - \frac{16}{3}\right)$$

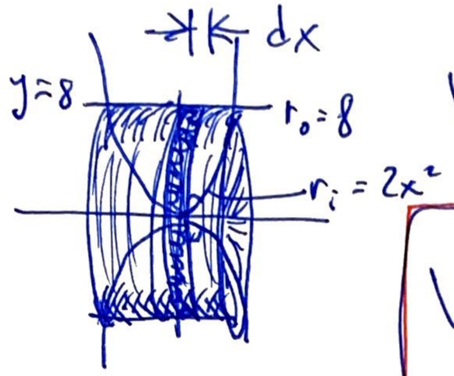
$$= \frac{5}{3} + \frac{22}{3} = \boxed{\frac{27}{3}} = \boxed{9}$$

:)

2. [5.2 Disks] (10 pts) Use the method of disks to determine the volume of the solid obtained by rotating the region bounded by the given curves about the given axis:

$y = 2x^2$, $y = 8$ and the y-axis about the x-axis.

$8 = 2x^2$
$\Rightarrow x = \pm 2$



$$V = \int \left[\pi r_o^2 - \pi r_i^2\right] dx$$

$$\boxed{V = \pi \int_{x=-2}^{2} \left[8^2 - (2x^2)^2\right] dx}$$

10

$$= \pi \left\{ 64x \Big|_{-2}^{2} - \frac{4x^5}{5} \Big|_{-2}^{2} \right\}$$

use symmetry

$$= 2\pi \left\{ 64x \Big|_0^2 - \frac{4}{5} x^5 \Big|_0^2 \right\}$$

$$= 2\pi \left\{ 128 - \frac{4 \cdot 32}{5} \right\}$$

$$= 2\pi \; \frac{640 - 128}{5}$$

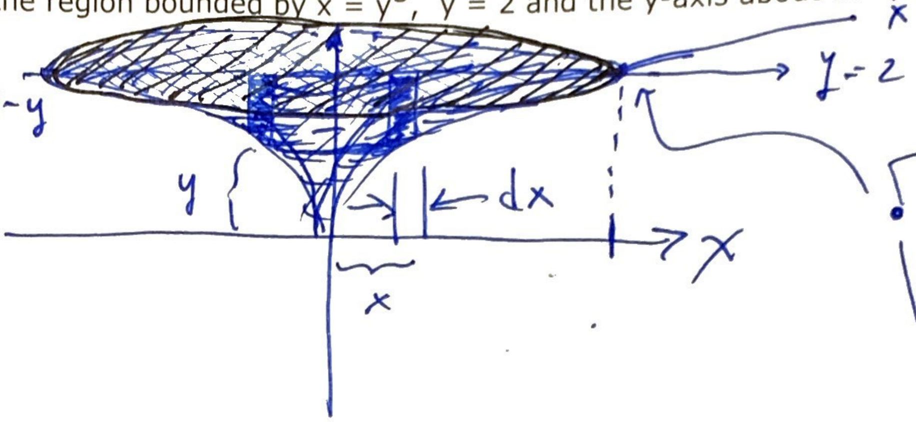
$$= \frac{2}{5} \pi \, 512$$

$$= \boxed{\frac{1024}{5} \pi}$$

3. [5.3 Shells] (10 pts) Use the method of cylinders to determine the volume of the solid obtained by rotating the region bounded by the given curves about the given axis:

Rotate the region bounded by $x = y^3$, $y = 2$ and the y-axis about the y-axis

$H = 2 - y$



intersection
$y = 2$ so
$x = 2^3 = 8$

$dV = C \cdot H \cdot dr$, $r = x$, $dr = dx$

$H(x) = 2 - y(x)$

$C = 2\pi x$

$dV = 2\pi x \cdot (2 - x^{1/3})\,dx$

$$\boxed{V = 2\pi \int_{x=0}^{8} x(2 - x^{1/3})\,dx}$$

10

$$= 2\pi \int_0^8 2x\,dx - 2\pi \int_0^8 x^{4/3}\,dx$$

$$= 2\pi \left[\frac{\cancel{2}x^2}{\cancel{2}}\right]_0^8 - 2\pi \left[\frac{3}{7}x^{7/3}\right]_0^8$$

$$= 2\pi \cdot 64 - 2\pi \cdot \frac{3}{7}\left[(\sqrt[3]{8})^7\right]$$

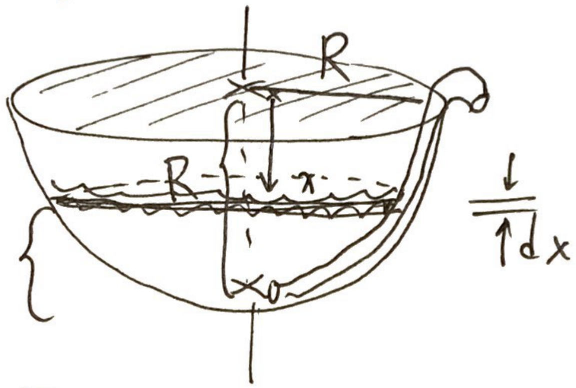
$$= 128\pi - \frac{6}{7}\pi \cdot 128 = 128\pi\left[1 - \frac{6}{7}\right] = \boxed{\frac{128}{7}\pi}$$

4. [5.4 Work] (10 pts) **[Set-Up Only!]** A tank is the shape of the lower half of a sphere of radius R = 6 meters. If the initial depth of the water is 4 meters how much work is required to pump all the water out via the rim of the tank. Assume that the density of water is 1000 kg/m³. Remember to calculate the work to lift each slab of water, at depth x below the rim, up to the rim (the tank's equator).

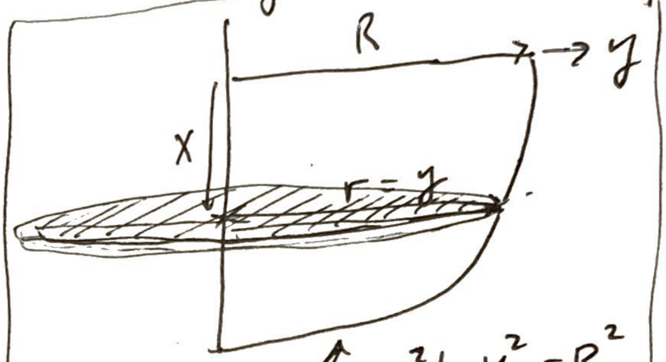
$$dW = x * dF_g, \quad dF_g = g * dM, \quad dM = \rho * dV, \quad dV = A * dx$$

Labeled diagram:

10



geometry



$x^2 + y^2 = R^2$

$\boxed{r = y = \sqrt{R^2 - x^2}}$

$$A(x) = \pi r^2 = \pi\left[\sqrt{R^2 - x^2}\right]^2$$

$$\boxed{A(x) = \pi\left[R^2 - x^2\right]}$$

$A(R) = 0$ ✓, $A(0) = \pi R^2$ ✓

$$dW = x\,dF_g = x\,g\,dM = x\,g\,\rho\,dV = x\,g\,\rho\,A\,dx$$

$$dW = g\rho\, x\,\pi\left[R^2 - x^2\right]dx$$

$$\boxed{W = \pi\rho g \int_{x=4}^{R} x\left[R^2 - x^2\right]dx}$$

$$\boxed{W = \pi\left(1000\,\frac{kg}{m^3}\right)\left(9.8\,\frac{m}{s^2}\right)\int_4^6 x\left[36 - x^2\right]dx}$$

5. [6.1/2 Exponentials] (10 pts) Differentiate the given function:

(a) $h(t) = 6^t - 4e^{3t}$

$= 6^t \ln(6) - 4e^{3t} \cdot \frac{d\,3t}{dt}$

5 $\boxed{h' = 6^t \ln(6) - 12e^{3t}}$

forget the formula?
• write $6^t = e^{\ln(6^t)}$
$= e^{t\ln(6)}$
Now diff't

(b) Find the tangent line to $f(x) = (1-8x)e^x$ at $x = -1$. $f(-1) = 9/e$

$f' = -8 \cdot e^x + (1-8x)e^x$ product rule

simplify $\boxed{f' = -8e^x x - 7e^x}$ @ $x=-1$: $m = -8 e^{-1}(-1) - 7e^{-1}$

$m = e^{-1} = \boxed{1/e}$

5 form: $y = \frac{1}{e}x + b$

point: $\frac{9}{e} = \frac{1}{e} \cdot (-1) + b$

solve: $\frac{10}{e} = b$

final: $\boxed{y(x) = \frac{1}{e}x + \frac{10}{e}}$

6. [6.3 Logs] (10 pts) Differentiate the given function:

$$U(z) = \log_4(z) - z^6 \ln(z)$$

change of base

$$\boxed{U = \frac{\ln(z)}{\ln(4)} - z^6 \ln(z)}$$

so

$$U' = \frac{1}{\ln(4)} \cdot \frac{1}{z} - \left(6z^5 \ln(z) + z^6 \cdot \frac{1}{z}\right)$$

$$U' = \frac{1}{z \ln(4)} - 6z^5 \ln(z) - z^5$$

or

$$\boxed{U' = -z^5 - 6z^5 \ln(z) + \frac{1}{z \ln(4)}}$$

7. [6.4 Derivatives Logs] (10 pts) Use logarithmic differentiation to find R' if

$$R(t) = [\sin(4t)]^{6t}$$

$$\ln R = 6t \ln(\sin(4t))$$

$$\Big(\qquad \Big)'$$

$$\frac{1}{R} R' = 6 \ln(\sin(4t)) + 6t \frac{1}{\sin(4t)} \frac{d \sin(4t)}{dt}$$

$$R' = R \left\{ 6 \ln(\sin(4t)) + \frac{6t}{\sin(4t)} \cdot \cos(4t) \cdot \frac{d4t}{dt} \right\}$$

$$\boxed{R' = [\sin(4t)]^{6t} \left\{ \ln(\sin^6(4t)) + \frac{24t\cos(4t)}{\sin(4t)} \right\}}$$