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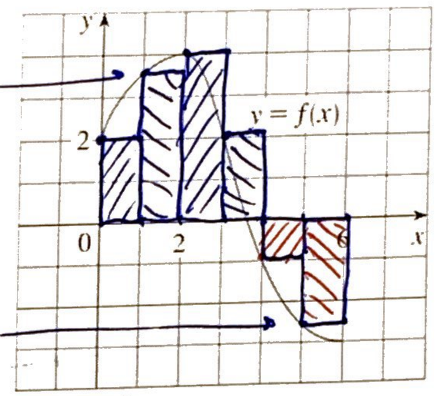
1. (10pts) [4.1 Riemann Sum] Use the graph of *f* to find the Riemann sum with six subintervals. Take the sample points to be Left endpoints. Draw the rectangles on the diagram.




R. Sum (LHS)

$= (2)(1) + (3.5)(1) + (4)(1) + (2)(1)$
$+ (-1)(1) + (-2.5)(1)$

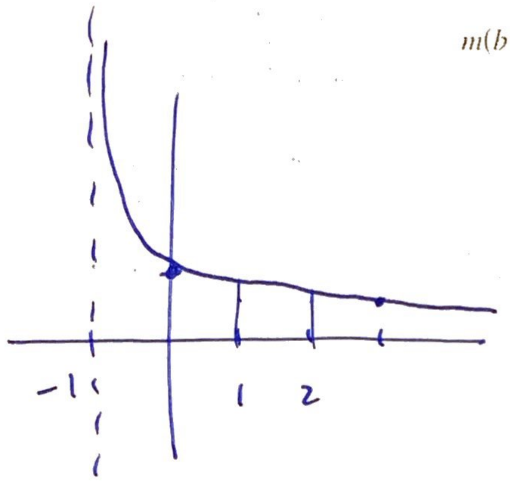
$= 11.5 - 3.5$

10 $= \boxed{8.0}$

2. (10 pts) [4.2 Def Integrals] Use Property 8 to estimate the value of $\int_1^2 \left(\frac{1}{\sqrt{1+x}}\right)dx$

**8.** If $m \le f(x) \le M$ for $a \le x \le b$, then

$$m(b-a) \le \int_a^b f(x)\,dx \le M(b-a)$$



$$f(x) = \frac{1}{\sqrt{1+x}}$$

$$\text{max}: \quad f(1) = \frac{1}{\sqrt{1+1}} = \frac{1}{\sqrt{2}}$$

$$\text{min}: \quad f(2) = \frac{1}{\sqrt{1+2}} = \frac{1}{\sqrt{3}}$$

$$b - a = 2 - 1 = 1$$

Prop 8:

$$\frac{1}{\sqrt{3}}(2-1) \le \int_1^2 \frac{dx}{\sqrt{1+x}} \le \frac{1}{\sqrt{2}}(2-1)$$

$$\boxed{\frac{1}{\sqrt{3}} \le \int_1^2 \frac{dx}{\sqrt{1+x}} \le \frac{1}{\sqrt{2}}}$$

7

$$0.5774 \le I \le 0.7071$$

- So average the two since the curve is pretty straight

$$\frac{0.5774 + 0.7071}{2} = \boxed{0.6422}$$

3

BTW: Wolfram Alpha: Exact $= 2\sqrt{3} - 2\sqrt{2} = 0.63567$

3. (5 pts) [4.3 Fundamental Thm] Use Part 1 of the Fundamental Theorem of Calculus to find the *derivative* of

$$g(r) = \int_0^r \sqrt{x^2+4}\, dx$$

$$\frac{d \int_0^r \sqrt{x^2+4}\, dx}{dr} = \boxed{\sqrt{r^2+4}}$$

5

4. (10pts) [4.4 Indef Integrals] Evaluate the following integrals:

(a) $\int (x^3 - 12x^2 + 1) dx$

$$= \frac{x^4}{4} - 12\frac{x^3}{3} + x + C$$

$$= \boxed{\frac{x^4}{4} - 4x^3 + x + C}$$

5

(b) $\int (\frac{1}{x^2} - \frac{1}{4\sqrt{x^7}}) dx$

$$= \int x^{-2} dx - \frac{1}{4}\int x^{-7/2} dx$$

$$= \frac{x^{-2+1}}{-2+1} - \frac{1}{4}\frac{x^{-7/2+1}}{-7/2+1} + C$$

5

$$= -\frac{1}{x} - \frac{1}{4}\frac{x^{-5/2}}{-5/2} + C$$

$$= -\frac{1}{x} + \frac{4 \cdot 2}{5}\frac{1}{x^{5/2}} + C$$

$$= \boxed{-\frac{1}{x} + \frac{8}{5}\frac{1}{(\sqrt{x})^5} + C}$$

$\boxed{5.}$ (5pts) [4.5 Substitution] Solve by using an appropriate substitution:

$$\int 7z^2\left(14+8z^3\right)^{-5}dz$$

let $u = 14 + 8z^3$

$du = 0 + 8 \cdot 3z^2 dz$

or

$du = 24z^2 dz \rightarrow \boxed{dz = \frac{du}{24z^2}}$

So...

$$I = \int 7\cancel{z^2}(u)^{-5}\left(\frac{du}{24\cancel{z^2}}\right)$$

5

$$= \boxed{\frac{7}{24}\int u^{-5}du}$$

$$= \frac{7}{24}\frac{u^{-5+1}}{-5+1} + C$$

$$= \frac{7}{24}\frac{u^{-4}}{-4} + C$$

$$= -\frac{7}{96}\frac{1}{u^4} + C$$

+3

$$= \boxed{-\frac{7}{96}\frac{1}{(14+8z^3)^4} + C}$$

6. (5pts) [4.5 Substitution] Solve by using an appropriate substitution:

$$\int_1^6 7\cos\left(\frac{\pi z}{2}\right)\left(4+\sin\left(\frac{\pi z}{2}\right)\right)^5 dz$$

- let $u = 4 + \sin\left(\frac{\pi}{2} z\right)$

$$du = 0 + \cos\left(\frac{\pi}{2} z\right)\left(\frac{\pi}{2}\right) dz \Rightarrow \boxed{dz = \frac{2\,du}{\pi\cos\left(\frac{\pi z}{2}\right)}}$$

- Limits:

$u(1) = 4 + \sin\left(\frac{\pi}{2}\cdot 1\right) = 4 + 1 = 5$

$u(6) = 4 + \sin\left(\frac{\pi}{2}\cdot 6\right) = 4 + 0 = 4$

- $$\int_5^4 7\cancel{\cos\left(\frac{\pi z}{2}\right)}(u)^5\left(\frac{2\,du}{\pi\cancel{\cos\left(\frac{\pi z}{2}\right)}}\right)$$

5

$$= \boxed{\frac{14}{\pi}\int_5^4 u^5\,du}$$

$$= \frac{14}{\pi}\frac{u^6}{6}\Bigg|_5^4$$

$$= \frac{14}{6\pi}\left[\frac{4^6}{6} - \frac{5^6}{6}\right]$$

+3

$$= \frac{14}{36\pi}\left[4096 - 15625\right]$$

$$= -\frac{7}{18}\pi\,[11529]$$

$2 \quad \cancel{3\cdot 3}\cdot 3\cdot 427$

$$= -\frac{7\pi\ 3\cdot 427}{2}$$

$$= \boxed{\frac{8967\pi}{2}}$$

7. (10 pts) [4.5 Substitution] Solve by using an appropriate substitution including substitution of the limits:

$$\int_3^5 \frac{4t}{2-8t^2}\,dt$$

$u = 2 - 8t^2$

$du = -8 \cdot 2t\,dt$

$\boxed{dt = \frac{du}{-16t}}$

4

- limits

$u(3) = 2 - 8(3)^2 = 2 - 72 = -70$

$u(5) = 2 - 8(5)^2 = 2 - 200 = -198$

- Integral

$$\int_{u=-70}^{-198} \frac{4\cancel{t}}{u}\left(\frac{du}{-16\cancel{t}}\right) = \boxed{-\frac{1}{4}\int_{-70}^{-198} \frac{du}{u}}$$

$$= -\frac{1}{4} \ln|u| \Big|_{-70}^{-198}$$

6

$$= -\frac{1}{4}\left[\ln|-198| - \ln|-70|\right]$$

$$= -\frac{1}{4}\ln\left(\frac{198}{70}\right)$$

$$= \boxed{\frac{1}{4}\ln\left(\frac{35}{99}\right)} = \ln\sqrt[4]{\left(\frac{35}{99}\right)}, \quad = \ln\left(\sqrt{\sqrt{\frac{35}{99}}}\right)$$

in case your calc does not have $\boxed{y^x}$

but $\sqrt[4]{\square} = \sqrt{\sqrt{\square}}$

8. (10pts) [4.5 Substitution] Solve by using an appropriate substitution including substitution of the limits:

$$\int_{-\pi}^{\frac{\pi}{2}} \underbrace{\cos(x)\,dx}_{du} \underbrace{\cos(\sin(x))}_{\cos(u)}$$

$u = \sin(x)$

3 $\boxed{du = \cos(x)\,dx}$

4 $\boxed{I = \int_{x=-\pi}^{\pi/2} \cos(u)\,du}$

$= \sin(u) \Big|_{x=-\pi}^{\pi/2}$

3 $= \sin[\sin(x)] \Big|_{x=-\pi}^{\pi/2}$

$= \sin[\sin(\frac{\pi}{2})] - \sin[\sin(-\pi)]$

$= \boxed{\sin(1)}$