Math 211 Su24 CoC R Erickson $\frac{80}{80}$ TEST 3: {ch 3} Name KEY

Show all work for FULL credit. No Calculators needed on this test.

1. **(10pts)** Find the absolute maximum and minimum values of the function

$$f(x) = x^3 - 3x^2 + 1 \qquad -\tfrac{1}{2} \leqslant x \leqslant 4$$

- $f'(x) = 3x^2 - 3\cdot 2x$

1 $\quad = 3x(x-2)$

- set $f'(x)=0$ for critical (candidate) points

2 $\quad f' = 0$ if $x=0$ or $x=2$

- Test $f''(x) = 6x - 6$
  - $\boxed{f''(0) = -6} \rightarrow$ max @ $x=0$

2
  - $f''(2) = 6(2)-6 = 12-6 = \boxed{6} \rightarrow$ min @ $x=2$

- Values:

1 $\quad f(0) = 0^3 - 3\cdot 0^2 + 1 = 1$

$\quad f(2) = 2^3 - 3\cdot 2^2 + 1 = 8 - 12 + 1 = -3$

- endpoints:

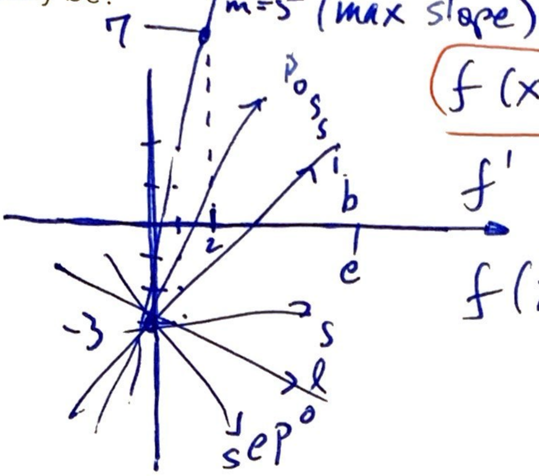
2 $\quad f(-\tfrac{1}{2}) = (-\tfrac{1}{2})^3 - 3(-\tfrac{1}{2})^2 + 1 = -\tfrac{1}{8} - \tfrac{3}{4} + \tfrac{8}{8} = \tfrac{1}{8}$

$\quad f(4) = (4)^3 - 3\cdot(4)^2 + 1 = 64 - 48 + 1 = 17$

- Absolute extrema

2 $\quad \boxed{\text{max} = 17 \text{ @ } x=4} \quad \boxed{\text{min} = -3 \text{ @ } x=2}$

2. **(10 pts)** Suppose that $f(0) = -3$ and $f\,'(x) \leq 5$ for all values of x. How large can $f\,(2)$ possibly be?



5

$$\boxed{f(x) = 5x - 3}$$

$$f' = 5$$

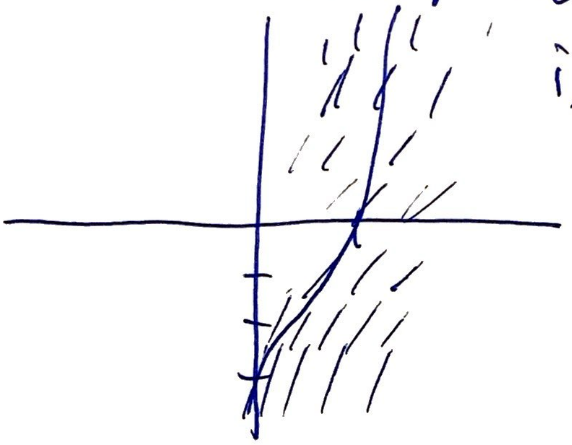
$$f(x=2) = 5 \cdot 2 - 3$$

$$= 10 - 3$$

$$= \boxed{7}$$

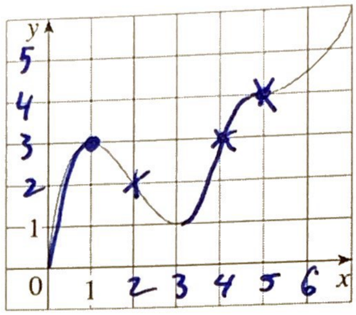
5

Consider a slope-field ... the highest trajectory is a straight line of slope 5



3. **(10pts)** Answer the following by observation:

(a) The open intervals on which f is increasing.
(b) The open intervals on which f is decreasing.
(c) The open intervals on which f is concave upward.
(d) The open intervals on which f is concave downward.
(e) The coordinates of the points of inflection.



2 (a) (0, 1), (3,5), (5, 7) ↑

2 (b) (1,3), ↓

2 (c) (2,4), (5,7) c.c. up.

2 (d) (0, 2), (4,5) c.c. down

2 (e) pt(2,2), pt(4, 3), pt(5, 4) I. Pts.

4. (10pts) Find the horizontal and vertical asymptotes of the graph of the function

$$f(x) = \frac{\sqrt{2x^2+1}}{3x-5}$$

Hint: For HA(s) divide both numerator and denominator by x

(i) HA:

$$f(x) = \frac{\sqrt{2x^2+1}\ (1/x)}{3x-5\ (1/x)}$$

$$= \frac{\sqrt{\frac{2x^2}{x^2}+\frac{1}{x^2}}}{\frac{3x}{x}+\frac{5}{x}}$$

7

$$= \lim_{x\to\pm\infty} \frac{\sqrt{2+\frac{1}{x^2}}}{3+\frac{5}{x}} \xrightarrow{x\to+\infty} \boxed{\frac{\sqrt{2}}{3}}$$

$$\xrightarrow{x\to-\infty} \boxed{\frac{\sqrt{2}}{-3}}$$

Denom

(ii) VA: $3x-5=0 \rightarrow \boxed{x=\frac{5}{3}}$

3

5. **(20pts)** Sketch the graph of $f(x) = \dfrac{x^3}{x^2+1}$

1 A: Domain: $\boxed{(-\infty, \infty)}$

1 B: Intercepts: $f(0) = \frac{0}{0+1} = 0 \rightarrow \boxed{(0,0)}$

$f(x) = \frac{x^3}{x^2+1} = 0 \Rightarrow x = 0 \rightarrow \boxed{(0,0)}$ ditto

1 C: Symmetry:

$\bullet\ f(-x) = \dfrac{(-x)^3}{(-x)^2+1} = -\dfrac{x^3}{x^2+1}$ $\boxed{\text{odd}}$

D1. Address HA: ~~Long~~ Divide: deg Top > degree Bot (b/c)

1 D2. Address VA: not ÷ by 0 so $\boxed{\text{No VA}}$

D3. Find the Slant Asymptote {Long Divide}:

$$\begin{array}{r} x \\ x^2+0x+1 \overline{) x^3+0x^2+0x+0} \\ -(x^3+0x^2+1x) \\ \hline 0 \quad 0 \quad -1x \end{array}$$

so

2 $$\boxed{\frac{x^3}{x^2+1} = x - \frac{x}{x^2+1}}$$

slant asymptote $\boxed{y = x}$

Math 211 Su24 CoC R Erickson TEST 3: {ch 3} Name KEY

E. Find the Critical Points using $f'(x)$ $f = \dfrac{x^3}{x^2+1}$

$$f' = \frac{3x^2(x^2+1) - x^3(2x)}{(x^2+1)^2}$$

2

$$= \frac{x^2[3x^2+3 \; -2x^2]}{(x^2+1)^2}$$

$$= \boxed{\frac{(x^2)(x^2+3)}{(x^2+1)^2}}$$

$f' = 0$ only @ $\boxed{x=0}$

F. What are the Increasing and Decreasing regions (and Extrema) based on $f'(x)$:

2

$f' = 0$ @ $x = 0$, but is otherwise $f' > 0$

Inc: $(-\infty, 0), (0, \infty)$, Decreasing No where.

G. Find the Concavity and Inflection Points $f''(x) \leq, \geq, = 0$ and summarize in a table on the next page:

$$f' = \frac{x^4 + 3x^2}{(x^2+1)^2}$$

$$f'' = \frac{(4x^3+6x)(x^2+1)^2 - (x^4+3x^2)(2(x^2+1)2x)}{(\;)^4}$$

3

$$= 2(x^2+1)\left[(2x^3+3x)(x^2+1) - (x^4+3x^2)(2x)\right] / (\;)^4$$

$$= 2(x^2+1)\left[\cancel{2x^5} + 3x^3 + 2x^3 + 3x - \cancel{2x^5} - 6x^3\right] / (\;)^4$$

$$= 2(x^2+1)\left[-x^3 + 3x\right] / (\;)^4$$

$$= -2x(x^2-3) / (\;)^3$$

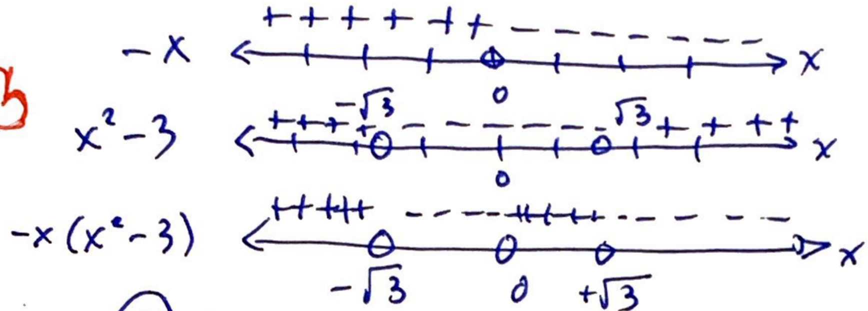
$$= \boxed{-\frac{2x(x^2-3)}{(x^2+1)^3}}$$

see next pg.

(G) (Cont): Table of Concavity

$$f'' = \frac{-2x(x^2-3)}{(x^2+1)^3}$$

3



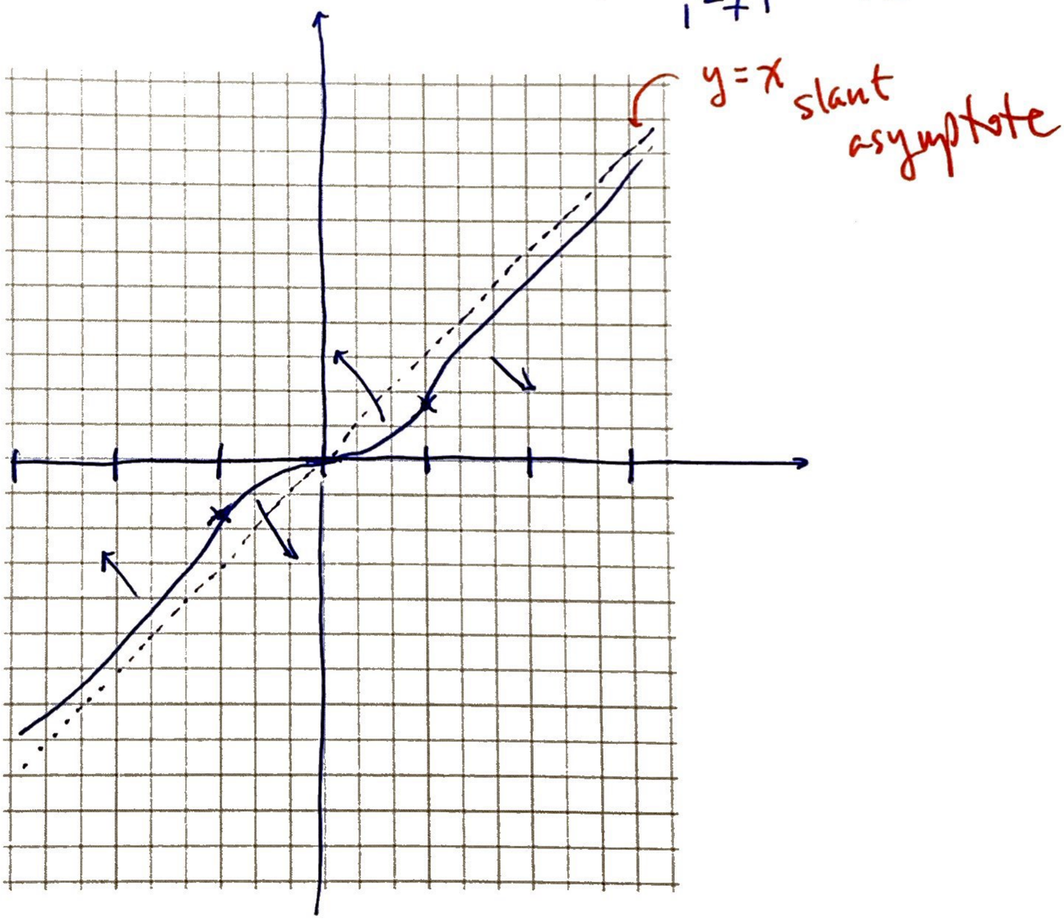
$(-\infty, -\sqrt{3})$ c.c. up
$(-\sqrt{3}, 0)$ c.c. down
$(0, \sqrt{3})$ c.c. up
$(\sqrt{3}, \infty)$ c.c. down

(H) Sketch using the knowledge derived from steps A - G

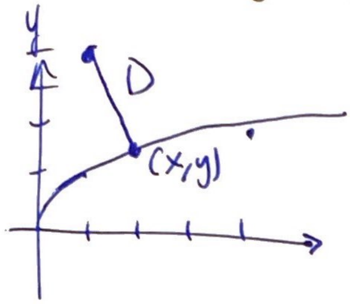
$$f(1) = \frac{1^3}{1^2+1} = 1/2$$

$$f(-1) = \frac{(-1)^3}{1^2+1} = -1/2$$

4



6. (10pts) Find the point on the parabola $y^2 = 2x$ that is closest to the point (1, 4).
{ Hint: Things are easier if you minimize d² vs d }



$D^2 = (x-1)^2 + (y-4)^2$

To avoid radicals let $x = \frac{y^2}{2}$

$$\boxed{\text{Then } D^2 = \left(\frac{y^2}{2} - 1\right)^2 + (y-4)^2}$$

3

4 $\left\{ \begin{array}{l} \frac{dD^2}{dy} = 2\left(\frac{y^2}{2} - 1\right)\left(\frac{2y}{2}\right) + 2(y-4) \\ 0 = (y^2 - 2)y + (2y - 8) \\ \boxed{0 = y^3 - 8} \longrightarrow \boxed{y = 2} \text{ then } 2^2 = 2(x) \Rightarrow \boxed{x = 2} \end{array} \right.$

3 The closest point to (1,4) is $\boxed{(x,y) = (2,2)}$

$\boxed{\text{Alt}}$ $D^2 = (x-1)^2 + (\sqrt{2x} - 4)^2$, $(x^{1/2})' = \frac{1}{2}x^{-1/2}$

$$\frac{dD^2}{dx} = \cancel{2}(x-1) + \cancel{2}(\sqrt{2}\sqrt{x} - 4)\left(\frac{\sqrt{2}}{2}\frac{1}{\sqrt{x}}\right)$$

$$0 = x - 1 + \frac{\cancel{\sqrt{2}}\cancel{\sqrt{x}}\cancel{\sqrt{2}}}{\cancel{2}\cancel{\sqrt{x}}} - \frac{4\sqrt{2}}{2}\frac{1}{\sqrt{x}}$$

$$0 = x - \cancel{1} + \cancel{1} - 2\sqrt{2}/\sqrt{x}$$

$$0 = x - 2\sqrt{2}/\sqrt{x}$$

$$0 = \frac{x^{3/2} - 2\sqrt{2}}{\sqrt{x}}$$

$$x^{3/2} = 2\sqrt{2}$$

$$x^3 = 2^2\sqrt{2}^2$$

$$x^3 = 4 \cdot 2$$

$$x^3 = 8 \longrightarrow x = 2 \text{ then } y^2 = 2 \cdot 2 \rightarrow y = 2$$

$$\boxed{(x,y) = (2,2)}$$

$\boxed{7.}$ **(10pts)** Starting with $x_1 = 2$, find the third approximation x_3 to the root of the equation $x^3 - 2x - 5 = 0$ using Newtons method:

let $\begin{cases} f(x) = x^3 - 2x - 5 \\ f' = 3x^2 - 2 \end{cases}$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= \boxed{2 - \frac{(2)^3 - 2(2) - 5}{3(2)^2 - 2}}$$

5 $$= 2 - \frac{8-4-5}{12-2}$$

$$= 2 - \frac{-1}{10}$$

$$\boxed{x_2 = 2.1}$$

5 $$x_3 = \boxed{2.1 - \frac{(2.1)^3 - 2(2.1) - 5}{3(2.1)^2 - 2}} = 2.1 - \frac{9.261 - 4.2 - 5}{11.23}$$

$$x_3 = 2.1 - \frac{0.061}{11.23} = \boxed{2.094568}$$