Math 211 Su24 CoC R Erickson $\frac{80}{80}$ TEST 2: {ch 2.2 - 2.9} Name KEY

Show all work for FULL credit. No Calculators needed on this test.

1. (10pts) Find $f'(x)$ from first principles, that is, directly from the definition of a derivative: $f(x) = \dfrac{4-x}{3+x}$

$$f'(x) = \lim_{h\to 0} \frac{\left(\frac{4-(x+h)}{3+(x+h)} - \frac{4-x}{3+x}\right)}{h}$$

$$= \lim_{h\to 0}\left(\frac{(4-x-h)(3+x)-(4-x)(3+x+h)}{(3+x+h)(3+x)h}\right)$$ don't expand denom

$$= \lim_{h\to 0}\left(\frac{(\cancel{12}-\cancel{3x}-3h+\cancel{4x}-\cancel{x^2}-\cancel{hx})-(\cancel{12}-\cancel{3x}+\cancel{4x}-\cancel{x^2}+4h-\cancel{xh})}{(3+x+h)(3+x)}\right)$$

10

$$= \lim_{h\to 0}\left(\frac{-3\cancel{h}-4\cancel{h}}{(3+x+h)(3+x)\cancel{h}}\right)$$

$$= \lim_{h\to 0}\left(\frac{-7}{(3+x+\cancel{h}^{\,0})(3+x)}\right)$$

$$= \boxed{-\frac{7}{(3+x)^2}}$$

Test $$f' = \frac{(4-x)'(3+x)-(4-x)(3+x)'}{(3+x)^2}$$

$$= \frac{-(3+x)-(4-x)(1)}{(3+x)^2}$$

$$= \frac{-7-x+x}{(3+x)^2}$$

$$= \frac{-7}{(3+x)^2}$$ ✓

2. **(5 pts)** Calculate y' using the quotient rule: $y = \dfrac{x^2 - x + 2}{\sqrt{x}}$

$$y' = \frac{(x^2-x+2)'(\sqrt{x}) - (x^2-x+2)(\sqrt{x})'}{(\sqrt{x})^2}$$

5

$$= \frac{(2x-1)\sqrt{x} - (x^2-x+2)\frac{1}{2}x^{-1/2}}{x}$$

$$= \boxed{\frac{(2x-1)\sqrt{x}}{x} - \frac{(x^2-x+2)}{x\,2\sqrt{x}}}$$

$$\frac{2\sqrt{x}}{2\sqrt{x}}$$

$$= \frac{2(2x-1)x - x^2+x-2}{2x^{3/2}} = \frac{4x^2-2x-x^2+x-2}{2(\sqrt{x})^3}$$

$$\boxed{\frac{3x^2 - x - 2}{2(\sqrt{x})^3}}$$

3. **(5pts)** Calculate y' using the chain rule: $y = tan^2(sin\theta)$

$$\frac{d[\tan(\sin\theta)]^2}{d\theta}$$

5

$$= 2\tan(\sin\theta)\frac{d\tan(\sin\theta)}{d\theta}$$

$$= 2\tan(\sin\theta)\cdot\sec^2(\sin\theta)\frac{d\sin\theta}{d\theta}$$

$$= \boxed{2\tan(\sin\theta)\cdot\sec^2(\sin\theta)\cdot\cos\theta}$$

4. **(5pts)** Find y' using implicit differentiating as taught in class: $\sin(xy) = x^2 - y$

$$\frac{d}{dx}(\sin(xy)) = \frac{d(x^2-y)}{dx}$$

$$\cos(xy)\,(xy)' = 2x - y'$$

5

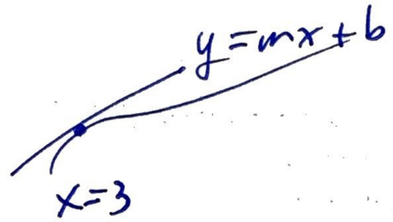
$$\Rightarrow \cos(xy)(x'y + xy') = 2x - y'$$

$$\Rightarrow (y + xy')\cdot\cos(xy) + y' = 2x$$

$$y'[x\cos(xy) + 1] = 2x - y\cos(xy)$$

$$\boxed{y' = \frac{2x - y\cos(xy)}{1 + x\cos(xy)}}$$

5. **(5pts)** Find the linear approximation to $f(x) = \sqrt{25 - x^2}$ near x = 3



5

$$f(3) = \sqrt{25 - 3^2} = \sqrt{16} = 4$$

$$f' = \frac{1}{2}(25 - x^2)^{-1/2}\cdot(-2x)$$

$$\boxed{f'(x) = \frac{-x}{\sqrt{25 - x^2}}}$$

$$f'(3) = \frac{-3}{\sqrt{25-9}} = \frac{-3}{4}$$

form $y = \frac{-3}{4}x + b$

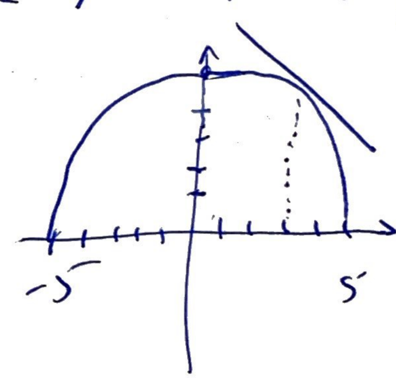
point $4 = \frac{-3}{4}\cdot 3 + b$

solve $4 + \frac{9}{4} = b \rightarrow b = \frac{16+9}{4} = \frac{25}{4}$

find $\boxed{y = \frac{-3}{4}x + \frac{25}{4}}$

So $\boxed{f(x) \approx \frac{-3}{4}x + \frac{25}{4}}$

near $x = 3$ only.



6. **(10pts)** Find y' if $y = \sin^2\left[\cos\sqrt{\sin \pi x}\right]$ Show all reasonable steps.

(i) $\dfrac{d\{\sin[\quad]\}^2}{dx} = 2\{\sin[\quad]\}^{2-1} \cdot \dfrac{d\sin[\,]}{dx}$

(ii) $\dfrac{d\sin(\cos\sqrt{\quad})}{dx} = \cos(\cos\sqrt{\quad}) \cdot \dfrac{d\cos\sqrt{\quad}}{dx}$

(iii) $\dfrac{d\cos\sqrt{\quad}}{dx} = -\sin\sqrt{\quad} \cdot \dfrac{d\sqrt{\quad}}{dx}$

(iv) $\dfrac{d\sqrt{\sin(\pi x)}}{dx} = \dfrac{1}{2\sqrt{\quad}} \dfrac{d\sin(\pi x)}{dx}$ $\cos(\pi x)\cdot(\pi x)'$

So $y' = \cancel{2}\sin\left[\cos(\sqrt{\sin(\pi x)})\right] \cdot \cos\left(\cos\sqrt{\sin \pi x}\right) \cdot -\sin\sqrt{\sin(\pi x)} \cdot \dfrac{\cos(\pi x)\pi}{\cancel{2}\sqrt{\sin \pi x}}$

$$\boxed{y' = -\pi\,\frac{\sin\left[\cos\left(\sqrt{\sin(\pi x)}\right)\right]\cos\left[\cos\sqrt{\sin(\pi x)}\right]\sin\sqrt{\sin(\pi x)}\,\cos(\pi x)}{\sqrt{\sin(\pi x)}}}$$

10

$\boxed{7.}$ **(10pts)** The position of an object at any time t is given by $s(t) = 3t^2 - 44t + 20$.

(a) Determine the velocity of the object at any time t.

$$v = \frac{ds}{dt} = 2 \cdot 3t - 44$$

$$\boxed{v(t) = 6t - 44}$$

3

(b) Does the ~~position of the~~ object ever stop moving?

$$v(t) = 0 \Rightarrow 6 \cdot t - 44 = 0 \Rightarrow t = \frac{44}{6}$$

2 $\boxed{V = 0 \quad @ \quad t = 7\tfrac{1}{3} \text{ sec.}}$

(c) When is the object moving to the left?

$$v < 0 \Rightarrow 6t - 44 < 0$$
$$6t < 44$$
$$t < \frac{44}{6}$$

$$\boxed{t < 7\tfrac{1}{3} \text{ sec}}$$

2

(d) What is the acceleration of the object

$$a(t) = \frac{dV(t)}{dt}$$

3

$$= \frac{d(6t - 44)}{dt}$$

$$\boxed{a = 6 \text{ constant}}$$

8. **(10pts)** Find a parabola $y = ax^2 + bx + c$ that passes through the point (1, 4) and whose tangent lines at x = −1 and x = 5 have slopes of 6 and −2 respectively.

- We need 3 eqns all with a, b, and/or c

3 eqns, 3 unknowns

(i) $4 = a\cdot 1^2 + b\cdot 1 + c \rightarrow \boxed{a+b+c=4}$

- $y' = 2ax + b$

6

(ii) @ $x=-1$ $m=6 \Rightarrow 6 = 2a(-1)+b \rightarrow \boxed{2a-b=-6}$

(iii) @ $x=5$ $m=-2 \Rightarrow -2 = 2a(5)+b \rightarrow \boxed{10a+b=-2}$

- Solve eqns

$$\begin{aligned} a+b+c &= 4 \\ 2a-b &= -6 \\ 10a+b &= -2 \\ \hline 12a &= -8 \\ a &= -8/12 = \boxed{-\tfrac{2}{3}} \end{aligned}$$

$* -5 \Rightarrow$

$$\begin{aligned} -10a+5b &= 30 \\ \oplus\; 10a+b &= -2 \\ \hline 6b &= 28 \\ \boxed{b=\tfrac{14}{3}} \end{aligned}$$

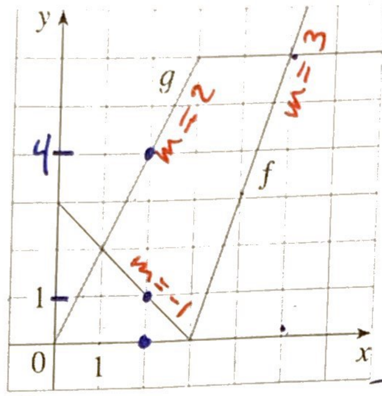
4

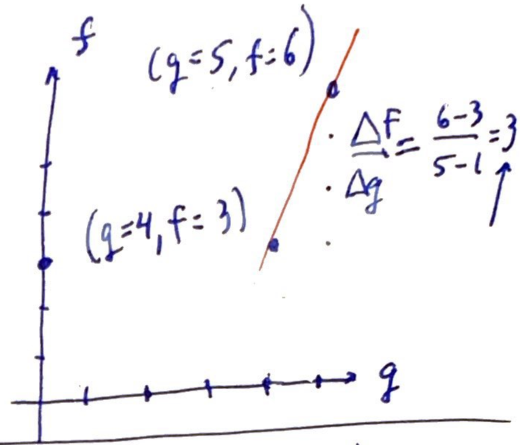
Top eqn $a+b+c=4 \rightarrow -\frac{2}{3}+\frac{14}{3}+c=4 \rightarrow \boxed{c=0}$

$(a,b,c) = \left(-\frac{2}{3}, \frac{14}{3}, 0\right)$

so $\boxed{y = -\frac{2}{3}x^2 + \frac{14}{3}x}$

$\boxed{9.}$ **(10pts)** If f and g are the functions whose graphs are shown, let $P(x) = f(x)g(x)$, $Q(x) = f(x)/g(x)$, and $C(x) = f(g(x))$. Find (a) $P'(2)$, (b) $Q'(2)$, and (c) $C'(2)$.





(a) $P'(2)$

4

$= (f \cdot g)'\Big|_{x=2}$

$= (f'g + fg')\Big|_{x=2}$

$= f'\Big|_{x=2} \cdot g\Big|_{x=2} + f\Big|_{x=2} \cdot g'\Big|_{x=2}$

$= (-1)\cdot(4) + (1)\cdot(2) = \boxed{-2}$

(b) $Q'(2)$

4

$= \left(\dfrac{f}{g}\right)'\Big|_{x=2}$

$= \dfrac{f'g - fg'}{g^2}\Big|_{x=2}$

$= \dfrac{(-1)(4) - (1)(2)}{4^2}$

$= \dfrac{-4-2}{16}$

$= \boxed{-3/8}$

(c) $C'(2) =$

2

$= (f(g))'$

$= \dfrac{df}{dg} \cdot \dfrac{dg}{dx}$

$= \dfrac{df}{dg}\Big|_{g(2)=4} \cdot \dfrac{dg}{dx}\Big|_{x=2}$

$= f'(g(x))\Big|_{x=2} \cdot 2$

$= f'(g(2)) \cdot 2$

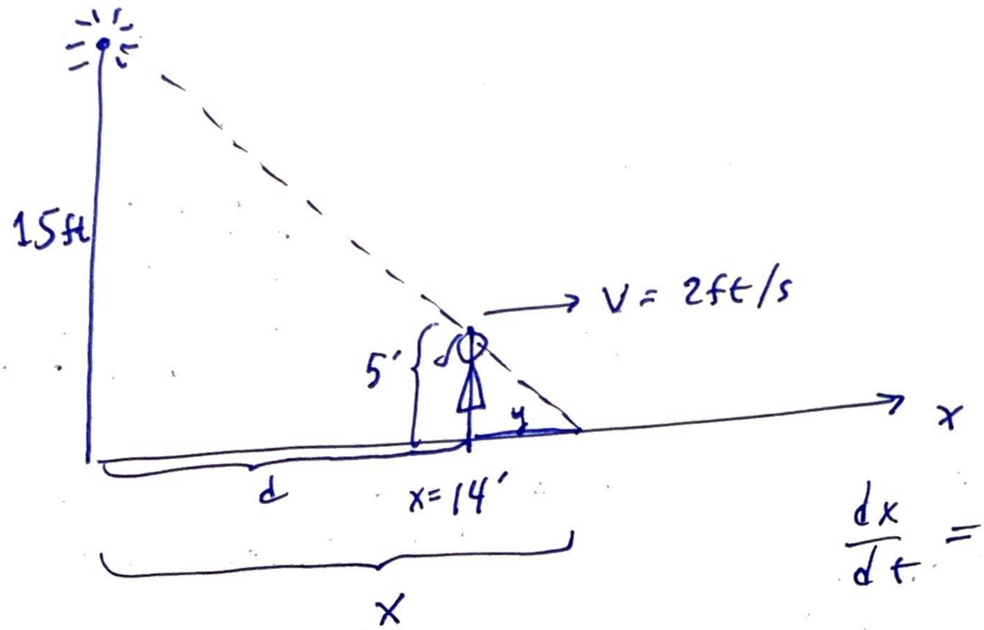
$= f'(4) \cdot 2$

$= 3 \cdot 2 = \boxed{6}$

10. **(10 pts)** A light is on the top of an 15 foot tall pole. A 5 foot tall woman starts at the pole and moves away from the pole at a rate of 2 ft/sec. After moving for 7 seconds what is the rate of the tip of her shadow moving away from the pole?

$(7s)(2\frac{ft}{s}) = 14 ft$

3



$\frac{dx}{dt} = ?$

3

• similar Δ's : 15' is to 5' as x is to y

$\frac{15}{5} = \frac{x}{y}$, but $y = x - d$

$\Rightarrow \quad 3 = \frac{x}{x-d} \quad \rightarrow \quad 3x - 3d = x$

$2x = 3d$

$\boxed{2x' = 3d'}$

$x' = \frac{3}{2} d'$

4

$= \frac{3}{2}(2 ft/s)$

$\boxed{x' = 3 ft/s}$