Math 211 Fa22 CoC R Erickson 65/65 Test 2 ch2.2-8 Name KEY

Show all work for FULL credit. NO NOTES, CALCULATORS, TEXT BOOKS NOR ANY OTHER ASSISTANCE IS PERMITTED WHILE YOU TAKE THE TEST.
All problem, or 'lettered' part therein, is 5 pts unless otherwise noted.

1. Find the derivative of the given functions

(a) $f(z) = z^{10} - 7z^5 + 2z^3 - z^2$

$$\boxed{f'(z) = 10z^9 - 35z^4 + 6z^2 - 2z}$$

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(b) $f(x) = 7\sqrt[9]{x^4} - 2\sqrt[2]{x^7} + \sqrt[3]{x^4}$

$$f = 7x^{4/9} - 2x^{7/2} + x^{4/3}$$

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$$\text{So } f' = 7\cdot\frac{4}{9}x^{-5/9} - 2\cdot\frac{7}{2}x^{5/2} + \frac{4}{3}x^{1/3}$$

$$\boxed{f' = \frac{28}{9}x^{-5/9} - 7x^{5/2} + \frac{4}{3}x^{1/3}}$$

$$\boxed{f' = \frac{28}{9}\frac{1}{\sqrt[9]{x^5}} - 7\sqrt{x^5} + \frac{4}{3}\sqrt[3]{x}}$$

(c) $h(x) = \sqrt{x}\,(1 - 9x^3)$

$$h' = (\sqrt{x})'(1-9x^3) + \sqrt{x}\,(1-9x^3)'$$

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$$\boxed{h' = \frac{1}{2\sqrt{x}}(1-9x^3) + \sqrt{x}(-27x^2)}$$

$$\boxed{h' = \frac{1-9x^3-27x^2(\sqrt{x})^2\cdot 2}{2\sqrt{x}}}$$

$$= \frac{1-9x^3-54x^3}{2\sqrt{x}}$$

$$= \boxed{\frac{1-63x^3}{2\sqrt{x}}}$$

2. The position of an object at any time t is given by $s(t) = 3t^2 - 44t + 20$.

(a) Determine the velocity of the object at any time t.

$$v = \frac{ds}{dt} = 6t - 44$$

(b) Does the position of the object ever stop moving?

instantaneous stops @ $v = 0$: $0 = 6t - 44 \Rightarrow t = \frac{44}{6} = \frac{22}{3}$

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$$\boxed{t = \frac{22}{3}}$$

(c) When is the object moving to the right and when is the object moving to the left?

(+)

$6t - 44 > 0$

$6t > 44$

$\boxed{t > \frac{22}{3} \text{ sec}}$

(−)

$6t - 44 < 0$

$6t < 44$

$\boxed{t < \frac{22}{3} \text{ sec}}$

3. Use the Quotient Rule to find the derivative of the given function.

$$V(t) = \frac{1 - 10t + t^2}{5t + 2t^3}$$

$$V' = \frac{(1 - 10t + t^2)'(5t + 2t^3) - (1 - 10t + t^2)(5t + 2t^3)'}{(5t + 2t^3)^2}$$

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$$= \frac{(-10 + 2t)(5t + 2t^3) - (1 - 10t + t^2)(5 + 6t^2)}{(5t + 2t^3)^2}$$

$$= \frac{(-50t + 10t^2 - 20t^3 + 4t^4) - [5 + 6t^2 - 50t - 60t^3 + 5t^2 + 6t^4]}{(5t + 2t^3)^2}$$

$$V' = \boxed{\frac{-2t^4 + 40t^3 - t^2 - 5}{(5t + 2t^3)^2}}$$

4. Prove that $\frac{d}{dx}(\cot(x)) = -\csc^2(x)$ using the definition of $cot(x)= \frac{\cos(t)}{\sin(t)}$

$$\frac{d\cot(x)}{dx}$$

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$$= \frac{d\left(\frac{\cos(x)}{\sin(x)}\right)}{dx}$$

$$= \frac{(\cos x)'\sin(x) - \cos(x)(\sin(x))'}{\sin^2 x}$$

$$= \frac{-\sin^2 x - \cos^2 x}{\sin^2 x}$$

$$= \frac{-1}{\sin^2 x}$$

$$= \underline{\underline{-\csc^2(x)}}$$

5. Differentiate using the chain rule

(a) $R(v) = (14v^2 - 3v)^{-2}$

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$$R' = -2(14v^2-3v)^{-3}(14v^2-3v)'$$

$$= \frac{-2}{(14v^2-3v)^3}\cdot(28v-3)$$

$$= \boxed{-\frac{56v-6}{(14v^2-3v)^3}}$$

(b) $g(z) = \cos(\sin(z) + z^2)$

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$$g' = -\sin(\sin(z)+z^2)\cdot(\sin z + z^2)'$$

$$= -\left[\sin(\sin(z)+z^2)\right]\left[(\cos z + 2z)\right]$$

$$= \boxed{-(\cos(z)+2z)\sin(\sin(z)+z^2)}$$

$$\frac{d\frac{\sin x}{\cos x}}{dx} = \frac{\sin'\cos - \sin x\cos'}{\cos^2 x} = \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \underline{\underline{\sec^2 x}}$$

(c) $f(x) = \tan^4(x) + \tan(x^4)$

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$$f' = 4\tan^3(x)\cdot(\tan x)' + \sec^2(x^4)\cdot(x^4)'$$

$$f' = 4\tan^3(x)\cdot\sec^2 x + \sec^2(x^4)\cdot 4x^3$$

$$\boxed{f' = 4\tan^3 x\cdot\sec^2 x + 4x^3\sec^2(x^4)}$$

6. Find y′ by implicit differentiation

(a) $\left(8xy + 2x^4y^{-3} = x^3\right)'$

$8(xy)' + 2(x^4y^{-3})' = 3x^2\frac{dx}{dx}$

5 $8(x'y + xy') + 2(x^4)'y^{-3} + 2x^4(y^{-3})' = 3x^2$

$8y + 8xy' + 8x^3y^{-3} + 2x^4(-3y^{-4})y' = 3x^2$

$y'[8x - 6x^4y^{-4}] = 3x^2 - 8x^3y^{-3} - 8y$

$\boxed{y' = \dfrac{3x^2 - 8x^3/y^3 - 8y}{8x - 6x^4/y^4}} \cdot \dfrac{y^4}{y^4}$

$= \dfrac{3x^2y^4 - 8x^3y - 8y^5}{8xy^4 - 6x^4}$

(b) $\sin\left(\frac{x}{y}\right) + x^3 = 2 - y^4$

$\cos\left(\frac{x}{y}\right)\cdot\left(\frac{x}{y}\right)' + 3x^2 = -4y^3y'$

5 $\cos\left(\frac{x}{y}\right)\left(\dfrac{x'y - xy'}{y^2}\right) + 3x^2 = -4y^3y'$

$\cos\left(\frac{x}{y}\right)\left(\dfrac{y - xy'}{y^2}\right) + 3x^2 = -4y^3y'$

$y\cos\left(\frac{x}{y}\right) - xy'\cos\left(\frac{x}{y}\right) + 3x^2y^2 = -4y^5y'$

$y'\left(4y^5 - x\cos\left(\frac{x}{y}\right)\right) = -3x^2y^2 - y\cos\left(\frac{x}{y}\right)$

$\boxed{y' = -\dfrac{3x^2y^2 + y\cos(x/y)}{4y^5 - x\cos\left(\frac{x}{y}\right)}}$

7. A light is on the top of a 18 foot tall pole. A 6 foot tall person starts at the pole and moves away from the pole at a rate of 2 ft/sec. After moving for 8 seconds at what rate is the tip of the shadow moving away from the pole?

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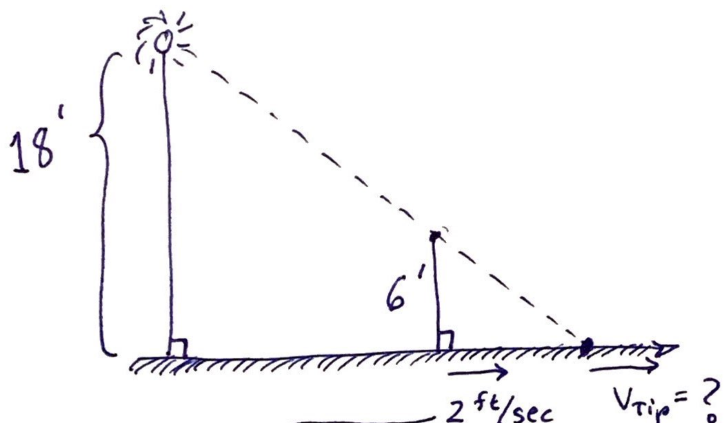
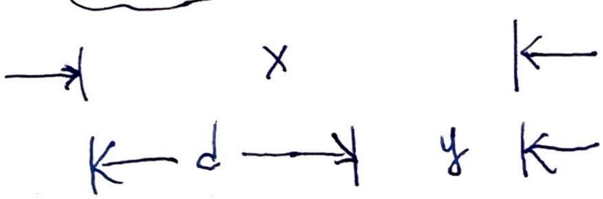


8sec = 16ft

$d = vt$
$= (2\,\tfrac{ft}{sec})\,8sec$
$d = 16\ ft$



• 18 to 6 as x to y

$\dfrac{18}{6} = \dfrac{x}{y}$

$18y = 6x$

$x = 3y$

• let d = length from pole to person

$d = x - y$

$\Rightarrow y = x - d$

So $x = 3(x - d)$

$x - 3x = -3d$

$-2x = -3d$

$\boxed{x = \frac{3}{2}d}$

16ft not needed

• $\dot{x} = \frac{3}{2}\dot{d} = \frac{3}{2}\left(2\frac{ft}{s}\right) = \boxed{3\frac{ft}{s}}$