Math 211 Su24 CoC R Erickson $\frac{75}{75}$ Test 1 1.4-2.1 Name KEY

Show all work for FULL credit.

1. **(5 pts)** For $f(x) = x^2 - x$ find the slope of the *secant line* between (1, f(1)) and (3, f(3)).

$f(1) = 1^2 - 1 = 0 \rightarrow (1, 0)$

$f(3) = 3^2 - 3 = 9 - 3 = 6 \rightarrow (3, 6)$

5

$$m = \frac{\Delta y}{\Delta x} = \frac{0-6}{1-3} = \frac{-6}{-2} = \boxed{3}$$

2. **(10 pts)** For the function $g(\theta)=\sin(3\theta)/3\theta$ complete the following table (compute accuracy to at least 6 decimal places). Make sure your calculator is set to radians for the computations.

θ	g(θ) = sin(3θ)/3θ	=
0.001	sin((0.001)(3))/(3)(0.001)	= 0.9999985
0.0001	sin(0.0003)/0.0003	= 0.999999985
-0.0001	sin(-0.0003)/-0.0003	= 0.999999985
-0.001	sin(-0.003)/-0.003	= 0.9999985

8

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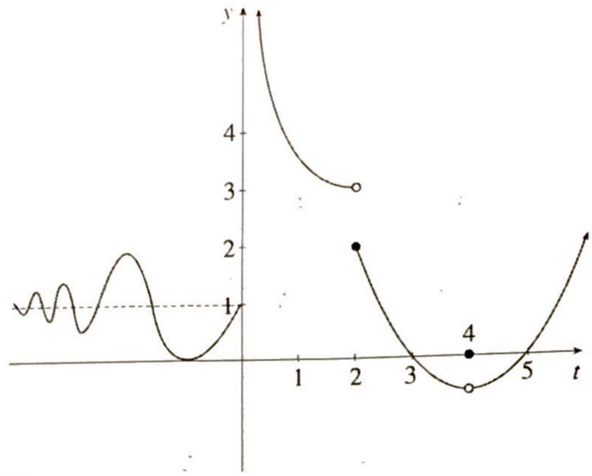
So what do you think the value of $\lim_{x \to 0} g(\theta)$ is? 1

15

3. **(10 pts)** Consider the following graph of f:



(a) What is $\lim_{t\to 0^+} f(t)$? $\lim_{t\to 0^-} f(t)$? $\lim_{t\to 2^-} f(t)$?

3 $\boxed{\infty \quad 1 \quad 3}$

(b) For what values of x does $\lim_{t\to x} f(t)$ exist? we need left limit = right limit

3 $\boxed{(-\infty, 0), (0,2), (2,\infty)}$ {all except 0, & 2}

(c) Does f have any vertical asymptotes? If so, where?

2 $\boxed{t=0, \text{ from the right}}$

(d) For what values of x is f discontinuous?

2 we need to lift our pencil at $\boxed{0, 2, 4}$

10

4. (10pts)

Let

$$f(x) = \begin{cases} \sqrt{3-x} & \text{if } x \le 1 \\ x^2 & \text{if } 1 < x < 3 \\ 27/x & \text{if } x \ge 3 \end{cases}$$

left limit $=\sqrt{2}$ right limit $=1$ $\}\neq$

(a) Evaluate each limit, if it exists.

(i) $\lim_{x\to 1^-} f(x)$ $\boxed{\sqrt{2}}$

(ii) $\lim_{x\to 1^+} f(x)$ $\boxed{1}$

(iii) $\lim_{x\to 1} f(x)$ $\boxed{DNE}$

(iv) $\lim_{x\to 3^-} f(x)$ $\boxed{3^2=9}$

8

(v) $\lim_{x\to 3^+} f(x)$ $\boxed{27/3=9}$

(vi) $\lim_{x\to 3} f(x)$ $\boxed{9}$

(vii) $\lim_{x\to 9} f(x)$ $\boxed{\frac{27}{9}=3}$

(viii) $\lim_{x\to -6} f(x)$ $\boxed{\sqrt{3-(-6)}=\sqrt{9}=3}$

(b) Where is f discontinuous?

2

left limit $\neq$ right limit @ $\boxed{x=1}$

10

5. **(5 pts)** Compute the limit. At each step clearly indicate, to the right, the limit or algebra property being used.

$$\lim_{h\to 0}\frac{(6+h)^2-36}{h}$$

$$\lim_{h\to 0}\frac{6^2+2\cdot 6\cdot h+h^2-36}{h}$$

$$=\lim_{h\to 0}\frac{12h+h^2}{h}$$

5

$$=\lim_{h\to 0} 12+h$$

$$=\boxed{12}$$

6. **(5 pts)** Evaluate the limit, if it exists, justifying your steps along the way. If it does not exist explain why not. {HINT: Rationalize the denominator}

$$\lim_{x\to 0}\frac{x}{3-\sqrt{x+9}}$$

$$=\frac{0}{0}\text{ form}$$

$$\lim_{x\to 0}\frac{x}{3-\sqrt{x+9}}\left(\frac{3+\sqrt{x+9}}{3+\sqrt{x+9}}\right)$$

$$=\lim_{x\to 0}\left(\frac{3x+x\sqrt{x+9}}{3^2-\sqrt{x+9}^2}\right)$$

5

$$=\lim_{x\to 0}\left(\frac{3x+x\sqrt{x+9}}{9-(x+9)}\right)$$

$$=\lim_{x\to 0}\left(\frac{3x+x\sqrt{x+9}}{-x}\right)$$

$$=\lim_{x\to 0}\left(-3-\sqrt{x+9}\right)$$

$$=\left(-3-\sqrt{0+9}\right)$$

$$=-3-3$$

$$=\boxed{-6}$$

10

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7. **(10pts)** Using the definition of slope of the tangent, find the slope of the tangent line to the curve $g(z) = 6z^2 - 9z + 1$ at $z = a$

7 $$m = \lim_{h \to 0} \frac{[6(z+h)^2 - 9(z+h) + 1] - [6z^2 - 9z + 1]}{h}$$

$$m = \lim_{h \to 0} \frac{6(z^2 + 2zh + h^2) - 9z - 9h + 1 - 6z^2 - 9z + 1}{h}$$

$$m = \lim_{h \to 0} \frac{\cancel{6z^2} + 12zh + 6h^2 - \cancel{9z} - 9h + \cancel{1} - \cancel{6z^2} - \cancel{9z} + \cancel{1}}{h}$$

$$m = \lim_{h \to 0} \frac{12zh + 6h^2 - 9h}{h}$$

$$m = \lim_{h \to 0} 12z + 6h - 9$$

3 $$m = \boxed{12z - 9}\Big|_{z=a}$$

$$= \boxed{\boxed{12a - 9}}$$

10

8. (10pts) Consider $f(x) = \dfrac{3e^{3x} + e^{-2x}}{7e^{3x} + 3e^{-2x}}$ show all pertinent steps

(a) Find $\lim\limits_{x\to+\infty} f(x)$ {Hint: First multiply numerator and denominator by e^{-3x}]

$\dfrac{\infty}{\infty}$

$e^{3x}e^{-3x} = e^{3x-3x} = e^{0} = 1$

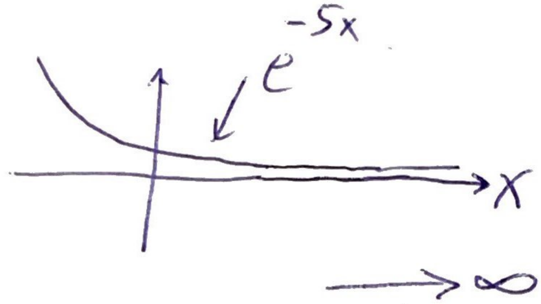
$$f = \frac{3e^{3x}+e^{-2x}}{7e^{3x}+3e^{-2x}}\left(\frac{e^{-3x}}{e^{-3x}}\right)$$

5

$$= \frac{3+e^{-5x}}{7+3e^{-5x}}$$

$$\Rightarrow \lim_{x\to\infty}\left(\frac{3+e^{-5x}}{7+3e^{-5x}}\right)$$

$$= \frac{3+0}{7+3\cdot 0} = \boxed{3/7}$$



(b) Find $\lim\limits_{x\to-\infty} f(x)$ {Hint: First multiply numerator and denominator by e^{2x}]

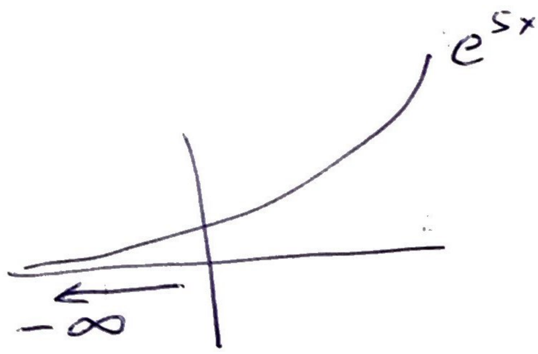
$$\frac{3e^{3x}+e^{-2x}}{7e^{3x}+3e^{-2x}}\left(\frac{e^{2x}}{e^{2x}}\right)$$

5

$$= \frac{3e^{5x}+1}{7e^{5x}+3\cdot 1}$$

$$\Rightarrow \lim_{x\to-\infty}\left(\frac{3e^{5x}+1}{7e^{5x}+3\cdot 1}\right)$$

$$= \frac{3\cdot 0+1}{7\cdot 0+3} = \boxed{\frac{1}{3}}$$



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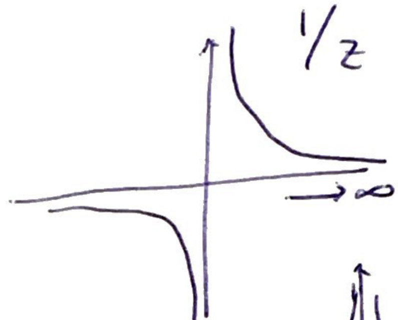
9. **(5 pts)** Evaluate by first dividing numerator and denominator by z^2

$$\lim_{z\to\infty} \ln\left(\frac{10z+8z^2}{z^2-1}\right)$$

$$\lim_{z\to\infty} \ln\left(\frac{(10z+8z^2)/z^2}{(z^2-1)/z^2}\right)$$



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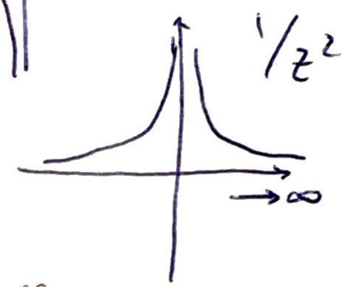
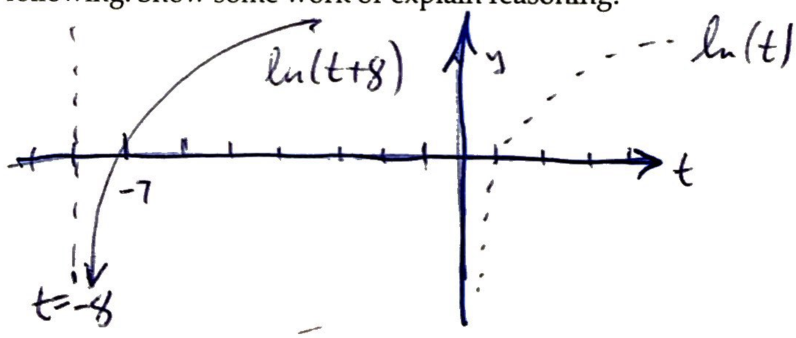
$$=\lim_{z\to\infty} \ln\left(\frac{\frac{10z}{z^2}+\frac{8z^2}{z^2}}{\frac{z^2}{z^2}-\frac{1}{z^2}}\right)$$



$$=\lim_{z\to\infty} \ln\left(\frac{\frac{10}{z}+8}{1-\frac{1}{z^2}}\right) = \ln\left(\frac{\overbrace{\lim_{z\to\infty}\frac{10}{z}}^{\to 0}+8}{1-\overbrace{\lim_{z\to\infty}1/z^2}^{\to 0}}\right)$$

$$=\boxed{\ln(8)}$$

10. **(5 pts)** For $W(t)= \ln(\,t+8\,)$ evaluate the following. Show some work or explain reasoning.



(a) $\lim\limits_{t\to -8^-} W(t)$

$\boxed{\text{DNE}}$

5

(b) $\lim\limits_{t\to -8^+} W(t)$

$\boxed{-\infty}$

(10)