2.7 Applications

I Kinematics

$$\begin{cases} \text{position} = f(t) \\ \text{velocity} = f'(t) \\ \text{acc'ln} = f''(t) \end{cases}$$

EX The position of a particle follows

$$S = f(t) = t^4 + 3t^2 + 2t - 1$$

diff't

(a) Find the velocity.

$$V = \frac{ds}{dt} = \underline{\underline{4t^3 + 6t + 2}}$$

(b) When is the particle at rest?

Ans: when $v = 0$

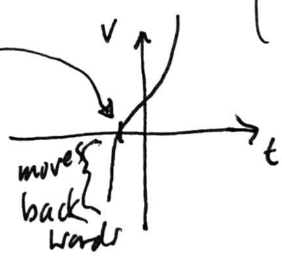
$$\Rightarrow \quad 0 = 4t^3 + 6t + 2$$

$$0 = 2t^3 + 3t + 1$$

$$\begin{array}{r|rrrr} -1 & 2 & 0 & 3 & 1 \\ & & -2 & 2 & -5 \\ \hline & 2 & -2 & 5 & \times \end{array}$$

$$\begin{array}{r|rrrr} -\tfrac{1}{2} & 2 & 0 & 3 & 1 \\ & & -1 & \tfrac{1}{2} & \\ \hline & 2 & -1 & \tfrac{7}{2} & \times \end{array}$$

plot: $\boxed{t = -0.3129 \text{ sec}}$



Ans: never if $t \geq 0$.

(c) When is the particle moving forward:

Ans: when $t \geq \underline{\underline{-0.3129 \text{ sec}}}$ (for $t \geq 0$ always)

(d) → over

(d) Find the acc'ln.

$$a(t) = \frac{dv(t)}{dt} = \frac{d(4t^3+6t+2)}{dt}$$

$$\boxed{a(t) = 12t^2+6}$$

(e) When is the particle speeding up {acc'ting}

Ans: when $a > 0$ thats is always for $t \geq 0$.

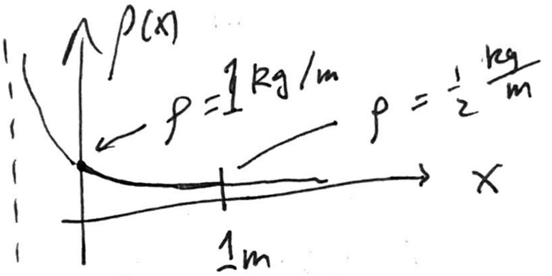
II Engineering Problem

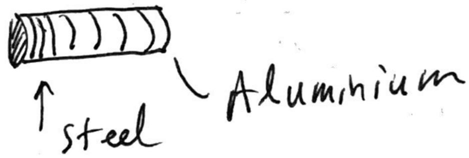
The <u>linear density</u>, the mass as a function of length, is denoted as ρ

Normally <u>volume density</u> is mass / volume

EX If A rod's density changes according to

$$\rho(x) = \frac{1}{x+1} \text{ for } x = [0,1] \text{ meter}$$





Q: At $x = 1/4$ m, what is the rod's density change w.r.t. x?

A: $\dfrac{d\rho}{dx}$

$$= \frac{d\,(x+1)^{-1}}{dx}$$

$$= -1(x+1)^{-2}$$

$$\boxed{\rho' = \frac{-1}{(x+1)^2}} \Bigg|_{x = 1/4}$$

so...

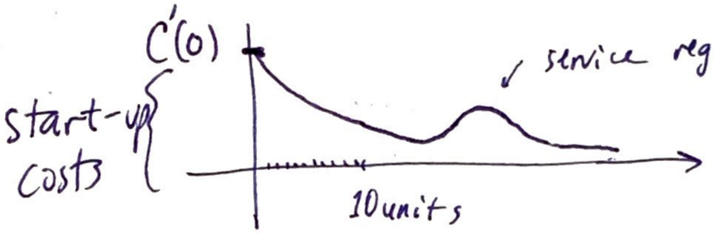
$$\rho' = \frac{-1}{\left(\frac{1}{4}+1\right)^2}$$

$$\boxed{\rho' = \frac{-16}{25} \text{ kg/m/m}}$$

III Economics Problem

The cost function is denoted as $C(x)$ and it yeilds the total cost needed to manufacture x units have been manufactured.

EX :



Cost per unit is the derivative of total cost $C(x)$

EX The cost function for production of widgets follows (Business Jet)

$$C(x) = 339 + 25x - 0.09x^2 + 0.0004x^3$$

in millions — start up, labor, tax incentive, material

a) interpret $C'(x)$ at $x = 100$.

$$\frac{dC(x)}{dx} = 25 - 0.18x + 0.0012x^2 \Big|_{x=100} = \frac{\$19 \text{ million}}{\text{Jet}}$$

This is the rate at which costs are changing after producing 100 Jets (in other words cost per unit)

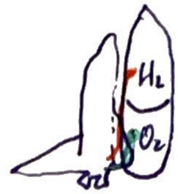
b) Cost of producing the 101st item

Same

$$C(101) - C(100) = 2538.03 - 2339 = \$19.03 \text{ million} \approx C'(100)$$

2.7 cont.

fuel



IV Chemistry

$$\underset{\text{fuel}}{2H_2} + \overset{\text{oxidizer}}{O_2} \rightarrow 2H_2O$$

The rate at which H_2 and O_2 is consumed is denoted as $\frac{d[H_2]}{dt}$, where $[H_2]$ = concentration

and $\left.\frac{d[O_2]}{dt}\right\}$

Conservation of atoms dictates that on the left we have 4H. and we have 4 Hydrogen on the right also.

* the average rate of reaction of a product C in the chemical reaction $\underbrace{A + B}_{\text{reactants}} \rightarrow \underbrace{C}_{\text{products}}$

is $$\frac{\Delta[C]}{\Delta t} = \frac{[C(t+\Delta t)] - [C(t)]}{\Delta t}$$

and in the limit we get

$$\boxed{\frac{d[C]}{dt} = \lim_{\Delta t \to 0} \frac{\Delta[C]}{\Delta t}}$$, the instantaneous rate of reaction

$\boxed{\text{EX}}$ Consider the reaction $A + B \to C$

where $[C]$ is analytically (theoretically) defined to be:

$$[C] = \frac{a^2kt}{akt+1}$$

quantity of $[C]$ after start of process (mixing)

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a) Find the rate of reaction @ t:

$$\frac{d[C]}{dt} = \frac{d\left(\frac{a^2kt}{akt+1}\right)}{dt}$$

$\left(\frac{f}{g}\right)' = \frac{f'g - g'f}{g^2}$

$(fg)' = f'g + fg'$

$$= \frac{(a^2kt)'(akt+1) - (a^2kt)(akt+1)'}{(akt+1)^2}$$

$$\frac{d[C]}{dt} = \frac{a^2k(akt+1) - a^2kt(ak)}{(akt+1)^2}$$

$$= \frac{\cancel{a^3k^2t} + a^2k - \cancel{a^3k^2t}}{(akt+1)^2}$$

$$\boxed{\frac{d[C]}{dt} = \frac{a^2k}{(akt+1)^2}}$$

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l , 1 ∅
7 seven
71
seven one

b) as time progresses, what is the concentration of $[C]$ as it is produced?

$$\lim_{t\to\infty}\left(\frac{a^2kt}{akt+1}\right) = \lim_{t\to\infty}\left(\frac{a^2kt}{akt+1}\right)\left(\frac{1/t}{1/t}\right) = \lim_{t\to\infty}\frac{a^2k}{ak+\frac{1}{t}} = \frac{a^2k}{ak} = \underline{\underline{a}}$$

(Initially we will let $[A]=a$ and this means $[B]=a$ also so not unexpected to see $[C]=a$

$\boxed{V}$ Biology

$\boxed{EX}$ the number of yeast cells in a lab culture is modelled by $\boxed{n = f(t) = \dfrac{a}{1+be^{-0.7t}}}$, t = hours.

- At $\underline{\underline{t=0}}$ $\underline{n = 20 \text{ cells}}$

and The cells are obsrved to be initially growing @ 12 cells/hr

Q: Find the values of "a" and "b"

$$\frac{dn}{dt} = \frac{d\left(\frac{a}{1+be^{-0.7t}}\right)}{dt}$$

$$\boxed{\frac{dy^n}{dx} = ny^{n-1}\frac{dy}{dx}}$$

$$= a\frac{d\,(1+be^{-0.7t})^{-1}}{dt}$$

$$\boxed{\begin{aligned}&\frac{de^{-0.7t}}{dt}\\ &= e^{-0.7t}(-0.7t)'\\ &= e^{-0.7t}(-0.7)\\ &= \underline{\underline{-0.7e^{-0.7t}}}\end{aligned}}$$

$$= a(-1)(1+be^{-0.7t})^{-2}(1+be^{-0.7t})'$$

$$= -a(1+be^{-0.7t})^{-2}\,b\,(-0.7e^{-0.7t})$$

$$\boxed{\frac{dn}{dt} = \frac{0.7abe^{-0.7t}}{(1+be^{-0.7t})^2}}$$

$$\boxed{\begin{aligned}&\frac{de^x}{dx} = e^x\\ &\int e^x dx = e^x\end{aligned}}$$

- Apply the "facts":

(i) $n(0) = \dfrac{a}{1+be^{-0.7(0)}}$ initial cell count

$$\downarrow$$

$$20 = \frac{a}{1+b}$$

(ii) $$\left.\frac{dn}{dt}\right|_{t=0} = \left.\frac{0.7ab\,e^{-0.7t}}{(1+b\,e^{-0.7t})^2}\right|_{t=0}$$

$$12 = \frac{0.7ab}{(1+b)^2}$$ Cell growth rate

- we have two eqns and two unknowns: lets substitute

(i) $$20 = \frac{a}{1+b} \Rightarrow \boxed{a - 20b = 20} \text{ or } a = 20 + 20b$$

(ii) $$12 = \frac{0.7ab}{(1+b)^2}$$

$$12 = \frac{0.7\,\overbrace{(20)(1+b)}^{a}\,b}{(1+b)^2} \Rightarrow 12 = \frac{0.7(20)\,b}{1+b}$$

Solve for b: $12 + 12b = 14b \Rightarrow 12 = 2b \Rightarrow \boxed{b = 6}$

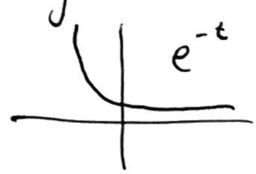
use (i) to get a: $a = 20 + 20b = 20(1+6) = \boxed{140 = a}$

- Finally

$$\boxed{n(t) = \frac{140}{1 + 6e^{-0.7t}}}$$

b) does the count level off as time goes by?

$$\lim_{t\to\infty} n(t) = \lim_{t\to\infty}\left(\frac{140}{1+\cancel{6e^{-0.7t}}}\right)$$



$= \boxed{140}$ cells is the long term limit.