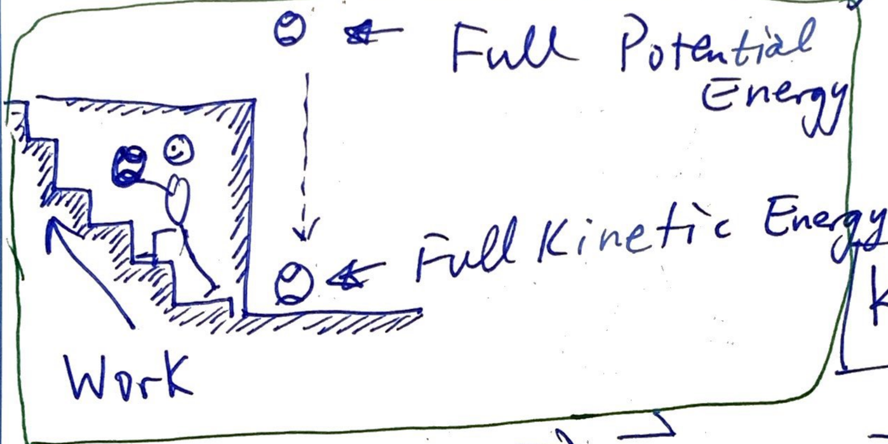
5.4 Work (Application #3)

In mechanics there is a quatity known as **Work** - Work is an energy type like Kinetic energy, potential energy, thermal energy atomic energy.

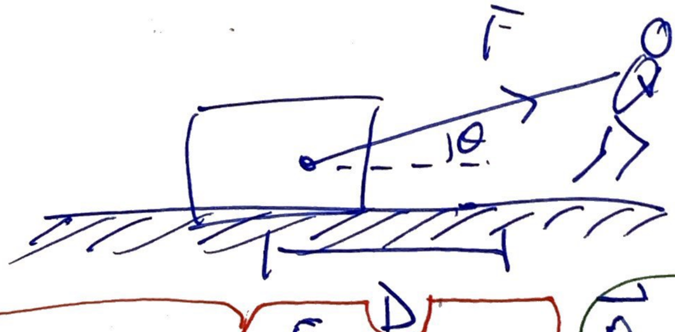
- Conservation of Energy says that we can transform energy from one type to another. But we cannot destroy energy (mass).



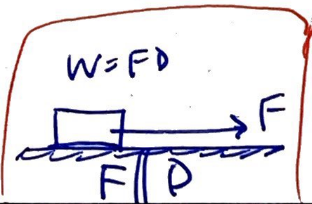
At any station (location)

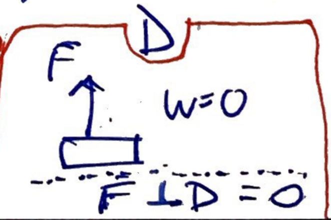
$$K_0 + P_0 = K_f + P_f + W_{performed}$$

⊗ Work is $\vec{F} \cdot \vec{D}$: $\vec{D}$ = displacement
$\vec{F}$ = Force applied.



$$\boxed{W = \vec{F} \cdot \vec{D}}$$

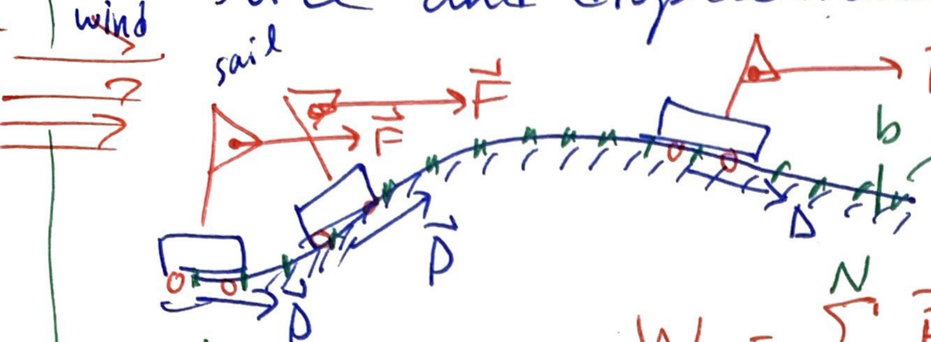




From Trig

$$\vec{A} \cdot \vec{B} = \|\vec{A}\| \, \|\vec{B}\| \cos\theta$$

- Over the course of transversal the force and displacement may change.



chop path up into pieces

$$W = \sum_{i=1}^{N} \vec{F} \cdot \Delta\vec{D}$$

Work done by wind

- To get most acc'y we take the limit

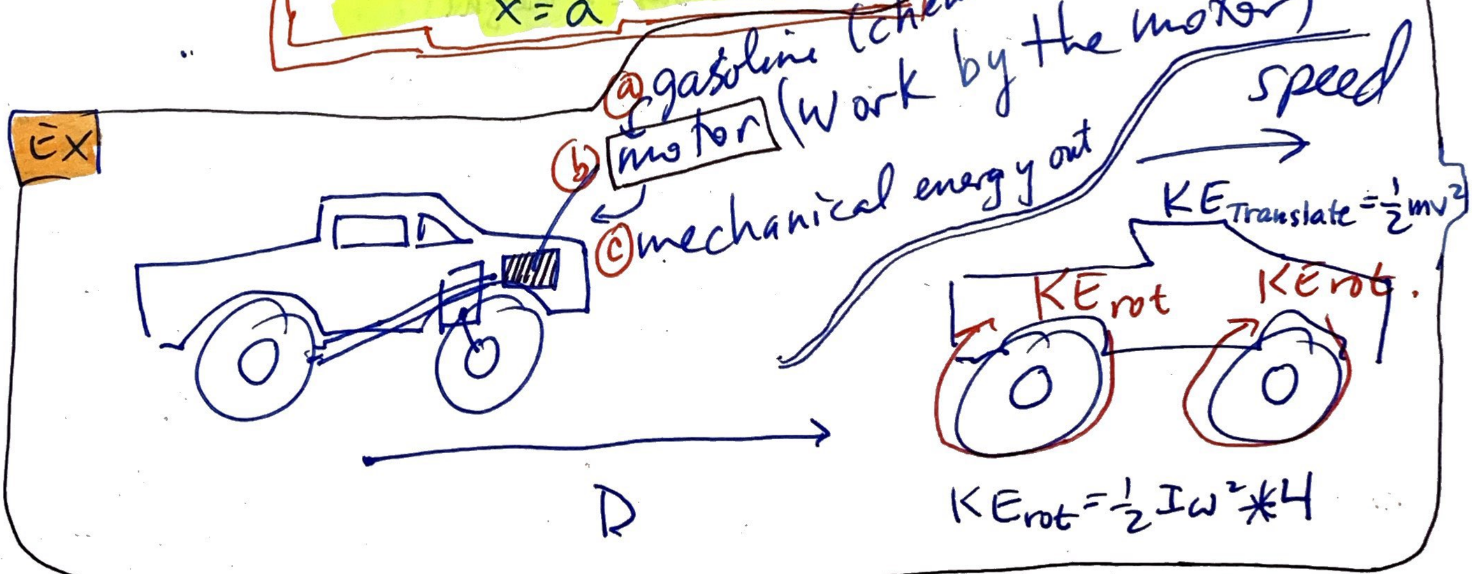
$$W = \int \vec{F}(x) \cdot d\vec{x}$$

line integral (calc III)

- For this section let $d\vec{x}$ always be in the <u>same direction</u> (vertical or horizontal)

So
$$\boxed{W = \int_{x=a}^{b} F(x)\,dx}$$

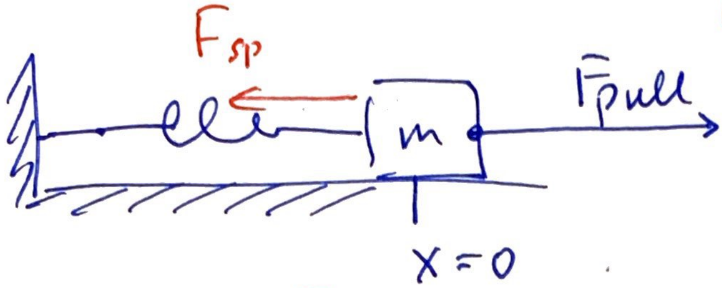
EX

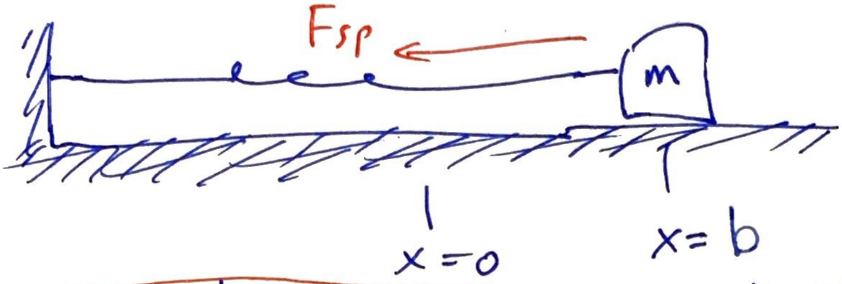


Ex Spring - mass

Hooke's Law

$$\boxed{F_{sp} = -k\,x}$$





Find the work done by the spring

let $b = 5\text{ cm} = \underline{0.05\text{ m}}$

let $k = 250\ N/m$

$$\boxed{W = \int_{x=0}^{b} F(x)\,dx}$$

$$= -\int_0^{0.05} k\,x\,dx$$

$$= -\int_0^{0.05} 250\,x\,dx$$

$$= -250\left.\frac{x^2}{2}\right|_0^{0.05}$$

$$= -250\frac{N}{m}\frac{(0.05\,m)^2}{2}$$

$$= 0.3125\ N\text{-}m$$

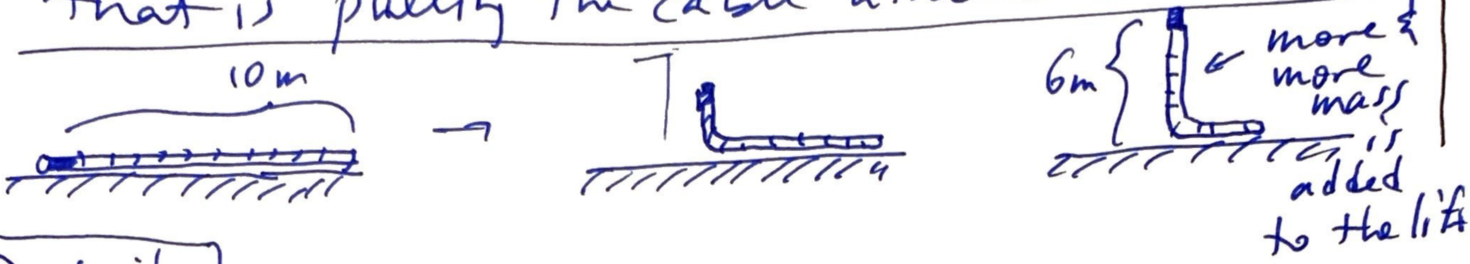
$\underline{F \cdot D}$

Joule

$$\boxed{0.3125\ J}$$

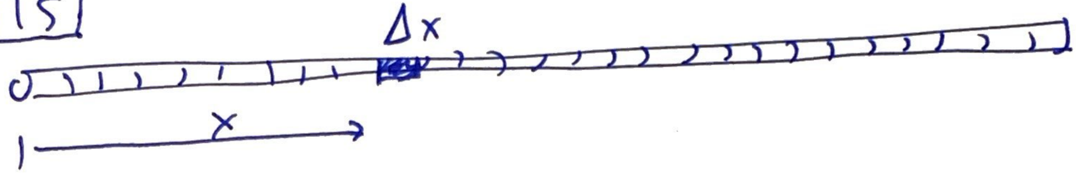
Ex lifting a chain from deck to ship

A 10m chain lies on a deck. The end is attached to a cable (massless) which lifts the cable end to a height of 6m. The weight of the chain is 8000kg. What work does the motor perform that is pulling the cable atached to the chain?



Details

- Chain:



- linear density = 8000kg / 10m = 800kg/m
- geometry

Newton's

$$\boxed{F_g = mg} \quad g = 9.8\,\frac{m}{s^2}$$

- work (vertical)

vertical height

horizontal position of chain element.

$\Delta W = \Delta F_g \cdot h$

$\Delta W = \Delta m \cdot g \cdot (6 - x)$ $\quad m = \rho x$

$\Delta W = \rho \Delta x \cdot g \cdot (6 - x)$

$$\boxed{W = \int_{x=0}^{6m} \rho g (6-x)\,dx}$$

$$\left(8000\frac{kg}{m}\right)\left(\frac{9.8\,m}{s^2}\right)\int_0^6 (6-x)\,dx$$

$$= 78400\frac{kg}{s^2}\left(6x - \frac{x^2}{2}\right)\Big|_0^6$$

$$= 78400\left[\left(36 - \frac{36}{2}\right) - (0-0)\right]$$

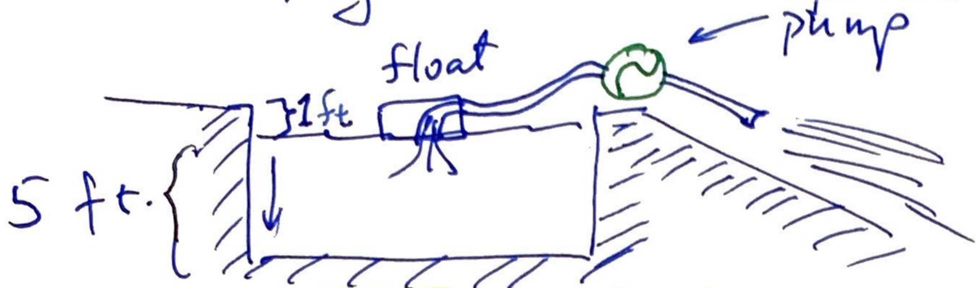
$$= 1411{,}200\ \frac{kg\text{-}m^2}{s^2}\ N\text{-}m$$

$$= \boxed{1.4\ \text{MJoules}}$$

EX Pumping water from a pool.

S.I. units: m, kg, s vs USCS: ft, lbs, s

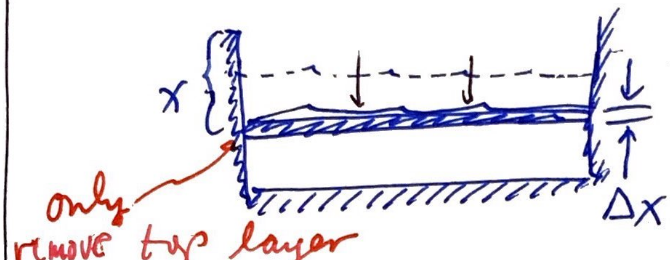
5



$A_O = \pi r^2$

$Circ = 2\pi r$

- Circular pool is 24 ft in diameter
- Density of water is 62.5 lbs / ft^3
- We Lift a horizontal slice x units high



$\Delta W = \Delta \text{weight} \cdot \text{height}$

$\Delta W = \rho \Delta Vol \cdot g \cdot x$

$\Delta W = \rho \cdot A \cdot \Delta x \cdot g \cdot x$

$A = \pi r^2$; $r = \text{fixed} = (Circ. / \pi)/2 = \frac{24ft}{2\pi} = \frac{12}{\pi} f^2$

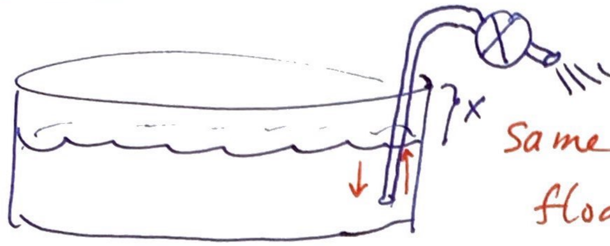
work

- $W = \sum_{\text{slices}} \Delta W \rightarrow W = \int_{x=1ft}^{5ft} \rho \cdot A(x) \cdot g \cdot x \cdot dx$

$$W = \underbrace{\left(62.5 \frac{lbs}{f^3}\right)\left(\pi 12^2 f^2\right)\left(\overset{g}{32.5 \frac{ft}{s^2}}\right)} \int_1^5 x\, dx$$

$$= \boxed{108{,}000\, \pi \text{ lb-ft}}$$

*Notes on draining fluids

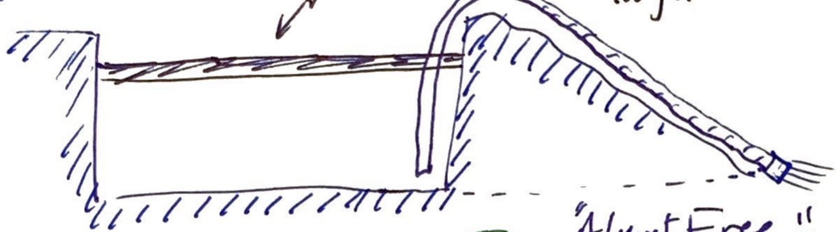


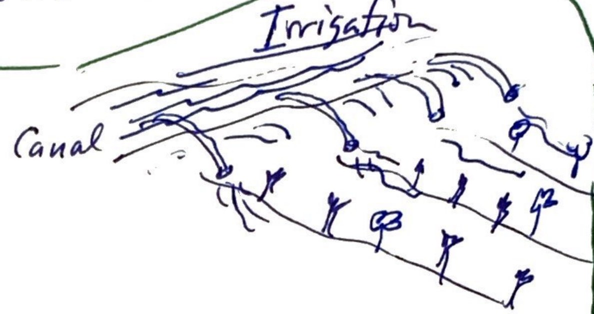
Work is only that of removing top slice.

Same answer as if the tube floats on the surface.

• extending the inlet to the bottom

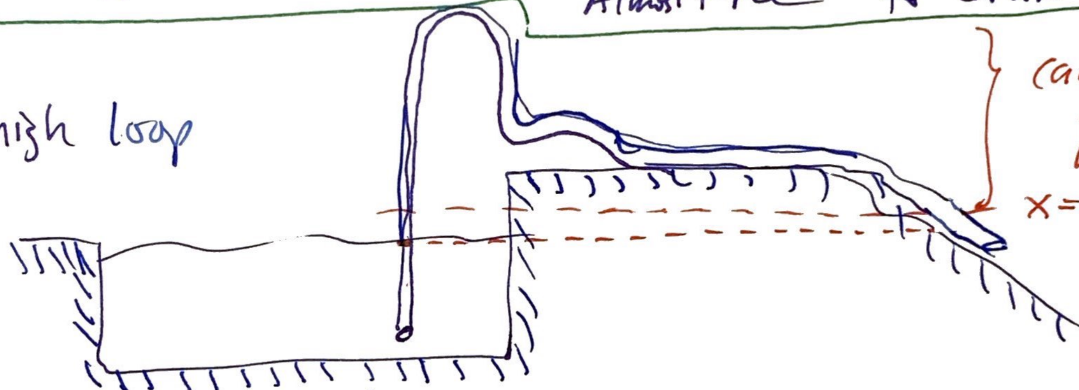
* syphon



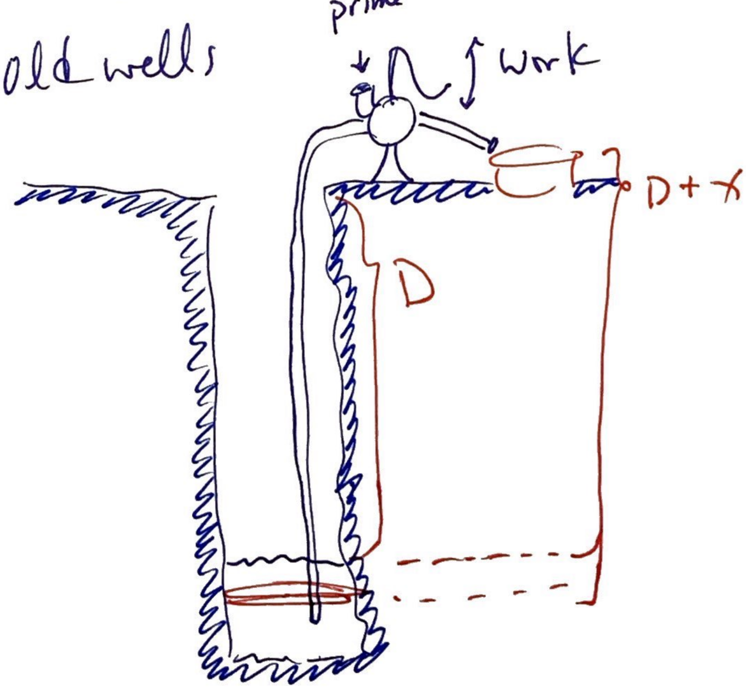


"Almost Free" to drain pool.

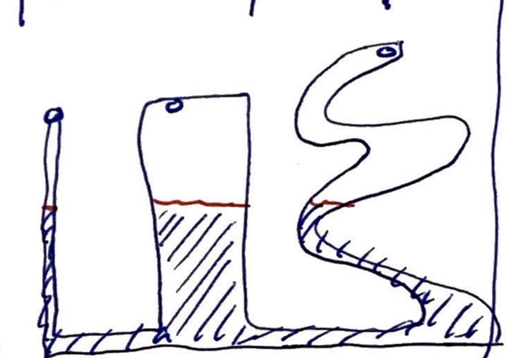
* high loop



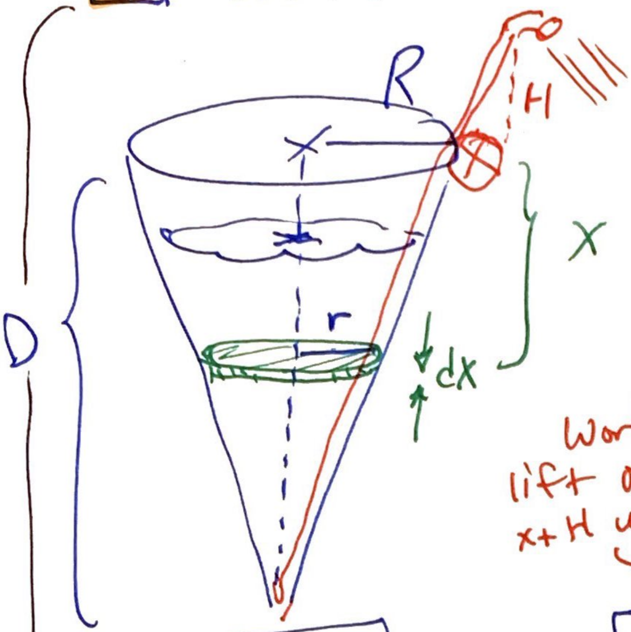
* old wells



Pascal's principle



Ex Draw a Conical Tank.



$$\Delta W = \Delta F \cdot \text{distance}$$
$$= \Delta F_g \cdot (x+H)$$
$$= \Delta m \cdot g \cdot (x+H)$$
$$= \rho \Delta V \cdot g \cdot (x+H)$$

Work to lift one slice $x+H$ units

$$\boxed{\Delta W = \rho\, A(x) \cdot \Delta x \cdot g \cdot (x+H)}$$

geometry

Cross-section

$\cdot\ A(x) = \pi r^2$

D, R as d, r so $\boxed{\dfrac{r}{d} = \dfrac{R}{D}}$

$d = D - x$

$$\boxed{r(x) = \frac{R}{D}(D-x)}$$

Work/slice

$$\cdot\ W = \rho g \cdot \int_{x=\text{depth }1}^{\text{depth }2} A(x)(x+H)\,dx$$

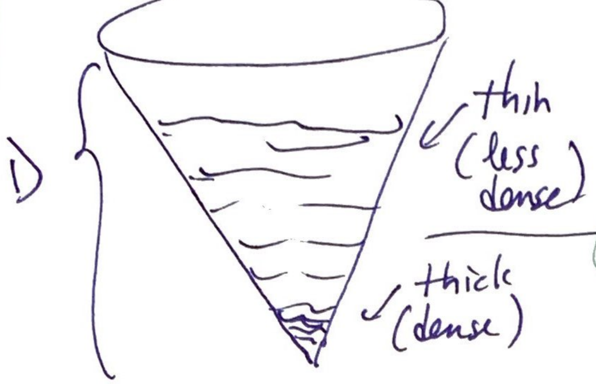
$$= \rho g \int \pi \left[\frac{R}{D}(D-x)\right]^2 (x+H)\,dx$$

$$\boxed{W = g\rho\pi R^2 \int_{\text{depth }1}^{\text{depth }2} \left(1-\frac{x}{D}\right)^2 (x+H)\,dx}$$

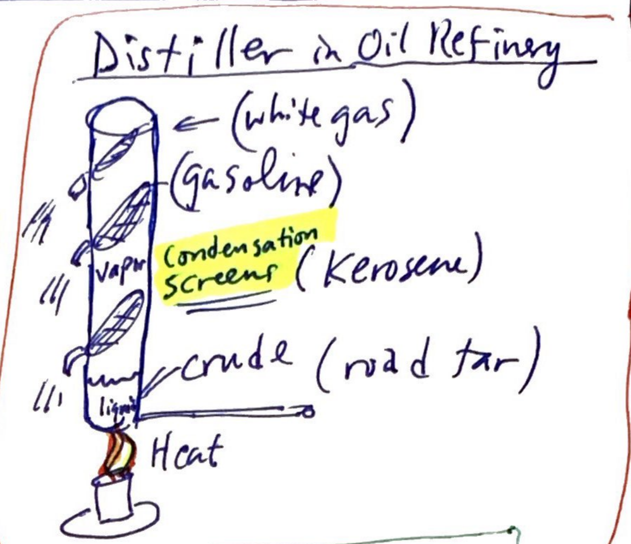
R, D, H

Formula Set-Up

EX lets vary the density: Assume now that the fluid in the conical tank has settled with the more dense portions towards the bottom



lets use $\rho(x) = \left(1 + \sqrt{\frac{x}{D}}\right)$ for example



Now our integral looks like this...

$$W = g\pi R^2 \int_{x_1}^{x_2} \underbrace{\left(1+\sqrt{\frac{x}{D}}\right)}_{\rho} \underbrace{\left(1-\frac{x}{D}\right)^2}_{\text{radius ratio}} (x+H)\, dx$$

EX Lets assume this tank has fuel in it and it is travelling next to a black hole, which has great changes in gravity. g = function of x also.

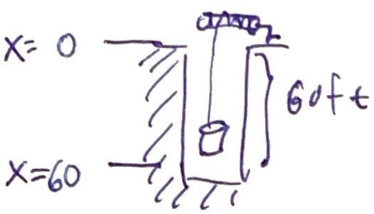
for ex. let $g = g_0 e^{x/D}$, this means g stays inside

$$W = \pi R^2 \int_{x_1}^{x_2} \underbrace{\left(g_0 e^{x/D}\right)}_{\text{gravity}} \underbrace{\left(1+\sqrt{\frac{x}{D}}\right)}_{\text{density}} \underbrace{\left(1-\frac{x}{D}\right)^2}_{\text{geometry}} \underbrace{(x+H)}_{\text{height}}\, dx$$

Practice Problem

A bucket weighs 70 lbs when filled with water and is lifted from a 60 ft deep well.

(a) what is the work? ✓ constant force



$$W = \int F(x)\,dx$$

$$W = \int_{x=0}^{x=60} (mg)\,dx$$

physics

$$\underline{W = mgh}$$

$$F = mg$$

$$W = F \cdot d = mg \cdot h$$

$$= 70 \cdot x \Big|_0^{60} = 70(60-0) = \boxed{\overset{mgh}{4200 \text{ ft-lb}}}$$

(b) The bucket leaks water! It weighs only 35 lbs by the time you get it to the top. What is the work now?

Assume the loss of water is linear. Lets try out different linear arrangements...

$$F(x) = 35\text{lbs}\left(1 + \frac{x}{60}\right)$$

TEST:

$x = 0$ @ top

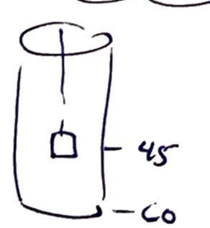
$$35\left(1 + \frac{0}{60}\right) = 35$$

$x = 60$ @ bottom

$$35\left(1 + \frac{60}{60}\right) = 70$$

So
$$W = \int_{x=0}^{60} \overbrace{35\left(1 + \frac{x}{60}\right)}^{\text{weight = Force}} dx$$

equivalence:



needed only 2/3 work

$$= 35\text{ lbs} \cdot \int_0^{60} \left(1 + \frac{x}{60}\right) dx$$

$$= 35\text{lbs}\left(x + \frac{x^2}{2 \cdot 60}\right)_0^{60} = 35\left(60 + \frac{60^{\cancel{2}}}{2 \cdot \cancel{60}} - 0\right)$$

$$= 35\text{ lbs} \cdot 90\text{ft} = \overset{\text{BTW}}{70\text{lbs} \cdot 45\text{ft}} = \boxed{3150 \text{ ft-lbs}}$$