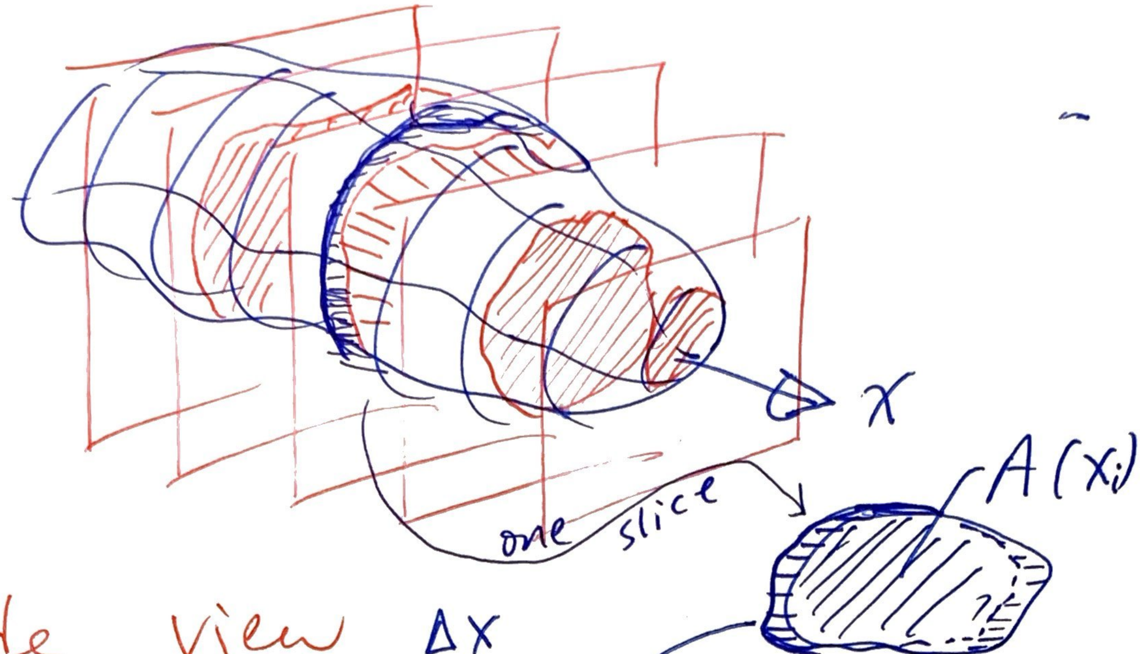
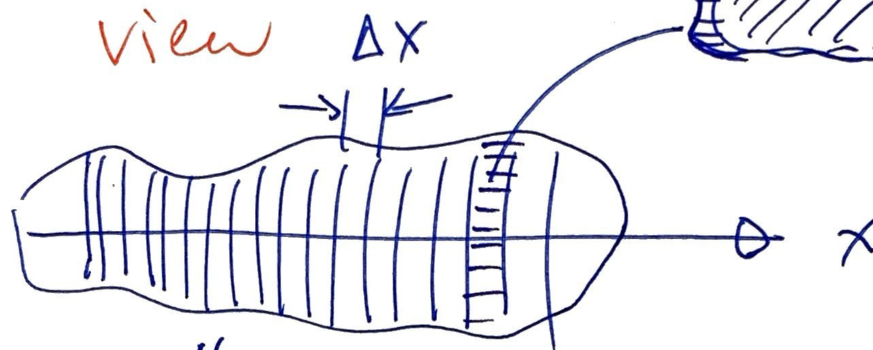
5.2 Review on Volumes



• side view



$$V \approx \sum_{i=1}^{N} \underbrace{A(x_i)\cdot\Delta x}_{V_i}$$

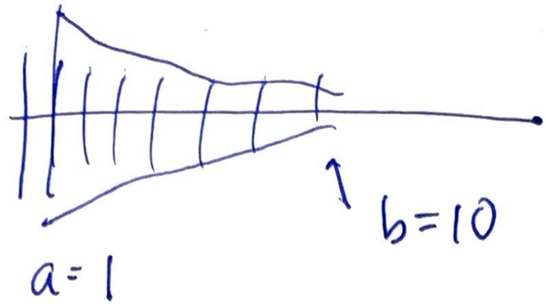
$$\Delta x = \frac{b-a}{N}$$

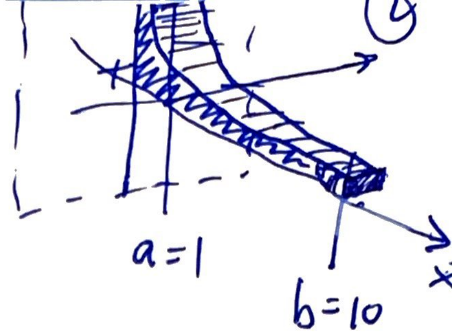
$$V = \lim_{N\to\infty} A(x_i)\cdot\Delta x$$

$$\boxed{V = \int_a^a A(x)\,dx}$$

EX

$$A(x) = \frac{1}{x+1}$$





$$\int \frac{1}{u}\,du = \ln|u| + c$$

$$V = \int_1^{10} \left(\frac{1}{x+1}\right) dx$$

$u = x+1 \rightarrow du = dx$, $\quad u(1) = 1+1 = 2$, $\quad u(10) = 10+1 = 11$

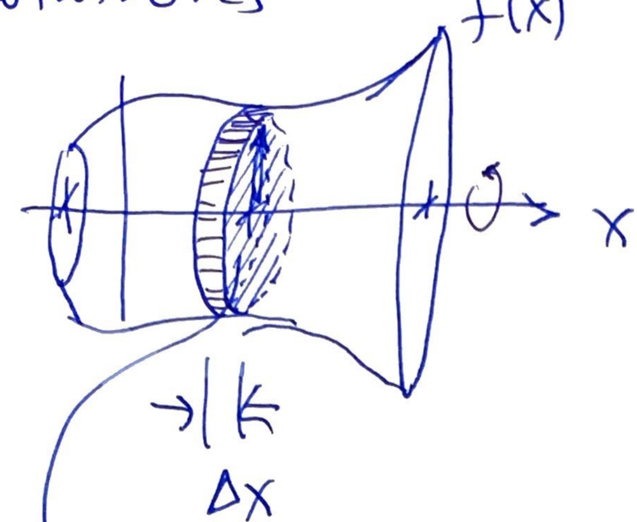
$$= \int_{u=2}^{11} \frac{1}{u}\,du$$

$$= \ln|u| \Big|_2^{11}$$

$$= \ln|11| - \ln|2|$$

$$= \boxed{\ln\left(\frac{11}{2}\right)}$$

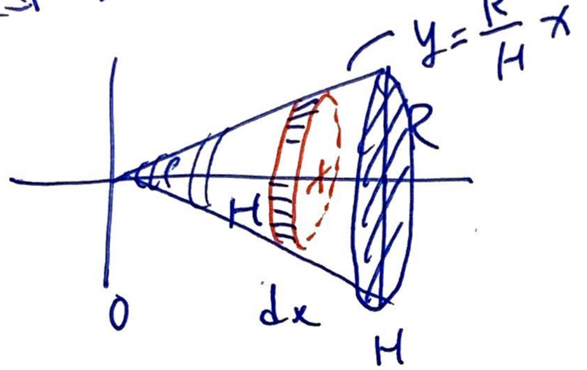
(*) Rotations



$$A(x) = \pi r^2 = \pi \left[f(x)\right]^2$$

$$V = \pi \int_{x=a}^{b} f^2 \, dx$$

Ex Volume of a cone



$$V = \int_0^H \underbrace{\pi \left[\frac{R}{H} x\right]^2}_{A(x)} dx$$

(like $y = \frac{R}{H} x$)

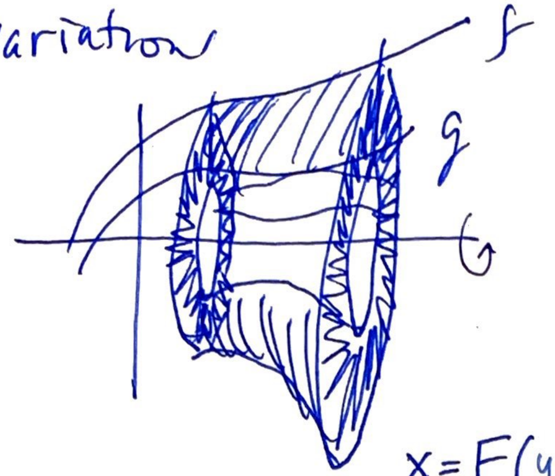
$$= \frac{\pi R^2}{H^2} \int_0^H x^2 \, dx$$

$$= \frac{\pi R^2}{H^2} \left. \frac{x^3}{3} \right|_0^H$$

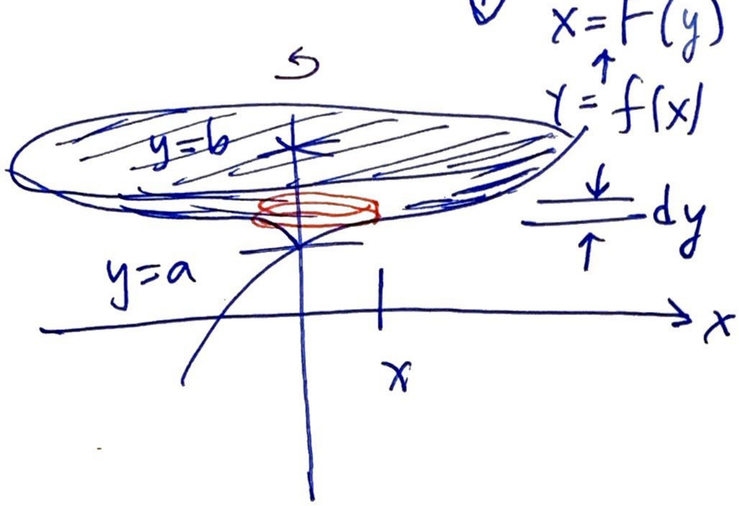
$$= \frac{1}{3} \pi R^2 \frac{H^3}{H^2}$$

$$\boxed{V = \frac{1}{3} \pi R^2 H}$$

- Variations



$$V = \int_a^b [\pi f^2 - \pi g^2]\, dx$$

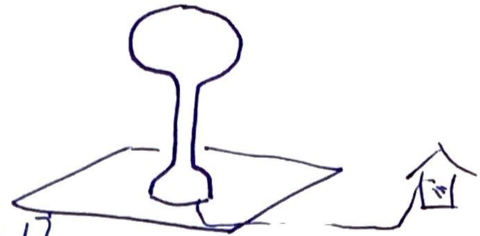


$$A(y) = \pi x^2$$

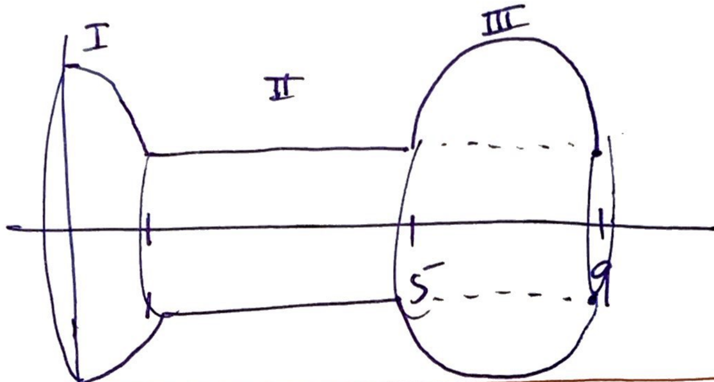
$$V = \int_{y=a}^{b} \pi [F(y)]^2 \, dy$$

Ex Volume of a water Tower

(Set up only)



let $f(x) = \begin{cases} 2 - x^2 & x \in [0,1] \\ 1 & x \in (1,5] \\ \sqrt{4-(x-7)^2}+1 & x \in [5,9] \end{cases}$



$$V = \pi \int f^2 \, dx$$

(radius → f)

$$V = \pi \int_0^1 (2-x^2)\,dx + \pi \int_1^5 1^2\,dx + \pi \int_5^9 \left[\sqrt{4-(x-7)^2}+1\right]^2 dx$$

$$= \boxed{\frac{323}{15}\pi + 4\pi^2} \approx 107.127 \text{ cubic units}$$