4.4 Indefinite Integrals, Net Change

- Def : $\boxed{\int_a^b f(x)dx = \text{number and is called a definite Integral}}$

- Def $\boxed{\int f(x)dx = \text{function and is called an indefinite integral}}$

I Indefinite integrals

$$\boxed{\int f(x)dx = F(x) + c}$$

F is the anti-derivative of f(x)

(a) Since $\frac{dx^n}{dx} = nx^{n-1}$ then $\frac{d\left(\frac{x^{n+1}}{n+1}\right)}{dx} = x^n$ $(n \neq -1)$

thus $\boxed{\int x^n dx = \frac{x^{n+1}}{n+1} + c}$

Anti-derivative of x^n is $\frac{x^{n+1}}{n+1}$

the integral of x^n is $\frac{x^{n+1}}{n+1}$

(b) $\boxed{\int k\,dx = kx + c}$ b/c $\frac{d(kx)}{dx} = k$

the integral is the antiderivative

Differentiation IN Reverse

(c) $\boxed{\int \sin(x)\,dx = -\cos(x) + c}$ b/c $\dfrac{d(-\cos(x))}{dx} = \sin(x)$

(d) $\boxed{\int \cos(x)\,dx = \sin(x) + c}$ b/c $\dfrac{d\sin(x)}{dx} = \cos(x)$

(e) $\boxed{\int \sec^2(x)\,dx = \tan(x) + c}$ b/c $\dfrac{d\tan(x)}{dx} = \sec^2(x)$

(f) $\boxed{\int \csc^2(x)\,dx = -\cot(x) + c}$ b/c $\dfrac{d-\cot(x)}{dx} = \csc^2(x)$

(g) $\boxed{\int \sec(x)\tan(x)\,dx = \sec(x) + c}$ b/c $\dfrac{d\sec(x)}{dx} = \sec(x)\tan(x)$

(h) $\boxed{\int \csc(x)\cot(x)\,dx = -\csc(x) + c}$ b/c $\dfrac{d-\csc(x)}{dx} = \csc(x)\cot(x)$

"Hey, where is $\int \tan(x)\,dx$?"

- Ans: chapter 7 but until then lets look at Stewarts' <u>table of Integrals</u> {end leafs}

simple complicated

$$\boxed{\int \tan(x)\,dx = \ln|\sec(u)| + c}$$

Do not derive or use until

Chpt 7 Calc II

Ex Integrate $\int \left(\frac{x^3 - 2\sqrt{x}}{x} \right) dx$

$$\int \left(\frac{x^3}{x} - \frac{2\sqrt{x}}{x} \right) dx$$

$$= \int \left(x^2 - 2x^{\frac{1}{2}-1} \right) dx$$

$$= \int x^2 dx - 2 \int x^{-1/2} dx$$

$$= \left(\frac{x^{2+1}}{2+1} + c \right) - 2 \left(\frac{x^{-1/2+1}}{-\frac{1}{2}+1} + c \right)$$

$$= \frac{x^3}{3} - 2 \frac{x^{1/2}}{1/2} + c$$

$$= \boxed{\frac{x^3}{3} - 4\sqrt{x} + c}$$

Math 211 C4c Name ______

1. Integrate $\int \frac{\sin(x)}{\cos^2(x)}\,dx$ {Hint. convert to sec, tan, etc}

2. Integrate $\int \sec(t)\,[\sec(t)+\tan(t)]\,dt$

3. $\int \frac{\sin(2x)}{\sin(x)}\,dx$ {Use an identity for $\sin(2x)$}

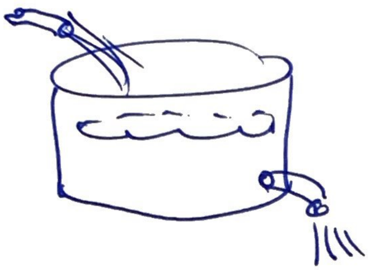
II Net Change

FTC · II

$$\boxed{\int_a^b \underset{\substack{\uparrow \\ \text{rate of change}}}{F'(t)}\,dt = \underbrace{F(b)-F(a)}_{\text{Net change}}}$$

⊛ Volume of Liquid in a tank.

- let $V(t)$ = volume of liquid in a tank at time "t"



Combined rate of change of the volume is $\dfrac{dV(t)}{dt}$

Then the volume change at a further time follows this formula (FTC II)

$$\boxed{V(t_2) = V(t_1) + \int_{t_1}^{t_2} V'(t)\,dt}$$

EX let $V'(t) = \dfrac{1}{t^2}$ if $t > 1$ min. Find the volume in the tank at $t = 3$ min, start @ $t_1 = 1$ m

(+) means tank will fill

$$V(3) = V(1) + \int_1^3 \left(\frac{1}{t^2}\right) dt$$

$$V(3) = \frac{1}{t^2}\bigg|_{t=1} + \int_1^3 t^{-2}\,dt$$

$$V(3) = \frac{1}{1^2} + \frac{t^{-2+1}}{-2+1}\bigg|_1^2$$

$$= 1 + \left(\frac{t^{-1}}{-1}\right)\bigg|_1^2$$

$$= 1 - \frac{1}{t}\bigg|_1^2$$

$$= 1 - \left(\frac{1}{2} - \frac{1}{1}\right)$$

$$= -1 = 3/2$$

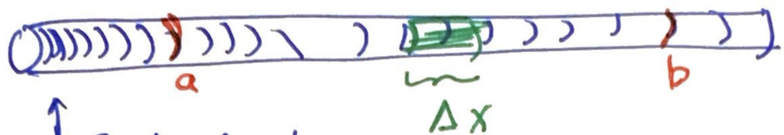
Density

- Linear Density = mass / length $\Rightarrow$ $\boxed{\rho = \dfrac{dm(x)}{dx}}$ ← Instantaneous density

Variable density $\rho = f(x)$

ex

50% lead
50% Al



80% lead
20% Al

$$\Delta m = \rho(x) \cdot \Delta x$$

So $\Delta m = \left(\dfrac{dm(x)}{dx}\right)\Delta x$

- Total mass between a & b $= \displaystyle\sum_{i=1}^{n} \rho(x_i) \cdot \Delta x$

- net mass @ $x = b$ is

$$\boxed{m(b) = m(a) + \int_a^b \rho(x)\,dx}$$

EX If a rod's density falls of as $\rho(x) = \dfrac{\rho_0}{\sqrt{x}}$

Find the mass between $a = 1\,cm$ & $b = 8\,cm$

$$\Delta m = \int_a^b \rho(x)\,dx$$

$$\Delta m = \int_1^8 \frac{\rho_0}{\sqrt{x}}\,dx$$

$$= \rho_0 \int_1^8 x^{-1/3}\,dx$$

$$\Delta m = \rho_0 \left.\frac{x^{-1/3+1}}{-1/3+1}\right|_{x=1}^{x=8}$$

$$\Delta m = \left.\frac{3\rho_0 x^{2/3}}{2}\right|_1^8$$

$$\Delta m = \frac{3}{2}\rho_0\left(\sqrt[3]{8}^{\,2} - \sqrt[3]{1}^{\,2}\right)$$

$$\Delta m = \frac{3}{2}\rho_0\left[4 - 1\right] = \boxed{\frac{9}{2}\rho_0}$$

Economics

If $C(x)$ is the cost to produce x widgets of a commodity, like cell phones, say, then the marginal cost is $C'(x)$ and represents the cost per unit after x widgets have been produced.

The net change of cost from x_1 widgets to x_2 widgets is then

$$\boxed{C(x_2) - C(x_1) = \int_{x_1}^{x_2} C'(x)\,dx}$$

Ex let $4 - 0.02x$ be the marginal cost to produce x toothbrushes. What is the cost expended from the 10th brush to the 30th brush

$C'(x) = 4 - 0.02x$

$$\Delta C = \int_{10}^{30} (4 - 0.02x)\,dx$$

$$= 4x\Big|_{10}^{30} - 0.02\frac{x^2}{2}\Big|_{10}^{30}$$

$$= 4(30-10) - \frac{0.02}{2}(30^2 - 10^2)$$

$$= 80 - 0.01(900 - 100)$$

$$= 80 - 0.01(800)$$

$$= 80 - 8$$

$$= \boxed{72\,\$}$$

⊛ Electrical Charge

Current in a wire or a capacitor is the amount of charge flowing across some reference point per unit time

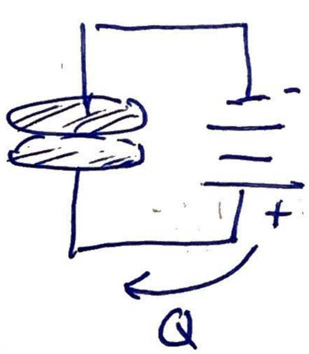
$$I \equiv \frac{\Delta Q}{\Delta t}$$

Instantaneous current is $\boxed{I = \frac{dQ}{dt}}$

What does $\displaystyle\int_{t=a}^{t=b} I(t)\,dt$ mean?

Ans: $\boxed{\displaystyle\int_a^b \left(\frac{dQ}{dt}\right)dt = Q(b) - Q(a)}$ via the FTC·II

EX A capacitor accumulates charge, and while doing so appears to have a current flowing across its plates.



Q: What is the net change of charge if $I(t) = \frac{I_0}{t^3}$ from $t = 1_s$ to $t = 3_s$

$$\Delta Q = \int_{t=1}^{3}\left(\frac{I_0}{t^3}\right)dt$$

$$= I_0 \int_1^3 t^{-3}$$

$$= I_0 \frac{t^{-2}}{-2}\Big|_1^3$$

$$= \frac{-I_0}{2}\left(\frac{1}{3^2} - \frac{1}{1^2}\right)$$

$$= \frac{-I_0}{2}\left(\frac{1}{9} - 1\right) = \frac{I_0}{\cancel{2}}\cdot\frac{\cancel{8}^{\,4}}{9}$$

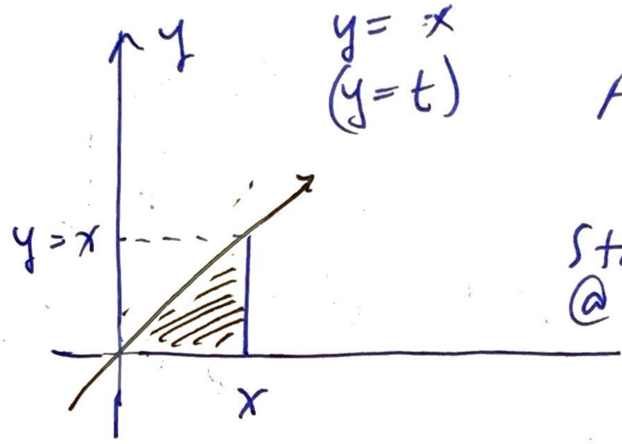
$$\boxed{\frac{I_0 4}{9}}$$

Area

Consider area under the line $y = x$

Q: How does area grow as we move x further out on the x-axis?

$y = x$
$(y = t)$



$$\text{Area} = \int_a^b f(t)\,dt$$

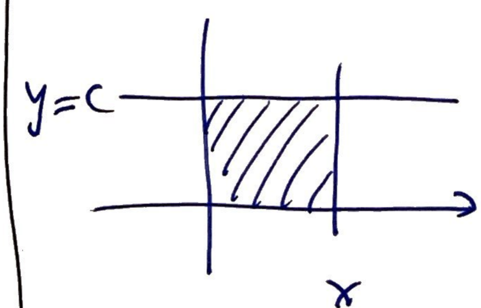
Start @ $x=0$ $$= \int_0^x (t)\,dt$$

$$= \frac{t^2}{2}\Big|_0^x$$

$$\boxed{A(x) = \frac{x^2}{2}}$$

$A_\Delta = \frac{1}{2} b \cdot h$ ($b = x$, $h = x$)

EX If $y = c$ what is $A(x)$?



$$A(x) = \int_0^x c\,dt$$

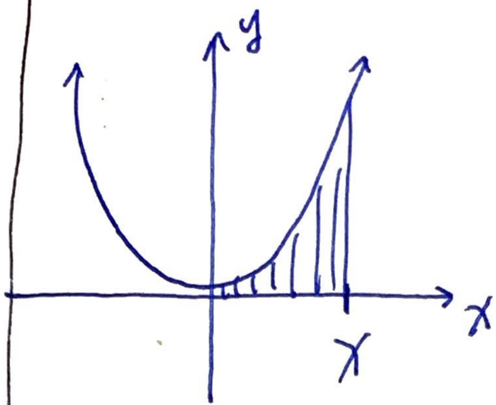
$$= ct\Big|_0^x$$

$$= c(x-0)$$

$$\boxed{A(x) = cx}$$

$A = W \cdot H$

Ex If $y = x^2$ what is $A(x)$?



$$A(x) = \int_0^x t^2\,dt$$

$$= \frac{t^3}{3}\Big|_0^x$$

$$= \frac{x^3}{3} - \frac{0^3}{3}$$

$$\boxed{A(x) = \frac{x^3}{3}}$$