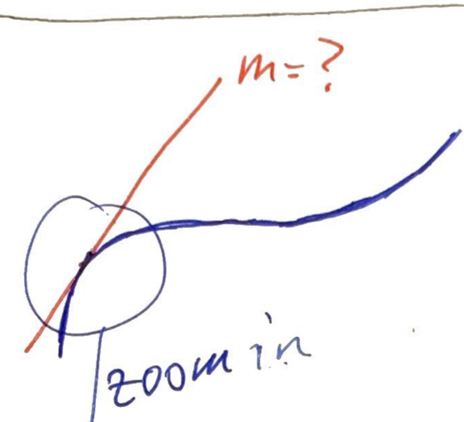
3.8 Numerical Analysis (so far)

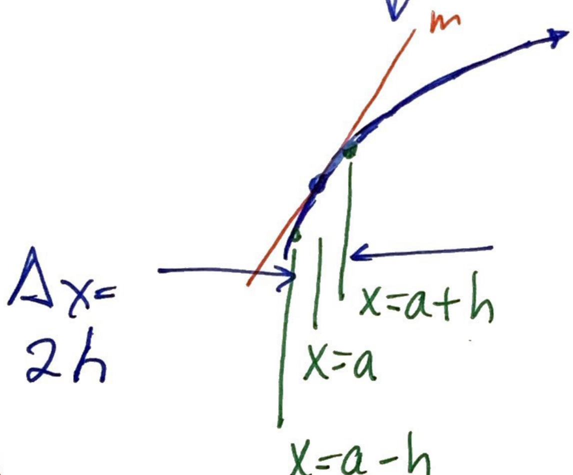
To approx a derivative numerically we siply compute the secant line of two neighboring points

Ex:



let say f' is very difficult to evaluate analytically either explicitly or implicitily

We can still get a numerical approximation

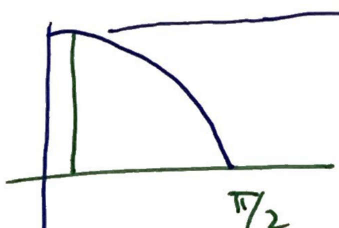


$$f'(a) \approx \frac{f(a+h) - f(a-h)}{(a+h) - (a-h)}$$

$$\boxed{f'(a) \approx \frac{f(a+h) - f(a-h)}{2h}}$$

use $h = 0.05$

Ex: Let $f(x) = \cos(x)$. approx $f'(x)$ @ $x = 0.4$



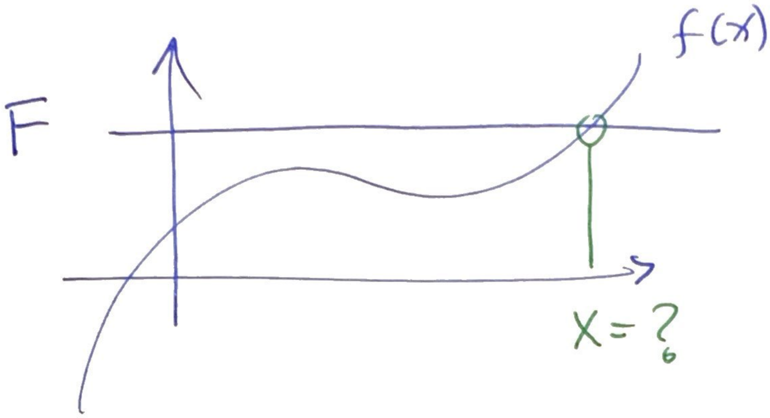
$$f'(0.4) \approx \frac{\cos(0.4 + 0.05) - \cos(0.4 - 0.05)}{2(0.05)}$$

$$\boxed{f'(0.4) \approx \frac{\cos(0.45) - \cos(0.35)}{0.1} \approx -0.3892}$$

Test: analytically

$$f'(x) = -\sin(x)\Big|_{0.4} = -\sin(0.4) = -0.3894$$

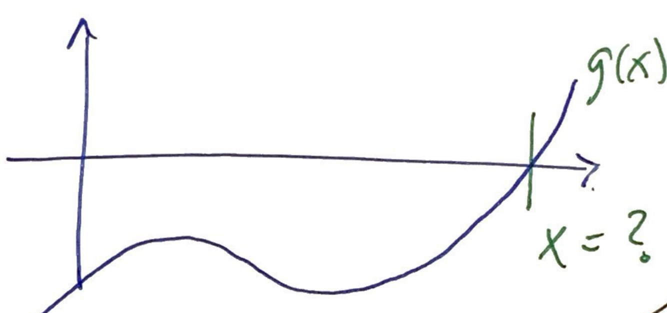
* Frequently we desire to know at what x value will $f(x)$ equal to a desired value, F



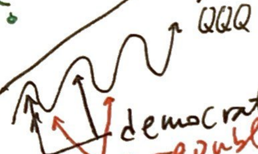
- we can recast the problem as a new function

$$g(x) = f(x) - F$$

we now ask when is $g(x) = 0$. I.E. Find the zero of $g(x)$.



Root Finding methods.



1st explaination
(variation due to capitalism)

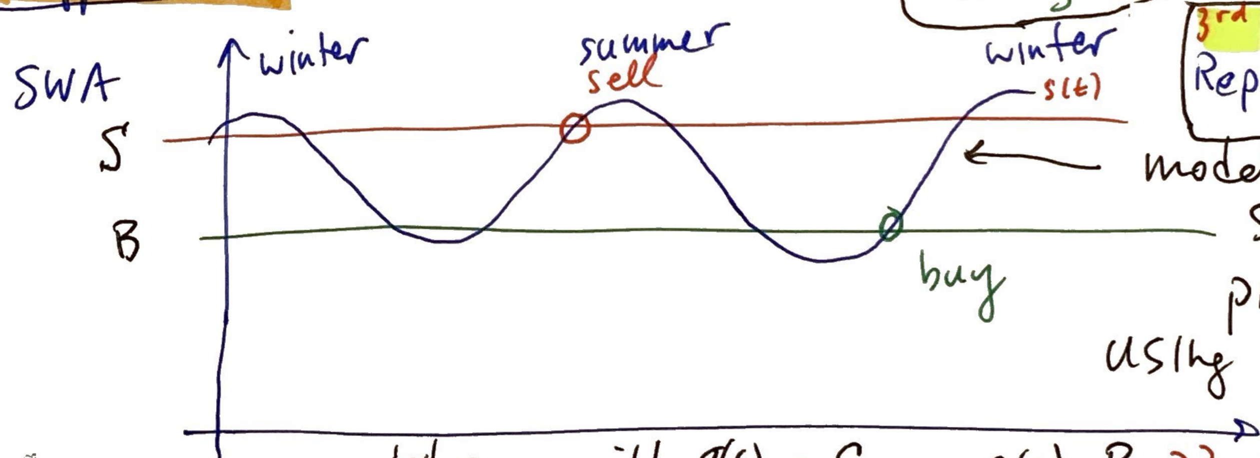
2nd explaination

"two santa clause theory"

3rd explaination
Repub's Crash and let the Dems build

Application

Stock market

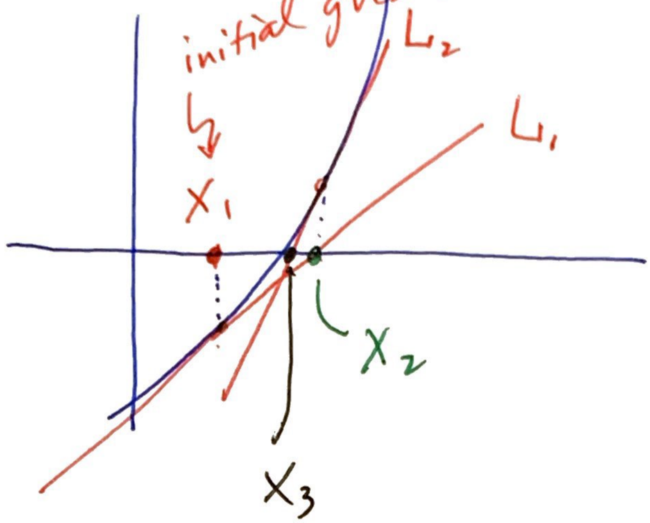


model the stock price by using a trig function

when will $s(t) = S$ or $s(t) = B$??

⊛ Newton's method

initial guess @ the crossing value (root)



X_1 = initial guess yeilds L_1

X_2 = tangent zero from L_1 line

X_3 = tangent zero of L_2

⋮

Algorithm: do while

1. compute tangent line at x_i
2. Find x-int. of the tangent line x_{i+1}
3. use that intercept to seed the next tangent line of $f(x)$

• tangent line

$$\boxed{y - f(x_1) = f'(x_1)(x - x_1)}$$

x-int: $y = 0$

next iterative point

$$0 - f(x_1) = f'(x_1)(x_2 - x_1)$$

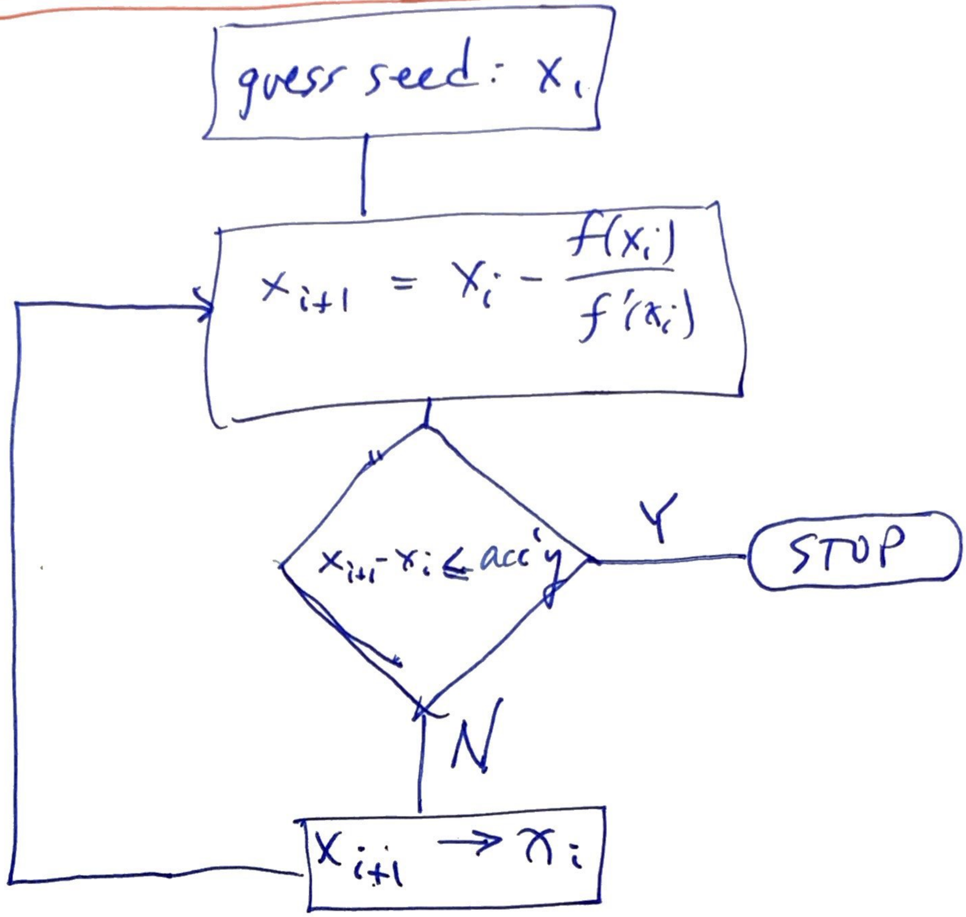
So

$$\boxed{x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}}$$

or

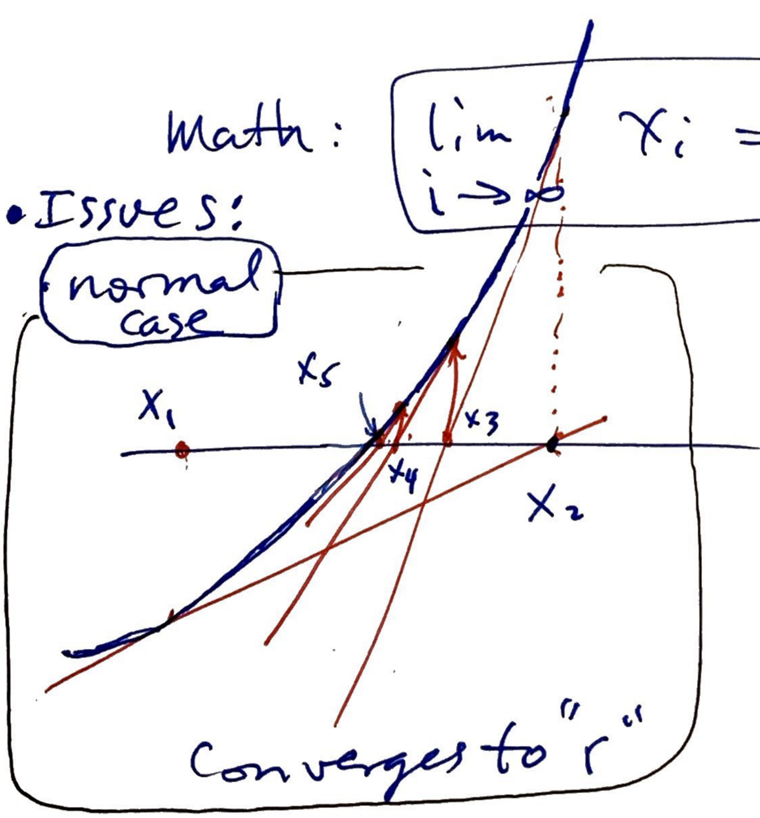
$$\boxed{x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}}$$

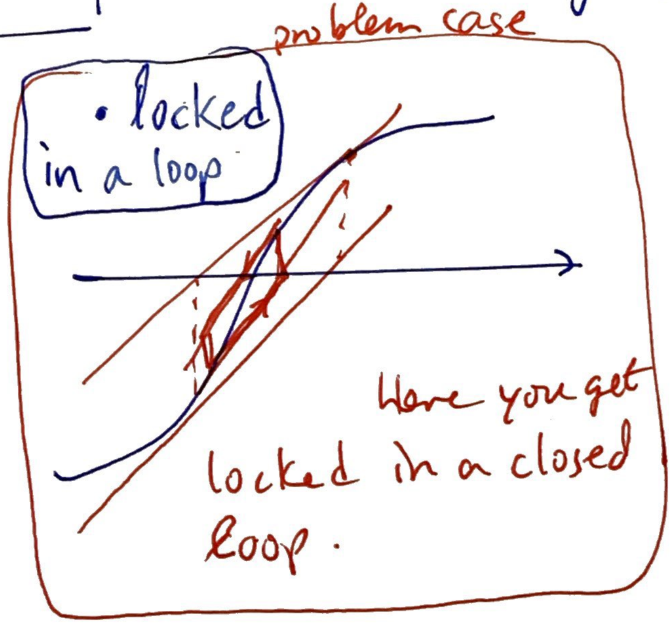
- Flow graph



Math: $\boxed{\lim_{i \to \infty} x_i = r}$ r = root of $g(x)$.

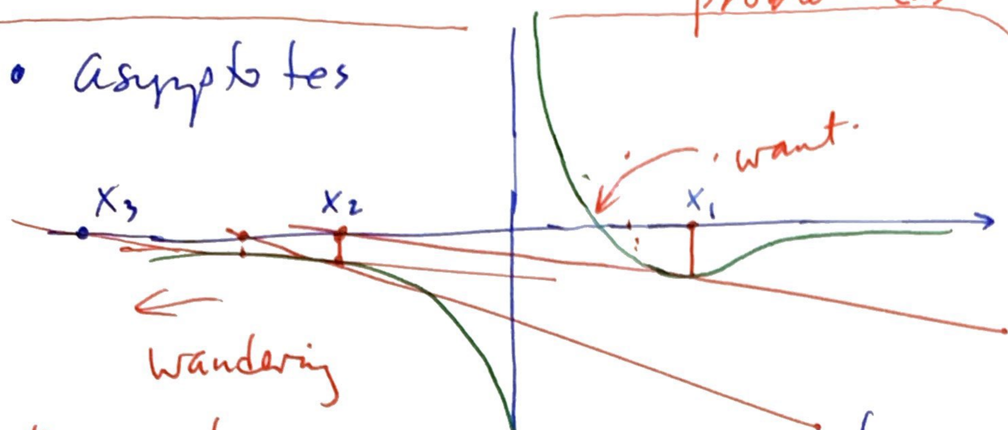
- Issues:





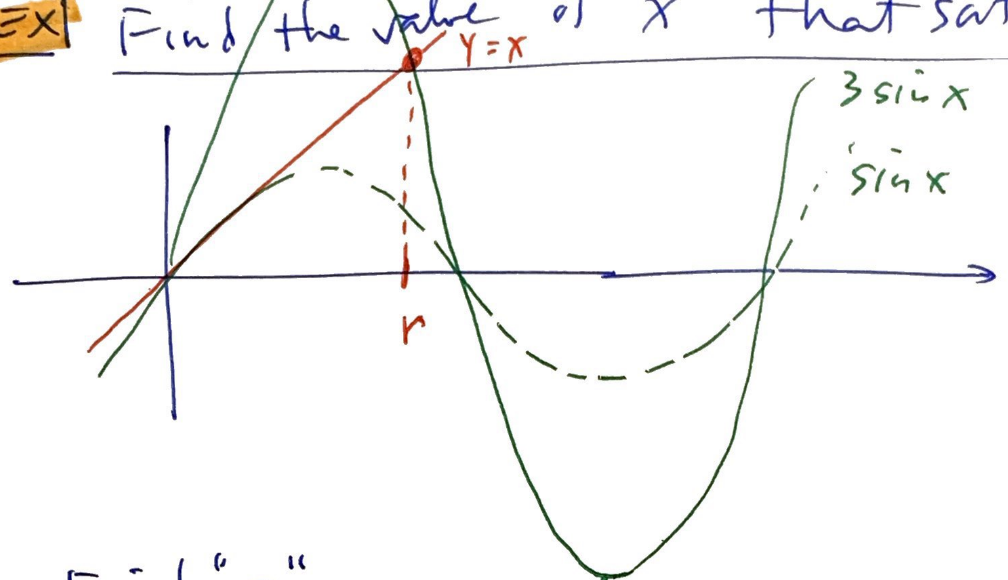
problem case

- asymptotes



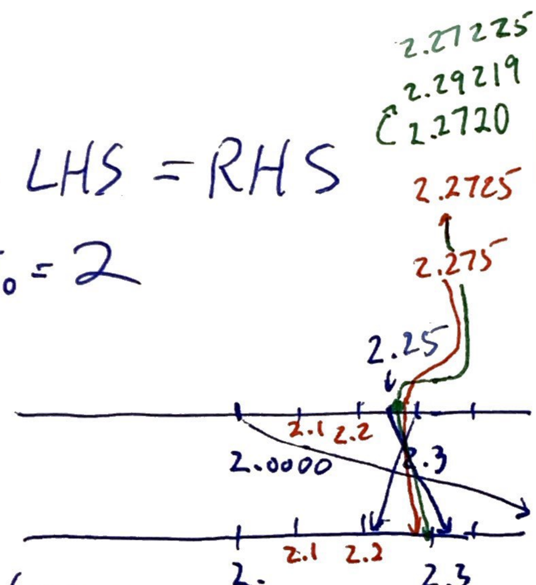
may not converge.. or converges to wrong root.

EX Find the value of x that satisfies $3\sin(x) = x$



Find "r":

- Hunt-n-peck method: make LHS = RHS



$\underline{3\sin(x) = x}$ let $x_0 = 2$

$\rightarrow 3\sin(2) \overset{?}{=} 2$

$2.7279 =$ NO

$\rightarrow 3\sin(2.3) \overset{?}{=} 2.30$

$2.23711 \overset{?}{=} 2.30$ no, but closer

$\rightarrow 3\sin(2. \ldots \overset{?}{=} 2.$

$\overset{?}{=}$ further away

$\rightarrow 3\sin(2.. \ldots) \overset{?}{=} 2.$

2.33

Laborious...

but OK as a last resort

EX

- Newton's method:

let
- $f(x) = 3\sin(x) - x$

$$\boxed{x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}}$$

- $f'(x) = 3\cos(x) - 1$

$$x_2 = x_1 - \frac{3\sin(x_1) - x_1}{3\cos(x_1) - 1}$$

- pick seed $x_1 = 2.0$

$$\rightarrow x_2 = 2.0 - \frac{3\sin(2.0) - 2.0}{3\cos(2.0) - 1}$$

$$x_2 = 2.3237320 61$$

$$\rightarrow x_3 = 2.323732 - \frac{3\sin(2.323732) - 2.323732}{3\cos(2.323732) - 1}$$

$$x_3 = 2.323732 -$$

$$x_3 = 2.279595$$

$$\rightarrow x_4 = 2.279595 - \frac{3\sin(2.279595) - 2.279595}{3\cos(2.279595) - 1}$$

6 digit accy!!

$$x_4 = 2.279595 - 0.000732$$

$$x_4 = 2.278863$$

$$\rightarrow x_5 = 2.278863 - \frac{3\sin(2.278863) - 2.278863}{3\cos(2.278863) - 1}$$

$$x = 2.2788633$$