Calculus I

Calculus is a branch of math that was developed to describe two major functions

(i) Rates of change → Tool: "Derivative"

(ii) Accumulation → Tool: "Integral" or anti-derivative

- For (i)

Chpts 1–3

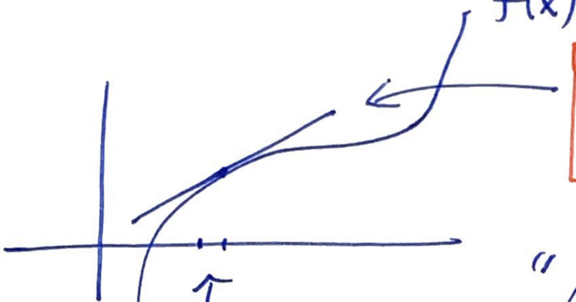
The Derivative

- Calc II

Chpt 7–8

- Calc III

Chpt 14 & 16



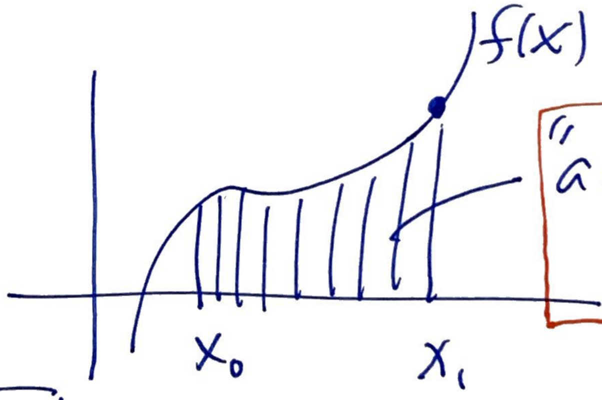
instantaneous rate of change of f(x)

⌊ we move "Δx", how much does "f" change?

- For (ii)

Chpt 4–5

The Integral



"area" or accumulation of f(x) between $x \in [x_0, x_1]$

"element of" — this means $x_0 \leq x \leq x_1$

- Calc II

Chpt 8–9, 10 → O.D.E.

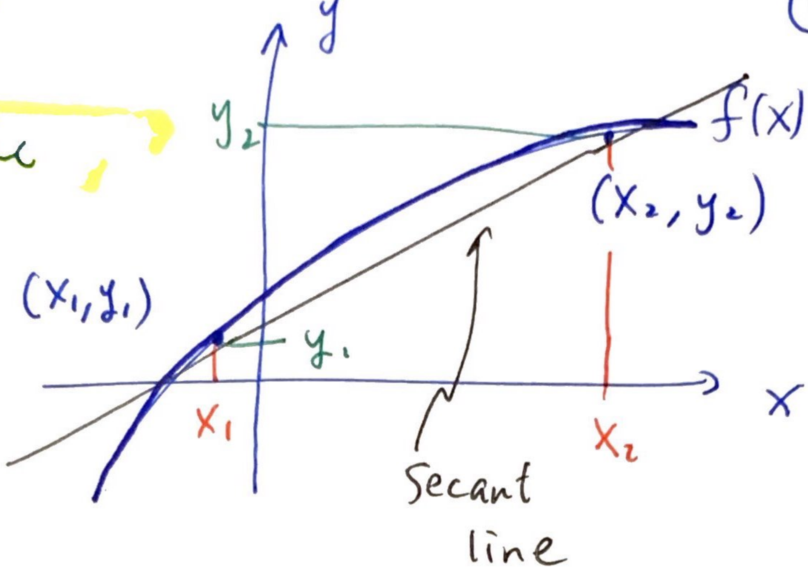
- Calc III

Chpt 15 & 16

The tangent

I tangent slopes

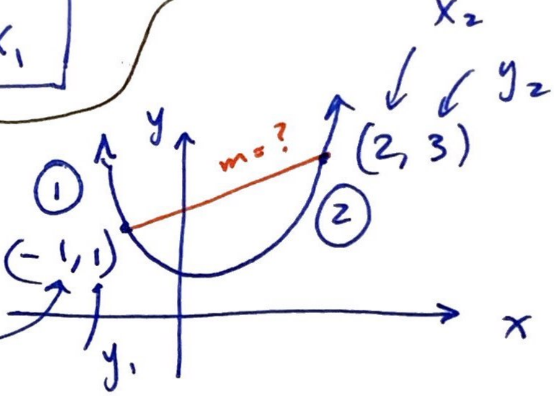
The secant Line



Slope of the secant line $= \dfrac{\text{rise}}{\text{run}} = \dfrac{\Delta y}{\Delta x}$

$$\boxed{m \equiv \frac{y_2 - y_1}{x_2 - x_1}}$$

Ex Find the slope

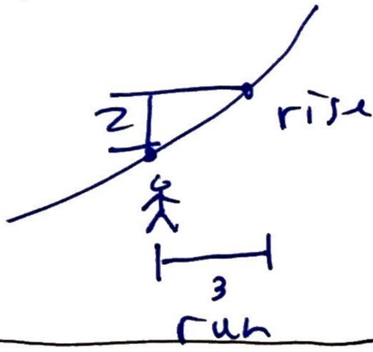


formula: $\dfrac{\Delta y}{\Delta x} = \dfrac{y_2 - y_1}{x_2 - x_1}$

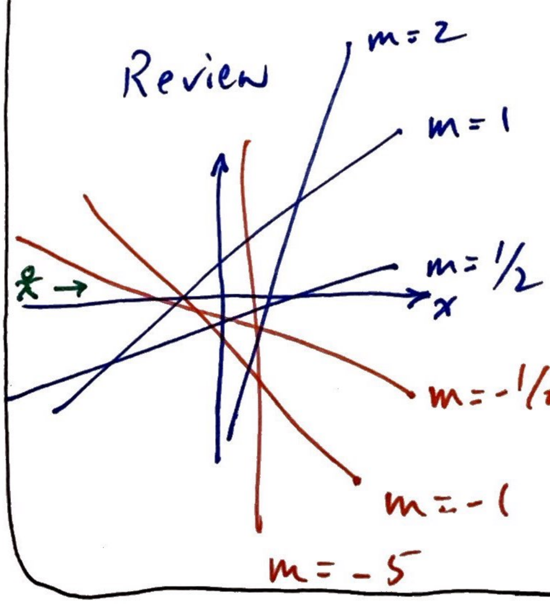
$= \dfrac{3 - 1}{2 - (-1)}$

$= \dfrac{2}{2 + 1}$

$m = \boxed{\dfrac{2}{3}}$



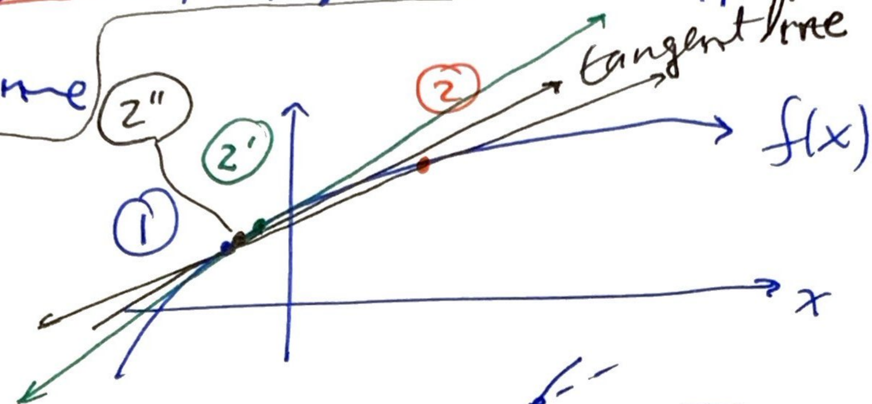
Review



⊛ tangent line

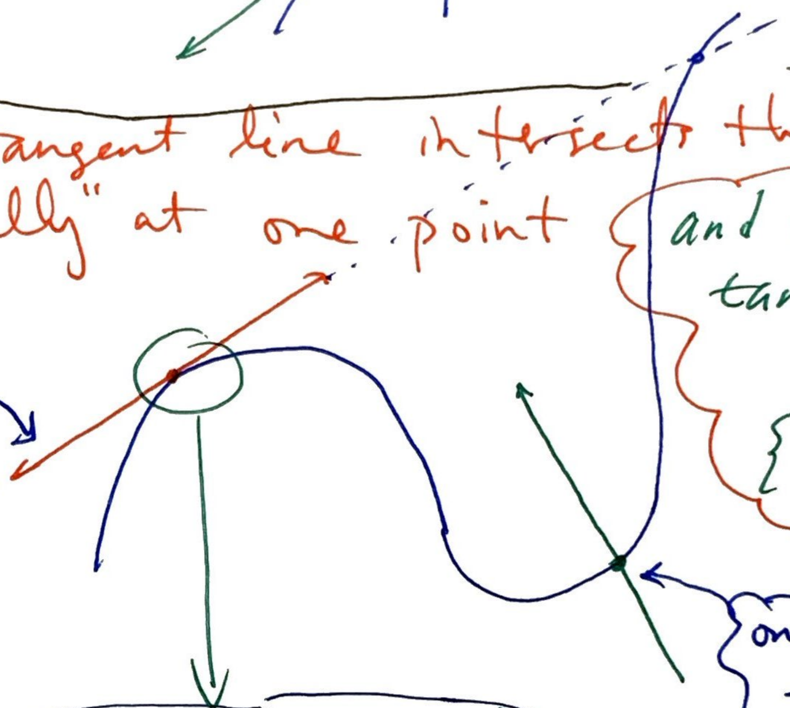
As we take a secant line, which is formed by two points on a curve, and move these two points close together we approach the tangent line

Ex

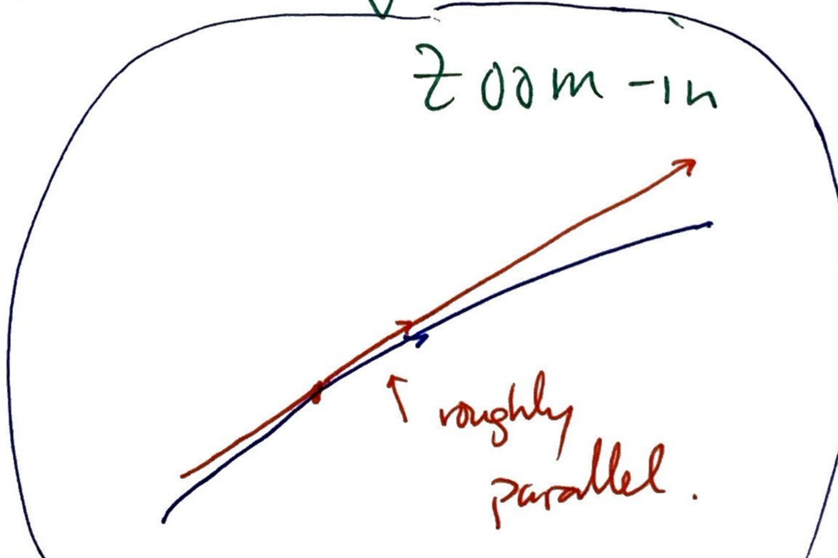


- The tangent line intersects the curve "locally" at one point and is locally tangent to the curve. {parallel}

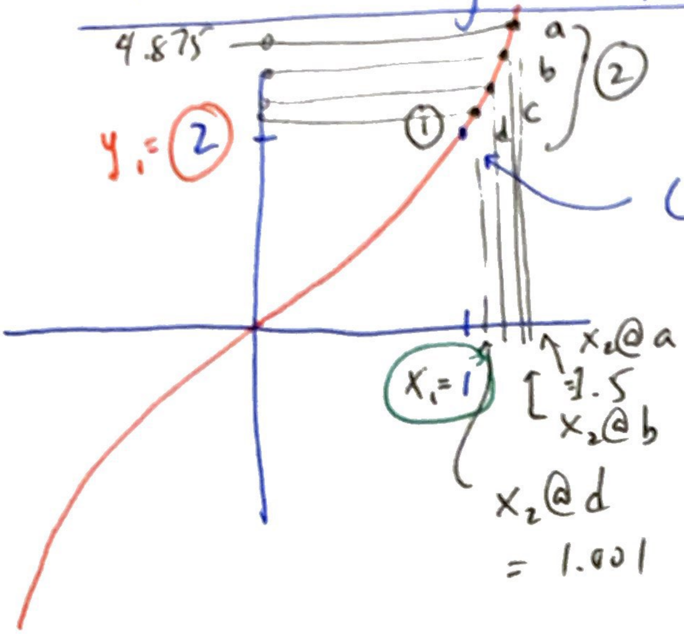
intersects twice but is tangent



once but not tangent



Ex Estimate the slope of the tangent line to curve $f(x) = x^3 + x$ at the point $(x, y) = (1, 2)$ ④



Table

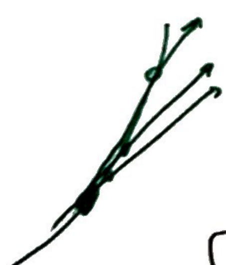
	x_1	x_2	$f = x^3 + x$	y_2
a)	1	1.5	$1.5^3 + 1.5$ =	4.875
b)	1	1.1	$1.1^3 + 1.1$ =	2.43
c)	1	1.01	$1.01^3 + 1.01$	= 2.0403
d)	1	1.001	$1.001^3 + 1.001$	= 2.004

• Secant Slope

$$m_a = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4.875 - 2}{1.5 - 1} = 5.8$$

$$m_b = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2.43 - 2}{1.1 - 1} = 4.3$$

$$m_c = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2.04 - 2}{1.01 - 1} = 4.01$$



$$m_d = \frac{2.004 - 2.}{1.001 - 1.} = \boxed{4.001}$$

$$\boxed{m_{\text{tangent line}} \approx 4.0}$$

(5)

- The slope of the tangent line is the "limit" of the ratio of $\boxed{\frac{\Delta y}{\Delta x} \text{ as } \Delta x \text{ approaches } 0}$.

{ We might think as $\Delta x \to 0$ (approaches) we will be dividing by 0 and so our ratio will be undefined (ie, ∞). However the value in the numerator is also approaching zero. It is the ratio that is the tangent slope }

we write this as

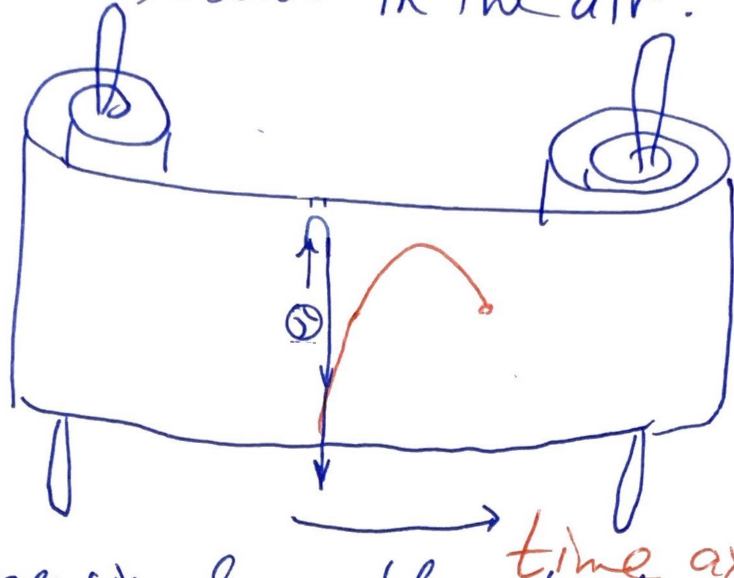
$$\boxed{m_{\text{tangent}} \equiv \lim_{x \to a} \frac{f(x) - f(a)}{x - a}}$$

Δy

Δx

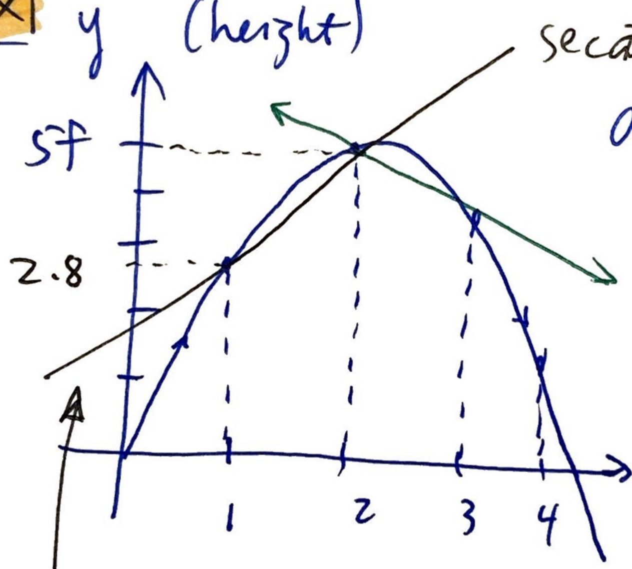
II Application : velocity: ave & instantaneous

Consider tossing a baseball in the air.



This is a one dimensional problem but we represent the path in 2-D (t, y) {vs (x, y)}

Ex



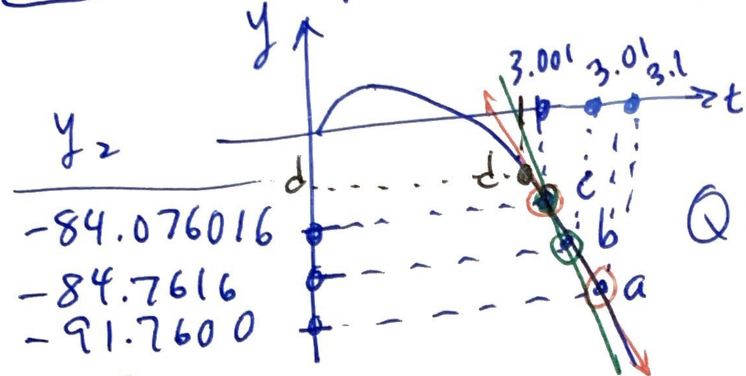
secant line : represents the average speed of the ball between two times.

$$V_{ave} = \frac{\Delta y}{\Delta t} = \frac{5f - 2.8ft}{2_s - 1_s} = \frac{1.2ft}{1s} = \boxed{1.2ft/s}$$

· Instantaneous speed is the limit as $\Delta t \to 0$ for the ratio $\frac{\Delta y}{\Delta t}$.

$$\boxed{V = \lim_{\Delta t \to 0} \left(\frac{\Delta y}{\Delta t} \right)}$$

EX our stopwatch & iphone measure seconds feet



$y_2(t) = 20t - 16t^2$

Q: Find the instantaneous speed at $t = 3s$.

	t	$y_2 = 20t - 16t^2$	$= y_2$
happens last a)	3.1	$= 20(3.1) - 16(3.1)^2$	-91.7600
b)	3.01	$20(3.01) - 16(3.01)^2$	-84.7616
c)	3.001	$20(3.001) - 16(3.001)^2$	-84.076016
comes 1st	3.00	$20(3.000) - 16(3.000)^2$	$= -84.0000$

· ave. speed

· $\boxed{V_{ad}} = \frac{\Delta y}{\Delta t} = \frac{-91.7600 - (-84.0000)}{3.1 - 3.00000}$

$$\boxed{V_{ad} = -77.6000 \text{ m/s}}$$

· $V_{bd} = \frac{\Delta y}{\Delta t} = \frac{-84.7616 - (-84.0000)}{3.01 - 3.00000}$

$$\boxed{V_{bd} = -76.16 \text{ m/s}}$$

- $V_{cd} = \dfrac{-84.076016 - (-84.0000)}{3.001 - 3.0000}$

$$\boxed{V_{cd} = -76.016}$$

Summary : $V: -77.6000 \to -76.16 \to \boxed{-76.016}$

$$\boxed{\lim_{t \to} \frac{\Delta y}{\Delta t} = -76.0 \text{ m/s}}$$

This is the instantaneous speed at $t = 3$.

{ Spoiler alert: $y = 20t - 16t^2$

using 2.1 technology

$y' = 20 - 2 \cdot 16t$

@ $t = 3$: $20 - 32(3)$

$= 20 - 96$

$= \boxed{-76 \text{ ft/s}}$ }