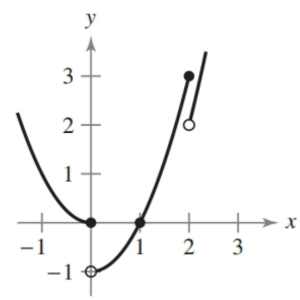
This test is VERY LONG - you probably won't finish. Go for the most points you are capable of achieving.
NO NOTES, CALCULATORS, TEXT BOOKS NOR ANY OTHER ASSISTANCE IS PERMITTED WHILE YOU TAKE THE TEST. Using such is deemed *CHEATING* and and you will be punished!
Show *all* work for FULL credit. Now, show me what you know. . . GO!

1. (5 pts) Use the graph to find the limit (if it exists). If the limit does not exist, explain why.

$$h(x) = \left[\!\left[-\frac{x}{2} \right]\!\right] + x^2$$



(a) $\lim_{x\to 2} h(x)$ (b) $\lim_{x\to 1} h(x)$

(a)

(b)

2. (5pts) Find the limit $\lim_{x\to 0} \frac{[1/(x+1)] - 1}{x}$

3. (5 pts) Complete the table and use the result to estimate the limit (write below the table):

x	-0.1	-0.01	-0.001	0	0.001	0.01	0.1
$f(x)$				?			

$$\lim_{x\to 0} \frac{\sqrt{x+4} - 2}{x}$$

4. (5pts) Find the limit (if it exists).
If it does not exist, explain why.

$$\lim_{x\to 1^+} g(x), \text{ where } g(x) = \begin{cases} \sqrt{1-x}, & x \le 1 \\ x+1, & x > 1 \end{cases}$$

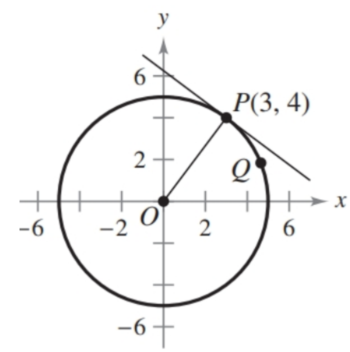
5. (5 pts) Find the value of c such that the function is continuous

$$f(x) = \begin{cases} x+3, & x \le 2 \\ cx+6, & x > 2 \end{cases}$$

6. (5 pts) Use the Intermediate Value Theorem to show that *f(x)* has a zero in the interval [1, 2].

$$f(x) = 2x^3 - 3$$

7. (10 pts) Let P(3, 4) be a point on the circle $x^2 + y^2 = 25$ (see figure).



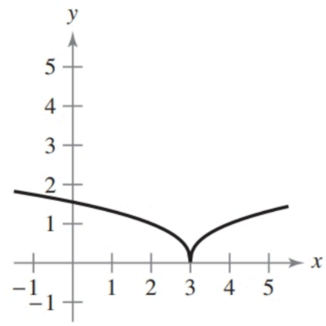
(a) Find an equation of the tangent line to the circle at P.

(b) Verify that these lines are perpindicular.

8. (5 pts) Find the derivative of the function by the definition of the derivative for $f(x) = \dfrac{6}{x}$

9. (5 pts) Describe the x-values at which f is differentiable

$f(x) = (x - 3)^{2/5}$



10. (10 pts) Use the rules of differentiation to find the derivative of the functions below:

(a) $h(x) = 6\sqrt{x} + 3\sqrt[3]{x}$

(b) $y = x\cos x - \sin x$

(c) $f(x) = \dfrac{2x + 7}{x^2 + 4}$

11. (10 pts) Find the second derivative of the function

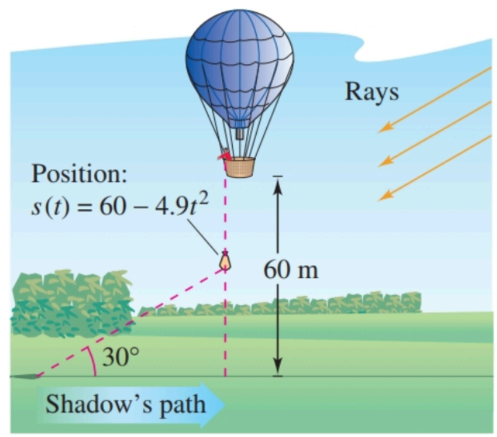
(a) $f(x) = 15x^{5/2}$

(b) $g(x) = 4 \cot x$

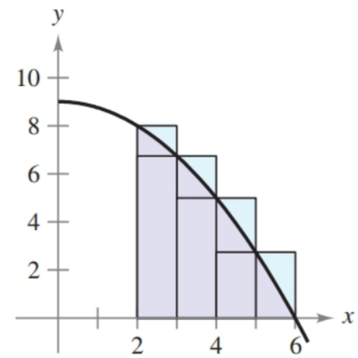
12. (5 pts) Find the derivative of $f(x) = \left(\dfrac{x}{\sqrt{x+5}}\right)^3$

13. (5 pts) Find dy/dx by implicit differentiation $x \sin y = y \cos x$

14. (10 pts) A sandbag is dropped from a balloon at a height of 60 meters when the angle of elevation to the sun is 30° (see figure). The position of the sandbag is $s(t) = 60 - 4.9t^2$. Find the rate at which the shadow of the sandbag is traveling along the ground when the sandbag is at a height of 35 meters.



15. (10 pts) Use upper and lower sums to approximate the area of the region using the given number of subintervals (of equal width.)



Upper:

Lower:

16. (10 pts) Find the indefinite integrals

(a) $$\int \frac{\cos\theta}{\sqrt{1-\sin\theta}}\,d\theta$$

(b) $$\int \sqrt{1+\sqrt{x}}\,dx$$

17. (10 pts) Evaluate the definite integral - show all steps in the evaluation!

(a) $$\int_0^3 \frac{1}{\sqrt{1+x}}\,dx$$

(b) $$2\pi \int_0^1 (y+1)\sqrt{1-y}\,dy$$

18. (5 pts) Find the derivative of $y = \ln\sqrt{\dfrac{x^2+4}{x^2-4}}$

19. (5 pts) Use logarithmic differentiation to find dy/dx for $y = \dfrac{x+2}{\sqrt{3x-2}}, \quad x > \dfrac{2}{3}$

20. (5 pts) Find the derivative of $f(x) = x^x$

21. (5 pts) Find the extrema and points of inflection (if any exist) of the function $f(x) = (x+1)e^{-x}$

22. (10 pts) Find the volume of the solid formed by revolving the region bounded by the graphs of $f(x) = 2 - x^2$ and $g(x) = 1$ about the line $y = 1$

(a) graph the region and rotation axis

(b) draw the disk orientation in the region

(c) circle the integration variable: x or y

(d) what will the radius of the disk be? r = ____________________

(e) set up but DO NOT EVALUATE the volume integral:

V =

____

23. (10 pts) Find the volume of the solid formed by revolving the region bounded by the graphs of $y = x^2 + 1$, $y = 0$, $x = 0$, and $x = 1$ about the y-axis

(a) graph the region and rotation axis

(b) draw the cylinderical shell orientation in the region

(c) circle the integration variable: x or y

(d) what will the radius of the shell be? r = ____________________

(e) set up but DO NOT EVALUATE the volume integral

V =

The Last Pages

Double-Angle Formulas

$\sin 2x = 2\sin x \cos x$

$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$

$\tan 2x = \dfrac{2\tan x}{1 - \tan^2 x}$

Half-Angle Formulas

$\sin^2 x = \dfrac{1 - \cos 2x}{2}$ $\cos^2 x = \dfrac{1 + \cos 2x}{2}$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$$

Mean Value Theorem

Suppose $f(x)$ is a function that satisfies both of the following.

1. $f(x)$ is continuous on the closed interval $[a, b]$.
2. $f(x)$ is differentiable on the open interval (a, b).

Then there is a number c such that $a < c < b$ and

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

Or,

$$f(b) - f(a) = f'(c)(b - a)$$

Definitions

Given a function, $f(x)$, an **anti-derivative** of $f(x)$ is any function $F(x)$ such that

$$F'(x) = f(x)$$

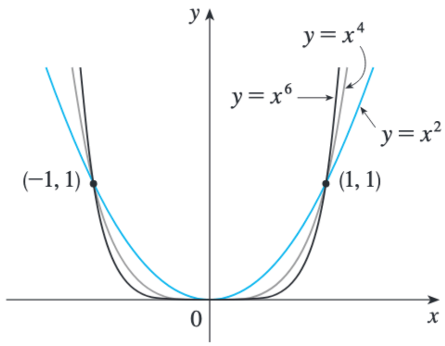
If $F(x)$ is any anti-derivative of $f(x)$ then the most general anti-derivative of $f(x)$ is called an **indefinite integral** and denoted,

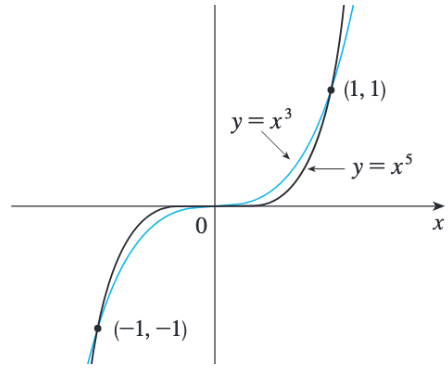
$$\int f(x)\,dx = F(x) + c, \qquad c \text{ is an arbitrary constant}$$

In this definition the $\int$ is called the **integral symbol**, $f(x)$ is called the **integrand**, x is called the **integration variable** and the "c" is called the **constant of integration**.

13. $\dfrac{d}{dx}(\sin x) = \cos x$ **14.** $\dfrac{d}{dx}(\cos x) = -\sin x$ **15.** $\dfrac{d}{dx}(\tan x) = \sec^2 x$

16. $\dfrac{d}{dx}(\csc x) = -\csc x \cot x$ **17.** $\dfrac{d}{dx}(\sec x) = \sec x \tan x$ **18.** $\dfrac{d}{dx}(\cot x) = -\csc^2 x$





$\boxed{10}$ **The Intermediate Value Theorem** Suppose that f is continuous on the closed interval $[a, b]$ and let N be any number between $f(a)$ and $f(b)$, where $f(a) \neq f(b)$. Then there exists a number c in (a, b) such that $f(c) = N$.

The Fundamental Theorem of Calculus Suppose f is continuous on $[a, b]$.

1. If $g(x) = \int_a^x f(t)\,dt$, then $g'(x) = f(x)$.

2. $\int_a^b f(x)\,dx = F(b) - F(a)$, where F is any antiderivative of f, that is, $F' = f$.

8. If $m \leqslant f(x) \leqslant M$ for $a \leqslant x \leqslant b$, then

$$m(b-a) \leqslant \int_a^b f(x)\,dx \leqslant M(b-a)$$

Summary of derivatives of cases with exp & bases

a) $\dfrac{d\,a^b}{dx} = 0$, a & b are constants.

b) $\dfrac{d}{dx}[f(x)]^b = b[f(x)]^{b-1} f'(x)$, b=const.

c) $\dfrac{d}{dx}(a^{g(x)}) = a^{g(x)}(\ln(a))\,g'(x)$, a=const.

d) $\dfrac{d}{dx}[f(x)]^{g(x)} = f(x)^{g(x)}\left[g(x)\cdot\dfrac{f'(x)}{f(x)} + g'(x)\ln f(x)\right]$

$\dfrac{de^x}{dx} = e^x$ $\qquad$ $\dfrac{d\ln(x)}{dx} = \dfrac{1}{x}$

$\dfrac{da^x}{dx} = \ln(a)\,a^x$ $\qquad$ $\dfrac{d\log_a(x)}{dx} = \dfrac{1}{\ln(a)}\cdot\dfrac{1}{x}$

$\int e^x\,dx = e^x + C$ $\qquad$ $\int \frac{1}{x}\,dx = \ln|x| + C$

$\int a^x\,dx = \dfrac{a^x}{\ln(a)} + C$ $\qquad$ $\int \ln(x)\,dx = x\ln(x) - x + C$