math270 SuC "matrixInversion" Roy Erickson

This approach shows how to avoid fractions and how we need not follow the traditional lower/ upper triangular strategy.

1. Invert $\left[\begin{array}{ccc} 1 & 2 & 3 \\ 3 & -1 & -2 \\ 4 & 5 & 6 \end{array}\right]$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 3 & -1 & -2 & 0 & 1 & 0 \\ 4 & 5 & 6 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} *-3\ ;\ *-4 \\ \\ \\ \end{array} \rightarrow \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & -7 & -11 & -3 & 1 & 0 \\ 0 & -3 & -6 & -4 & 0 & 1 \end{array}\right] \begin{array}{l} \\ *3 \\ *-7 \end{array}$$

$$\rightarrow \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & -21 & -33 & -9 & 3 & 0 \\ 0 & 21 & 42 & 28 & 0 & -7 \end{array}\right] + \rightarrow \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 0 & 9 & 19 & 3 & -7 \\ 0 & -3 & -6 & -4 & 0 & 1 \end{array}\right] \begin{array}{l} *3 \\ \\ *3 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 3 & 6 & 9 & 3 & 0 & 0 \\ 0 & 0 & 9 & 19 & 3 & -7 \\ 0 & -9 & -18 & -12 & 0 & 3 \end{array}\right] \begin{array}{l} \\ *-1;\ *2 \\ \\ \end{array} \left[\begin{array}{ccc|ccc} 3 & 6 & 0 & -16 & -3 & 7 \\ 0 & 0 & 9 & 19 & 3 & -7 \\ 0 & -9 & 0 & 26 & 6 & -11 \end{array}\right] \begin{array}{l} *3 \\ \\ \\ \end{array}$$

$$\left[\begin{array}{ccc|ccc} 9 & 18 & 0 & -48 & -9 & 21 \\ 0 & 0 & 9 & 19 & 3 & -7 \\ 0 & -9 & 0 & 26 & 6 & -11 \end{array}\right] \begin{array}{l} \\ \\ *2 \end{array} \rightarrow \left[\begin{array}{ccc|ccc} 9 & 0 & 0 & 4 & 3 & -1 \\ 0 & 0 & 9 & 19 & 3 & -7 \\ 0 & -9 & 0 & 26 & 6 & -11 \end{array}\right] \begin{array}{l} \div 9 \\ \div 9 \\ \div -9 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 4/9 & 3/9 & -1/9 \\ 0 & 0 & 1 & 19/9 & 3/9 & -7/9 \\ 0 & 1 & 0 & -26/9 & -6/9 & 11/9 \end{array}\right] \rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 4/9 & 3/9 & -1/9 \\ 0 & 1 & 0 & -26/9 & -6/9 & 11/9 \\ 0 & 0 & 1 & 19/9 & 3/9 & -7/9 \end{array}\right]$$

The inverse is

$$\boxed{\left[\begin{array}{ccc} 4/9 & 3/9 & -1/9 \\ -26/9 & -6/9 & 11/9 \\ 19/9 & 3/9 & -7/9 \end{array}\right]}$$