Counting Outline

I. Identify the Activities involved.

Example1: Find the number of ways to award one 1st,one 2nd, and three 3rd place ribbons to 12 contestants.
Solution: Three activities: A_1-choosing 1st, A_2-choosing 2nd, A_3-choosing three 3rd

Example2: A student is allowed the privilege of checking out 4 books from either of two presidential libraries. The first library has 12 available books. The second has 7 available books. How many different ways can the selection be made?
Solution: Two activities: A_1-choosing 4 books from Library1, A_2-choosing 4 books from library2.

II.Identify the counting method for each Activity. It will be one of the following:

i) Simple Selection (#): Number of ways to select one thing from a group of n.
ii) Addition (+): One arrangement *or* the other (not both).
iii) Multiplication (*): One arrangement, followed by another (and then another, …).
iv) Permutation (P(n,k)): Number of ways to arrange k things out of n, w/ regard to order.
v) Repetitive elements (n!/r!): Number of ways to arrange n things where one element is repeated r times. Order is important.
vi) Combination (C(n,k)): Number of ways to arrange k things out of n, w/o regard to order.

Example1 (cont): A_1- Simple Selection, 1 out of 12,
A_2- Simple Selection, 1 out of remaining11,
A_3- Combination – Selecting 3 out of remaining10 w/o regard to order i.e. C(10,3)

Example2 (cont): A_1- Combination – Selecting 4 from 12 w/o regard to order i.e. C(12,4).
A_2- Combination – Selecting 4 from 7 w/o regard to order i.e. C(7,4).

III.Identify the interaction between Activities.

Example1 (cont): A_1, A_2 and A_3 all exist at once => multiplication rule:
$A_1 * A_2 * A_3 = 12 * 11 * C(10,3)$

Example2 (cont): A_1 or A_2 => addition rule:
$A_1 + A_2 = C(12,4) + C(8,4)$