

9.8 Determinants and Cramer's Rule

①

A determinant of a matrix A is a scalar #.

• 2×2 : $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

then $\det(A) =$

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = \underline{\underline{a \cdot d - c \cdot b}}$$

↙ main diag
↘ anti-diag

EX

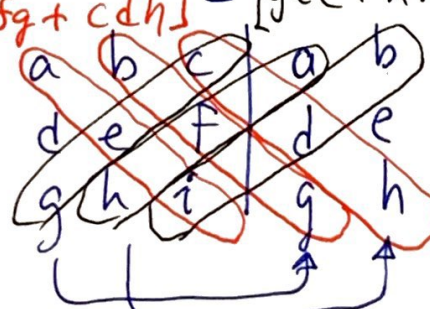
$$\begin{vmatrix} 5 & -6 \\ 2 & 1 \end{vmatrix} = 5 \cdot 1 - 2 \cdot (-6) = 5 + 12 = \underline{\underline{17}}$$

⊗ When we solve a 2×2 system: $ax + by = e$
 $cx + dy = f$

via substitution the answer has the determinant in the denominator.

• 3×3 : $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ then

$$[a \cdot e \cdot i + b \cdot f \cdot g + c \cdot d \cdot h] - [g \cdot e \cdot c + h \cdot f \cdot a + i \cdot d \cdot b] = \det(A)$$

$$\det(A) = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$$


[EX]

Find the determinant of

$$\begin{vmatrix} 5 & 1 & -1 \\ 2 & 3 & 1 \\ 3 & -6 & -3 \end{vmatrix} - \begin{vmatrix} 5 & 1 \\ 2 & 3 \\ 3 & -6 \end{vmatrix}$$

$$= -30 - (-45)$$

$$= -30 + 45$$

$$= \boxed{15}$$

* Cramer's Rule

The solution to a 2×2 system of equations:

$$ax + by = e$$

$$cx + dy = f$$

is, via Cramer's Rule

$$x = \frac{\begin{vmatrix} e & b \\ f & d \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a & e \\ c & f \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}$$

[EX]

Solve $x - 3y = 4$
 $2x + y = 1$

$$x = \frac{\begin{vmatrix} 4 & -3 \\ 1 & 1 \end{vmatrix}}{\begin{vmatrix} 1 & -3 \\ 2 & 1 \end{vmatrix}} = \frac{7}{7} = 1; \quad y = \frac{\begin{vmatrix} 1 & 4 \\ 2 & 1 \end{vmatrix}}{7} = \frac{-7}{7} = -1$$

$$(x, y) = (1, -1)$$

- graph
- substit
- elimin

- Gauss
- Gauss-Jordan
- Inverse

EX

Solve (set up only)

3

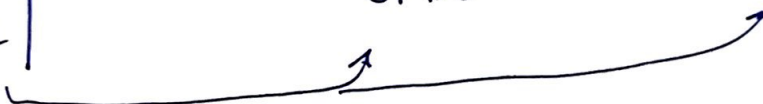
$$-5x + 2y - 4z = -3$$

$$4x - 3y - z = -1$$

$$3x - 3y + 2z = 0$$

$$x = \frac{\begin{vmatrix} -3 & 2 & -4 \\ -1 & -3 & -1 \\ 0 & -3 & 2 \end{vmatrix}}{\begin{vmatrix} -5 & 2 & -4 \\ 4 & -3 & -1 \\ 3 & -3 & 2 \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} -5 & -3 & -4 \\ 4 & -1 & -1 \\ 3 & 0 & 2 \end{vmatrix}}{\begin{vmatrix} -5 & 2 & -4 \\ 4 & -3 & -1 \\ 3 & -3 & 2 \end{vmatrix}}, \quad z = \frac{\begin{vmatrix} -5 & 2 & -3 \\ 4 & -3 & -1 \\ 3 & -3 & 0 \end{vmatrix}}{\begin{vmatrix} -5 & 2 & -4 \\ 4 & -3 & -1 \\ 3 & -3 & 2 \end{vmatrix}}$$

ditto ditto



end of 9.8