$\boxed{9.8}$ Determinants and Cramer's Rule

A determinat of a matrix $\mathbb{A}$ is a scalar #.

• 2X2: $\mathbb{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

main diag

anti-diag.

then $\det(\mathbb{A}) = \boxed{\begin{vmatrix} a & b \\ c & d \end{vmatrix} = \underline{\underline{a \cdot d - c \cdot b}}}$

$\boxed{\text{Ex}}$

$$\begin{vmatrix} 5 & -6 \\ 2 & 1 \end{vmatrix} = 5 \cdot 1 - 2 \cdot (-6) = 5 + 12 = \underline{\underline{17}}$$

(*) when we solve a 2x2 system: $\begin{aligned} ax + by &= e \\ cx + dy &= f \end{aligned}$

via substitution the answer has the determinat in the denominator.

• 3X3: $\mathbb{A} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ then

$[a \cdot e \cdot i + bfg + cdh] - [gec + hfa + idb] = \det(\mathbb{A})$

$$\det(\mathbb{A}) = \left|\begin{array}{ccc|cc} a & b & c & a & b \\ d & e & f & d & e \\ g & h & i & g & h \end{array}\right.$$

EX Find the determinant of

$$(-45+3+12)-(-9-30-6)$$

$$\left|\begin{array}{ccc|cc} 5 & 1 & -1 & 5 & 1 \\ 2 & 3 & 1 & 2 & 3 \\ 3 & -6 & -3 & 3 & -6 \end{array}\right.$$

$$= -30 - (-45)$$
$$= -30 + 45$$
$$= \boxed{15}$$

* Cramer's Rule

The solution to a 2x2 system of equations:

$$ax + by = e$$
$$cx + dy = f$$

is, via Cramer's Rule

$$x = \frac{\begin{vmatrix} e & b \\ f & d \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a & e \\ c & f \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}$$

EX Solve

$$x - 3y = 4$$
$$2x + y = 1$$

$$x = \frac{\begin{vmatrix} 4 & -3 \\ 1 & 1 \end{vmatrix}}{\begin{vmatrix} 1 & -3 \\ 2 & 1 \end{vmatrix}} = \frac{7}{7} = 1; \quad y = \frac{\begin{vmatrix} 1 & 4 \\ 2 & 1 \end{vmatrix}}{7} = \frac{-7}{7} = -1$$

$$\boxed{(x, y) = (1, -1)}$$

- graphin
- substit
- elimin
- Gauss
- Gauss-Jordan
- Inverse

$\boxed{\text{EX}}$ Solve (set up only)

$$
\begin{aligned}
-5x + 2y - 4z &= -3 \\
4x - 3y - z &= -1 \\
3x - 3y + 2z &= 0.
\end{aligned}
$$

$$
x = \frac{\begin{vmatrix} -3 & 2 & -4 \\ -1 & -3 & -1 \\ 0 & -3 & 2 \end{vmatrix}}{\begin{vmatrix} -5 & 2 & -4 \\ 4 & -3 & -1 \\ 3 & -3 & 2 \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} -5 & -3 & -4 \\ 4 & -1 & -1 \\ 3 & 0 & 2 \end{vmatrix}}{\text{ditto}}, \quad z = \frac{\begin{vmatrix} -5 & 2 & -3 \\ 4 & -3 & -1 \\ 3 & -3 & 0 \end{vmatrix}}{\text{ditto}}
$$

end of 9.8