9.5 Matrix Operations

We need to get ready for 9.6 which will require some matrix knowledge.

* A matrix is an array of numbers

EX $A = \begin{bmatrix} 1 & 5 & -6 \\ 0 & 14 & -3 \end{bmatrix}$ 2x3

EX $B = \begin{bmatrix} 1 & 5 \\ 6 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 1 \end{bmatrix}$ 1x1

2x2 ← cols "columns"
↖ rows

* matrix addition: matrices of the same dimension (size) can be add, element by element

EX let $A = \begin{bmatrix} 1 & 5 & -6 \\ 0 & 14 & -3 \end{bmatrix}$ let $B = \begin{bmatrix} 3 & 11 & 0 \\ 5 & 6 & -8 \end{bmatrix}$

So now $A+B = \left[\begin{array}{c|c|c} 1+3 & 5+11 & -6+0 \\ \hline 0+5 & 14+6 & -3-8 \end{array}\right] = \begin{bmatrix} 4 & 16 & -6 \\ 5 & 20 & -11 \end{bmatrix}$

$C = A+B$

* scalar multiplication: when a scalar is multiplied to a matrix, all elements are multiplied by the scalar: $4A$

$$4A = 4\begin{bmatrix} 1 & 5 & -6 \\ 0 & 14 & -3 \end{bmatrix} = \begin{bmatrix} 4\cdot1 & 4\cdot5 & 4\cdot(-6) \\ 4\cdot0 & 4\cdot14 & 4\cdot(-3) \end{bmatrix} = \begin{bmatrix} 4 & 20 & -24 \\ 0 & 56 & -12 \end{bmatrix}$$

* subtraction: $/\!/A - /\!/B \equiv /\!/A + (-1 \cdot /\!/B)$

let $/\!/A = \begin{bmatrix} 1 & -4 \\ 5 & 2 \\ 3 & 0 \end{bmatrix}$, let $/\!/B = \begin{bmatrix} 11 & 0 \\ -10 & 12 \\ 13 & 3 \end{bmatrix}$

then $5/\!/A - 3/\!/B$, for example, would be

$$= 5\begin{bmatrix} 1 & -4 \\ 5 & 2 \\ 3 & 0 \end{bmatrix} - 3\begin{bmatrix} 11 & 0 \\ -10 & 12 \\ 13 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -20 \\ 25 & 10 \\ 15 & 0 \end{bmatrix} + \begin{bmatrix} -33 & 0 \\ 30 & -36 \\ -39 & -9 \end{bmatrix}$$

$$= \begin{bmatrix} 5-33 & -20+0 \\ 25+30 & 10-36 \\ 15+(-39) & 0+(-9) \end{bmatrix} = \boxed{\begin{bmatrix} -28 & -20 \\ 55 & -26 \\ -24 & -9 \end{bmatrix}}$$

* matrix multiplication

EX

$$\begin{bmatrix} 3 & 6 & 4 \\ -8 & 0 & 12 \end{bmatrix} * \begin{bmatrix} 4 & 10 \\ -2 & 6 \\ 5 & 9 \end{bmatrix}$$

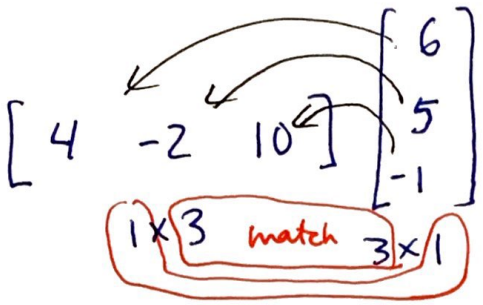
2×3 must match 3×2

size of product

$$= \left[\begin{array}{c|c} 3\cdot 4 + 6(-2) + 4\cdot 5 & 3\cdot 10 + 6\cdot 6 + 4\cdot 9 \\ \hline -8\cdot 4 + 0(-2) + 12\cdot 5 & -8\cdot 10 + 0\cdot 6 + 12\cdot 9 \end{array}\right]$$

$$= \left[\begin{array}{c|c} 20 & 102 \\ \hline 28 & 28 \end{array}\right] = \boxed{\begin{bmatrix} 20 & 102 \\ 28 & 28 \end{bmatrix}}$$

EX



"dot product"

$= 4 \cdot 6 + (-2) \cdot 5 + 10 \cdot (-1) = 24 - 10 - 10 = \underset{1\times 1}{[4]}$

* Powers of matrices

$A^2 \equiv AA$ No-No: we do not square individual elements!!!

* Completes 9.5 *