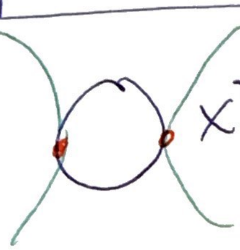
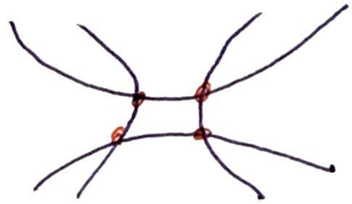
$\boxed{9.3}$ Two-dimensional Non-Linear Systems

exponents are not 1 so these are non-linear equations



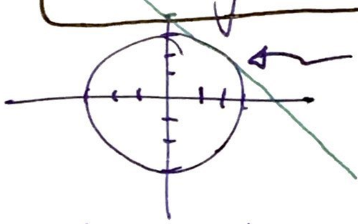
$x^2+y^2=1$, $x^2-y^2=1$



$\circ\circ\circ$

* using $\boxed{\text{substitution}}$ to solve non-lin. eqns.

$\boxed{\text{EX}}$ $\boxed{x^2+y^2=3^2}$ intersects? $\boxed{x+y=4}$



intersects? twice? once?

Lets solve the system using substitution

Advice: { consider substitution when one of the equations is linear }

(i) • Solve the linear eqn for one variable:

$$x = 4-y$$

(ii) • Substitute into the non-linear eqn:

$$(4-y)^2+y^2=3^2$$

(iii) • Solve for the remaining variable:

$$16-8y+y^2+y^2=9$$

$$2y^2 - 8y + 7 = 0$$

- Factor? $(2y \quad)(y \quad) = 0$ No Love...
- Quadratic Formula:

$$y = \frac{-(-8) \pm \sqrt{(-8)^2 - 4 \cdot 2 \cdot 7}}{2 \cdot 2}$$

$$= \frac{8 \pm \sqrt{64-56}}{4} = \frac{8 \pm 2\sqrt{2}}{4} = 2 \pm \frac{\sqrt{2}}{2}$$

(iv) we have two y values $\boxed{\overset{(a)}{y = 2 - \frac{\sqrt{2}}{2}}, \; \overset{(b)}{y = 2 + \frac{\sqrt{2}}{2}}}$

Back substitute

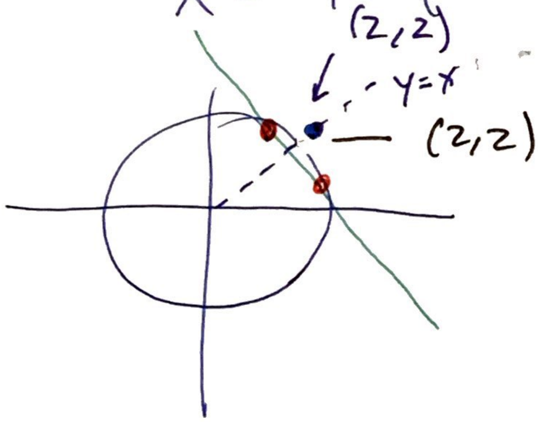
- place this y into the linear to get the corresponding x for each y

$x + y = 4$

$\boxed{x = 4 - y}$

(b) $x = 4 - \left(2 + \frac{\sqrt{2}}{2}\right) = 2 - \frac{\sqrt{2}}{2}$: $\boxed{\left(2 - \frac{\sqrt{2}}{2}, 2 + \frac{\sqrt{2}}{2}\right)}$

(a) $x = 4 - \left(2 - \frac{\sqrt{2}}{2}\right) = 2 + \frac{\sqrt{2}}{2}$ $\boxed{\left(2 + \frac{\sqrt{2}}{2}, 2 - \frac{\sqrt{2}}{2}\right)}$



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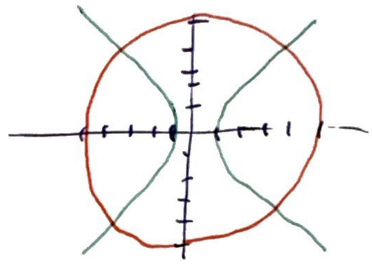
$\boxed{\text{Elimination}}$ If both eqns have x^2 & y^2 terms, try to eliminate by adding or subtracting

$\boxed{EX}$ $\begin{cases} x^2+y^2=25 \\ x^2-y^2=1 \end{cases}$



anticipate 4 solutions

(i) +

$2x^2 = 26$

$x^2 = 13$

$x = \pm\sqrt{13}$

(ii) plug each into any of the eqns : try $x^2-y^2=1$

$\boxed{x=\sqrt{13}}$

$x^2-y^2=1$

$(\sqrt{13})^2-y^2=1$

$13-y^2=1$

$y^2=12$

$\underline{y=\pm\sqrt{12}}$

$\boxed{(\sqrt{13},\sqrt{12})}$ QI

$\boxed{(\sqrt{13},-\sqrt{12})}$ QIV

$\boxed{x=-\sqrt{13}}$

$x^2-y^2=1$

$(-\sqrt{13})^2-y^2=1$

$13-y^2=1$

$y=\pm\sqrt{12}$

$\boxed{(-\sqrt{13},\sqrt{12})}$ QII

$\boxed{(-\sqrt{13},-\sqrt{12})}$ QIII

* A version of substitution

$\boxed{\text{EX}}$ $\begin{cases} 16x^2 - 9y^2 + 144 = 0 \\ x^2 + y^2 = 16 \end{cases}$

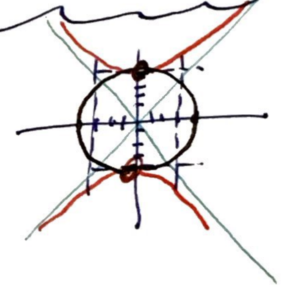
$\div 144$: $\dfrac{16x^2}{144} - \dfrac{9}{144}y^2 = -1$

$\dfrac{x^2}{\frac{144}{16}} - \dfrac{y^2}{\frac{144}{9}} = -1$

$\dfrac{x^2}{9} - \dfrac{y^2}{16} = -1$

$\dfrac{1}{\frac{a}{b}} = \dfrac{b}{a}$

$\dfrac{y^2}{4^2} - \dfrac{x^2}{3^2} = 1$



(i) Solve the 2^{nd} eqn for x^2:

$$x^2 = 16 - y^2$$

(ii) Substitute into the other:

$$16(16 - y^2) - 9y^2 + 144 = 0$$

(iii) Solve

$$256 - 16y^2 - 9y^2 + 144 = 0$$

$$-25y^2 = -144 - 256$$

$$y^2 = 400/25$$

$$y^2 = 16$$

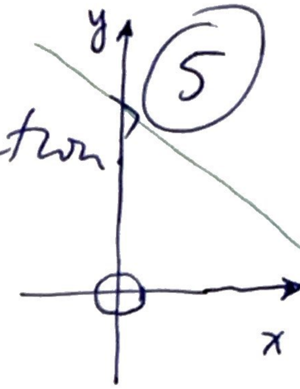
$$\boxed{y = \pm 4}$$

(iv) ~~Back substitute into either eqn~~. { use the bottom eqn }

Use the original substitution form...

$\boxed{y=+4}$	$\boxed{y=-4}$
$x^2 = 16 - (+4)^2$	$x^2 = 16 - (-4)^2$
$x^2 = 0$	$x = 0$
$x = 0$	$\boxed{(0,-4)}$
$\boxed{(0,4)}$	

* What happens if there is no intersection?



$\boxed{\text{EX}}$ $x^2+y^2=1$, $\boxed{y=-x+10}$

(i) substitution

$$x^2+(10-x)^2=1$$

(ii) solve

$$x^2+100-20x+x^2=1$$

$$2x^2-20x+99=0$$

$\sqrt{b^2-4ac}$

$$x=\frac{-(-20)\pm\sqrt{(-20)^2-4\cdot 2\cdot 99}}{2\cdot 2}$$

$$x=\frac{20\pm\sqrt{400-792}}{4}$$

non-real #'s

$\boxed{\text{No Real Solutions}}$

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* Inequalities

$\boxed{\text{EX}}$ Graph $x^2+y^2<4$

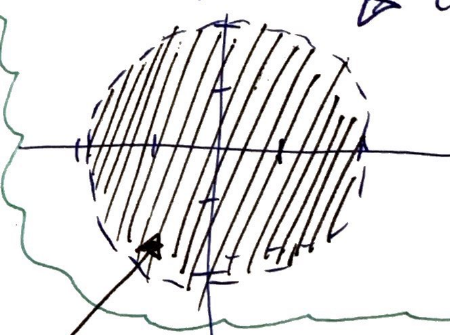
cannot include the boundary

(i) boundary is (open):

$x^2+y^2=4$

or $x^2+y^2=2^2$

circle radius 2



(ii) Shade inside or outside?

Test Point: $(0,0)$

$0^2+0^2 \overset{?}{<} 4$

$\boxed{\text{yes}}$

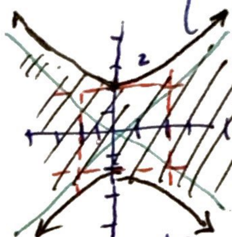
(iii) so shade inside

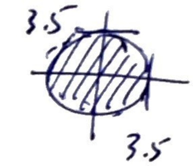
* Now apply to a system of non-linear inequalities:

$\boxed{\text{EX}}$ #44

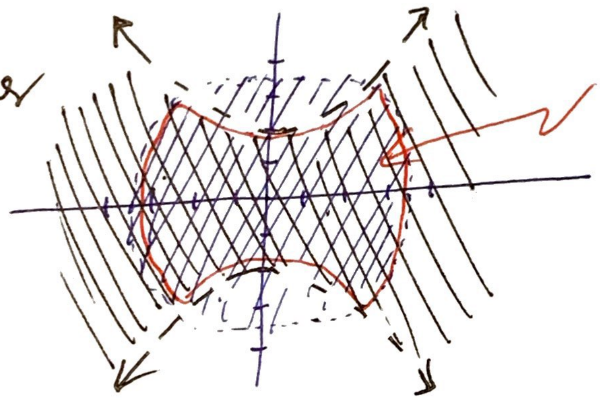
$$x^2 - y^2 > -4 \quad * -1 \Rightarrow \begin{cases} y^2 - x^2 < 4 & \div 4 \\ x^2 + y^2 < 12 \end{cases}$$
$$x^2 + y^2 < 12$$

$$\Rightarrow \begin{cases} \dfrac{y^2}{2^2} - \dfrac{x^2}{2^2} < 1 \Rightarrow \\ x^2 + y^2 < \left(\sqrt{12}\right)^2 \rightarrow \text{radius} = \sqrt{12} \approx 3.5 \end{cases}$$



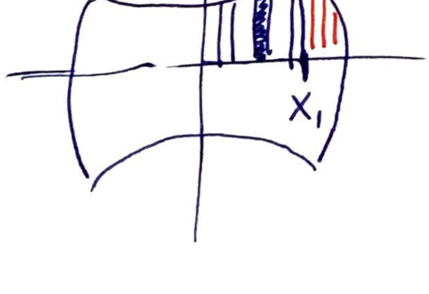


Together



solution "space"
Any (x, y) in this region satisfies BOTH inequalities!

* We need both the intersection & regions to solve for area in calculus.



$$A = \int_{x=0}^{x=x_1} \sqrt{4+x^2}\, dx + \int_{x=x_1}^{x=\sqrt{6}} \sqrt{6-x^2}\, dx \quad \times 4$$

all four sections