$\boxed{8.6}$ Parametric Equations

Some curves are not functions but we still need to describe them. This is the job of a parametric equation:

$\boxed{\text{Ex}}$ $y = 3x + 4$

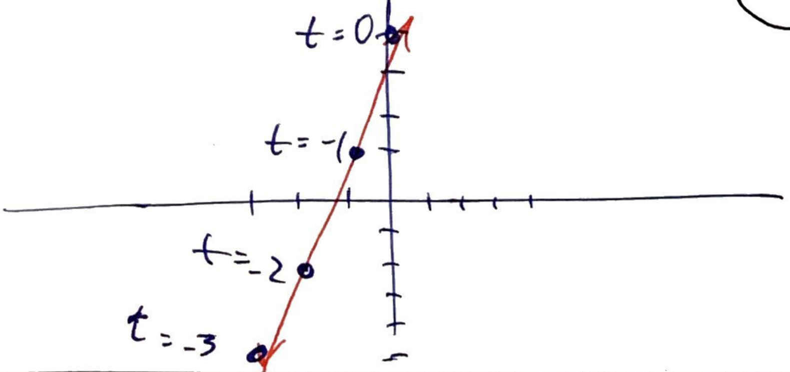
let $x = t$ then $y = 3t + 4$ and the order pair of any point on this line can be described as

$$\underbrace{(x, y) = (t, 3t+4)}_{\text{parmetric form of a line}}$$

Z	zee
Ƶ	two
O	zero
Ø	ohh

* graphing a parametric line.

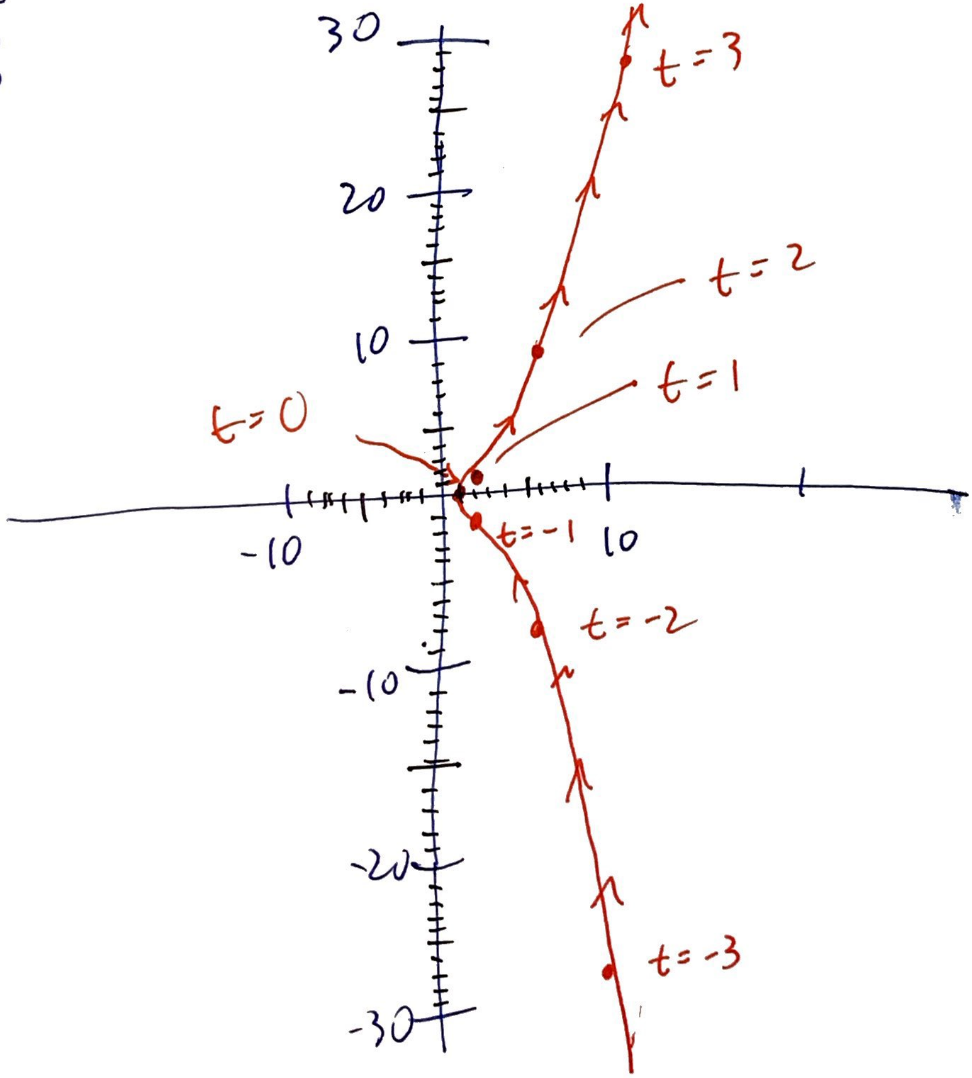
t	$x(t) = t$	$y(t) = 3t+4$	(x,y)
-3	-3	$3(-3)+4 = -5$	$(-3,-5)$ ✓
-2	-2	$3(-2)+4 = -2$	$(-2,-2)$
-1	-1	$3(-1)+4 = 1$	$(-1,1)$
0	0	4	$(0,4)$



$\boxed{EX}$

$$\begin{cases} x(t) = t^2 + 1 \\ y(t) = t^3 \end{cases}$$

t	$x = t^2+1$	$y = t^3$	(x, y)
-3	$(-3)^2+1=10$	-27	$(10, -27)$
-2	$(-2)^2+1=5$	-8	$(5, -8)$
-1	$(-1)^2+1=2$	-1	$(2, -1)$
0	1	0	$(1, 0)$
1	$1^2+1=2$	1	$(2, 1)$
2	$4+1=5$	8	$(5, 8)$
3	$9+1=10$	27	$(10, 27)$
⋮			



8.6 cont.

* Converting parametric eqns to cartesian eqns:

EX $x(t) = 2t+1$ → solve one for the parameter and substitute it into the other

$y(t) = 3\sqrt{t}$

let solve $x = 2t+1$ for "t": $t = \frac{x-1}{2}$

put into $y(t) = 3\sqrt{t}$

$$\boxed{y = 3\sqrt{\frac{x-1}{2}}}$$

EX $x(t) = e^{2t}$

$y(t) = e^{6t}$

"observation" y is x^3 $\boxed{y = x^3}$

$x = e^{2t}$

$\ln(\quad)$

$\ln x = \ln e^{2t}$

See: $y(t) = (\overbrace{e^{2t}}^{x})^3$

Formally: $\ln x = 2t\,\overset{1}{\cancel{\ln(e)}}$ → $t = \frac{1}{2}\ln(x)$

now $y = e^{6[\frac{1}{2}\ln(x)]}$

$y = e^{3\ln(x)}$

$y = e^{\ln x^3}$

$$\boxed{y = x^3}$$

EX
#22

$$\left\{\begin{array}{l} x(t) = \cos(t)+4 \\ y(t) = 2\sin^2 t \end{array}\right\}$$ Convrt to cartesian

$y = f(x)$

I notice $\sin^2(t)$ so I can write it as $\underline{\underline{1-\cos^2(t)}}$

solve for cosine instead of "t"

$$\begin{cases} X = \cos(t)+4 \longrightarrow \cos(t) = X-4 \\ y = 2[1-\cos^2(t)] = 2[1-(X-4)^2] \end{cases}$$

$$\Rightarrow \quad \boxed{y = -2(X-4)^2+2}$$

* go in reverse : lets convert a cartesian eqn into a parameterized eqn:

EX
#30

$\underline{\underline{y = 3x^2+3}} \xrightarrow{\text{easiest}} \boxed{\begin{array}{l} X(t) = t \\ y(t) = 3t^2+3 \end{array}}$

- but there are other choice $\begin{cases} x^2 = t \\ y = 3t+3 \end{cases}$

$\boxed{\begin{array}{l} X(t) = \sqrt{t} \\ y(t) = 3t+3 \end{array}}$ for $t \geq 0$ $\quad \boxed{\begin{array}{l} X(t) = -\sqrt{|t|} \\ y(t) = 3t+3 \end{array}}$ for $t<0$

EX
#32

$x = 3\log y + y$ let $\boxed{\begin{array}{l} y = t \\ X = 3\log(t)+t \end{array}}$

8.7 Plotting Parameterized Eqns

we saw how to build a table in 8.6

EX graph $\begin{cases} x(t) = 5 - |t| \\ y(t) = t - 2 \end{cases}$

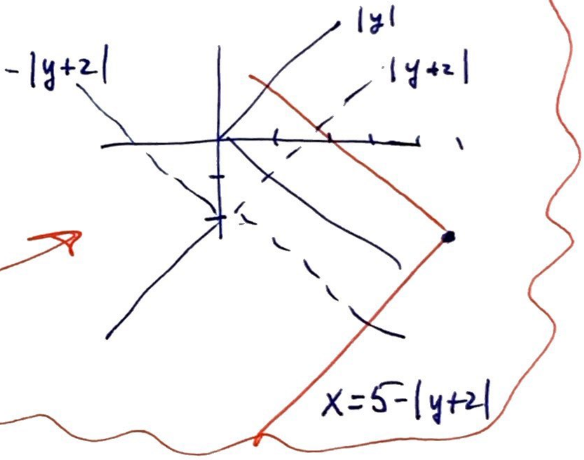
* approach #1 : eliminate "t" : • Solve $y = t - 2$ for t : $t = y + 2$

• insert this into x

$$\Rightarrow \boxed{x = 5 - |y+2|}$$

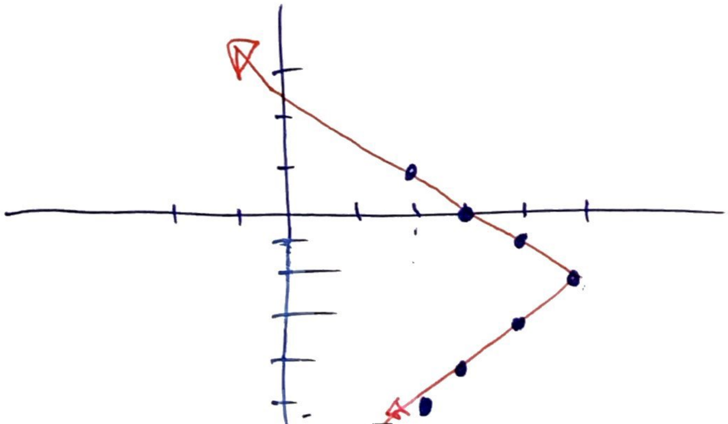
$x = f(y)$

"side ways"



Lets build a table

t	$x = 5 - \lvert t \rvert =$	$y = t - 2$	(x, y)
-3	$5 - \lvert -3 \rvert = 2$	$= -3 - 2 = -5$	$(2, -5)$ ✓
-2	$5 - \lvert -2 \rvert = 3$	$= -2 - 2 = -4$	$(3, -4)$ ✓
-1	$5 - \lvert -1 \rvert = 4$	$= -1 - 2 = -3$	$(4, -3)$ ✓
0	5	-2	$(5, -2)$ ✓
1	$5 - \lvert 1 \rvert = 4$	$1 - 2 = -1$	$(4, -1)$
2	$5 - \lvert 2 \rvert = 3$	$2 - 2 = 0$	$(3, 0)$
3	$5 - \lvert 3 \rvert = 2$	$3 - 2 = 1$	$(2, 1)$



For easy problems we build the table.
But for more involved problems we use an online tool desmos.com or fooplot: some computer platforms have tools also {mac's have "grapher" built in}

* desmos.com

To plot $\begin{cases} x(t) = e^{2t} \\ y(t) = \cos(t) \end{cases}$

- start a new entry: "+" but instead of writting $y =$ some function of x we do...
- Open parenthesis $($ x(t) stuff, y(t) stuff $)$ close paren.

ex: $(e^{2t}, \cos(t))$

- set the limits for "t" down below the ordered pair

ex: $\underline{-10} \leq t \leq \underline{10}$

* play around with diffrent values, have fun