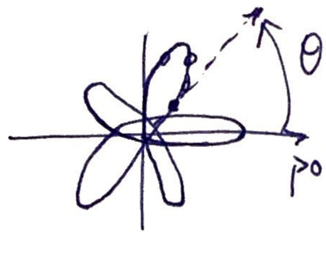
$\boxed{8.4}$ Polar Graphs

In calculus we need to calculate both the arc length and area enclosed by polar graphs.

$\boxed{\text{EX}}$



Q: Find the arc length?
Q: Find the area enclosed?
Q: Find the slope of the tangent line to the graph @ $\theta = \pi/8$

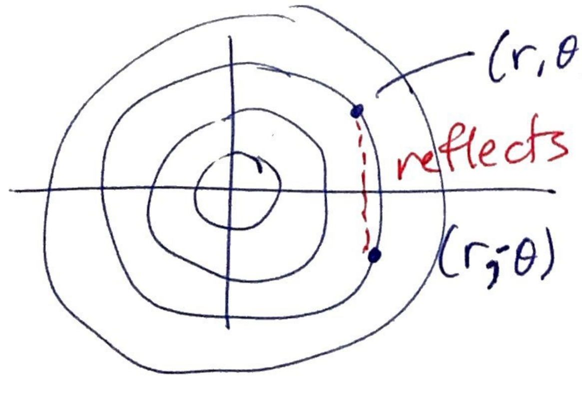
CALC II

* Symmetry of polar graphs

$\boxed{\text{I}}$ Symmetry about the x-axis (aka the polar axis)

Test: replace (r, θ) with $(r, -\theta)$ and check to see if you can recover the function

- if so we have symmetry
- if not we don't have symmetry



$\boxed{\text{EX}}$ Q: is $r = 2\sin\theta$ sym about x-axis?

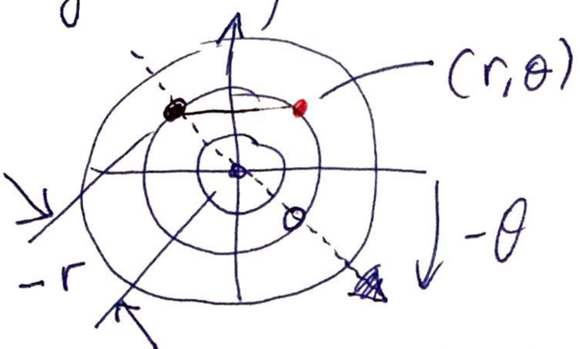
$r = 2\sin\theta$

Test: $r \stackrel{?}{=} 2\sin(-\theta)$

$r = -2\sin(\theta)$

No Sym about x-axis.

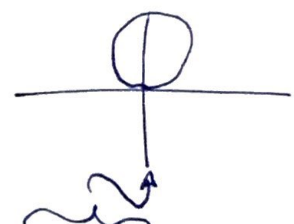
II Symmetry about the y-axis (aka the line $\theta = \pi/2$)



To check that there is a point on the opposite side of the $\theta = \pi/2$

$\boxed{\text{replace } \theta \text{ with } -\theta \text{ and replace } r \text{ with } -r.}$

EX $r = 2\sin\theta$ original



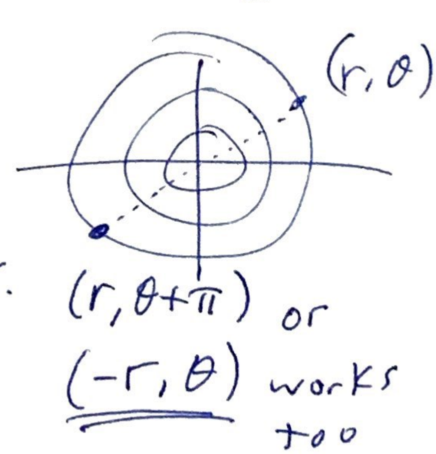
Test $-r \stackrel{?}{=} 2\sin(-\theta)$) identity

$-r \stackrel{?}{=} -2\sin(\theta)$) $* -1$

$r = 2\sin(\theta)$ ✓

So $r = 2\sin\theta$ has sym about the $\theta = \frac{\pi}{2}$ line.

III Symmetry about the origin



- replace r with $-r$ and check that the eqn works out ok.

EX $r^2 = a^2 \sin 2\theta$

Test: $(-r)^2 = a^2 \sin 2\theta$) perform $(\ )^2$

$r^2 = a^2 \sin 2\theta$

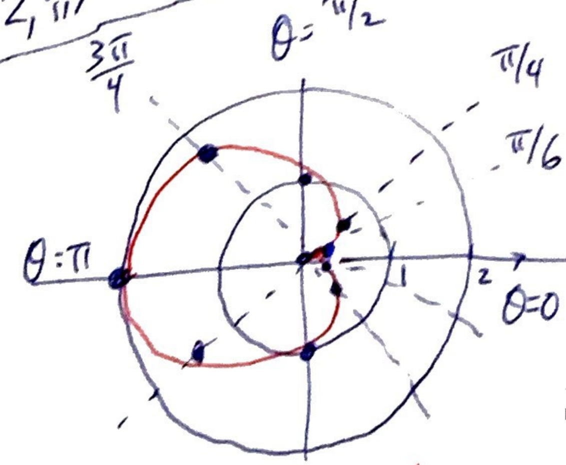
We recover the original eqn

* Methods of plotting polar graphs

$r(\theta) = f(\theta)$, $\boxed{ex}$ $r(\theta) = 1 - \cos(\theta)$

Table:

θ	$r = 1 - \cos\theta$	(r, θ)
0	$1 - \cos 0 = 0$	$(0, 0)$
$\pi/6$	$1 - \frac{\sqrt{3}}{2} = 0.15$	$(0.15, \frac{\pi}{6})$
$\pi/4$	$1 - \frac{\sqrt{2}}{2} = 0.3$	$(0.3, \pi/4)$
$\pi/2$	$1 - 0 = 1$	$(1, \pi/2)$
$\frac{3\pi}{4}$	$1 - (-\frac{\sqrt{2}}{2}) = 1.7$	$(1.7, 3\pi/4)$
π	$1 - (-1) = 2$	$(2, \pi)$



Q: x-axis ($\theta = 0$) symmetry?

replace $r(\theta)$ w/ $r(-\theta)$

original: $r(\theta) = 1 - \cos(\theta)$

test: $r(-\theta) = 1 - \cos(-\theta)$

$r(-\theta) = 1 - \cos\theta$

$r(-\theta) = r(\theta)$ ✓

* Test for zeros ($r = 0$): $0 = 1 - \cos(\theta)$

$\Rightarrow 1 = \cos(\theta)$

$\Rightarrow \underline{\theta = 0, 2\pi}$

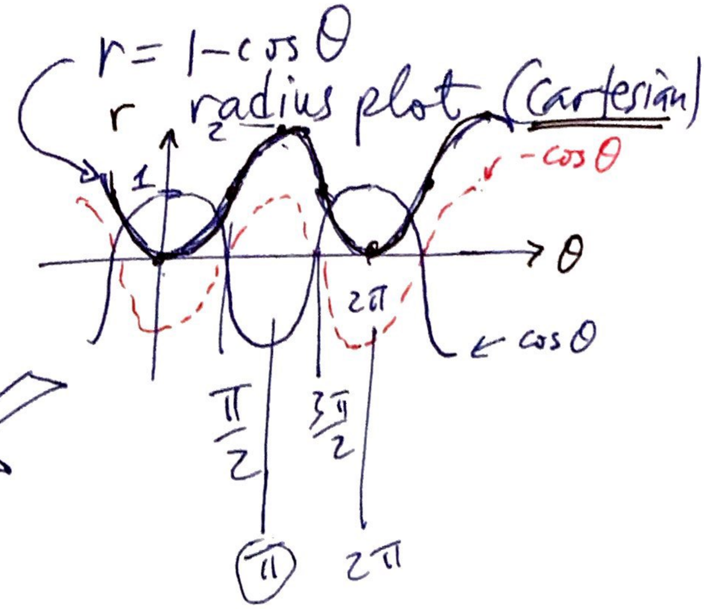
* Test for extrema $\left(\begin{array}{l} \cos = 1 \text{ or } -1 \\ \sin = 1 \text{ or } -1 \end{array} \right)$

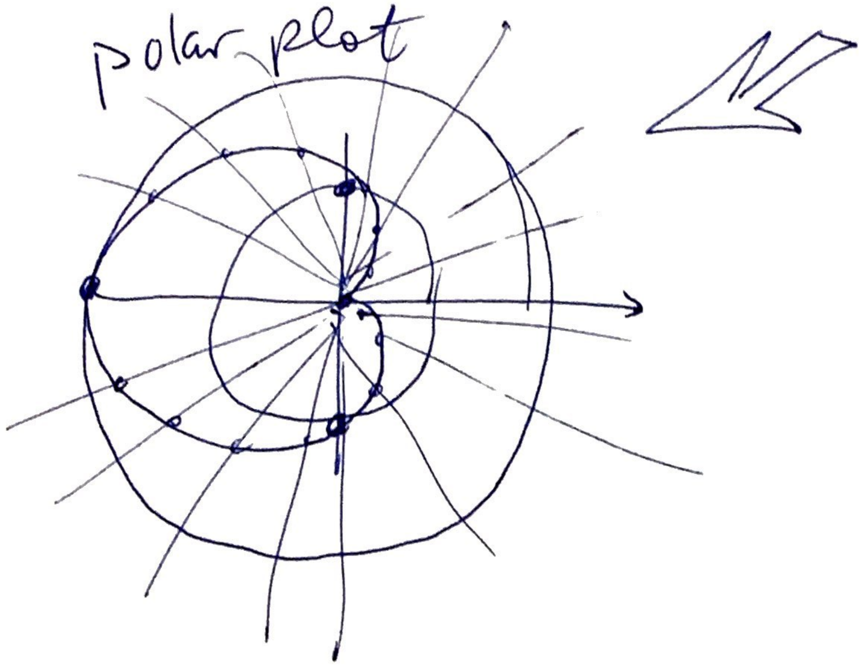
	$1 - \cos\theta$
$\cos\theta = 1$ @ $\theta = 0, 2\pi$ so	$1 - 1 = 0$
$\cos\theta = -1$ @ $\theta = \pi$	$1 - (-1) = 2$

* methods (cont.)

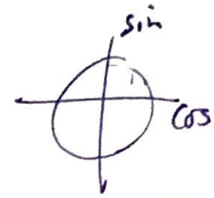
- radius plot

  $r(\theta) = 1 - \cos\theta$



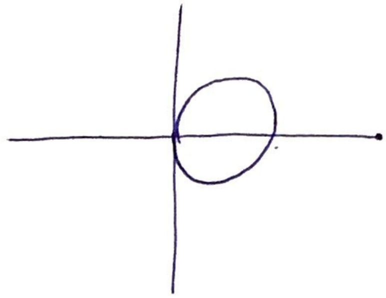


* Popular Polar Graphs

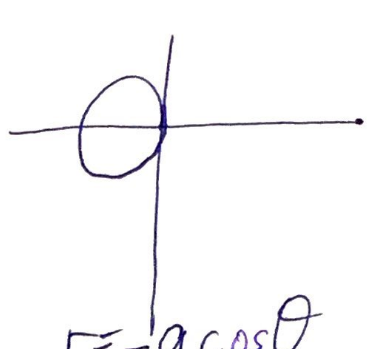


Trig Circle

- Circles

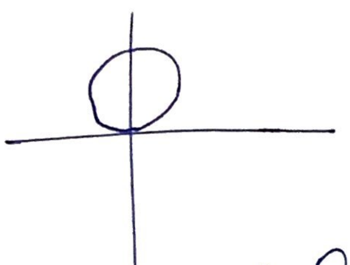


$r = a\cos\theta$

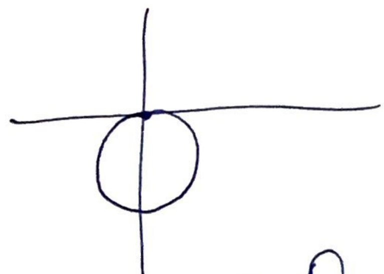


$r = -a\cos\theta$

- sym about pole axis (x-axis)



$r = a\sin\theta$



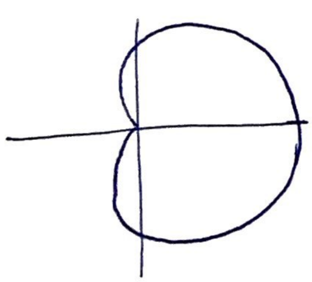
$r = -a\sin\theta$

- sym about $\theta = \frac{\pi}{2}$ (y-axis)

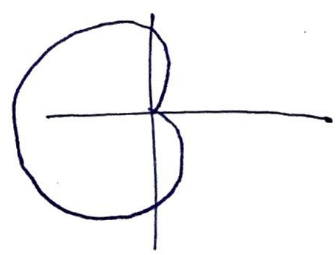
- Cardiods

$\boxed{r(\theta) = a \pm b\begin{Bmatrix}\sin\theta \\ \cos\theta\end{Bmatrix}}$

(a=b for cardiods)

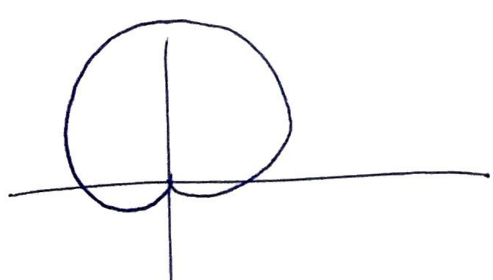


$r = 1 + \cos\theta$

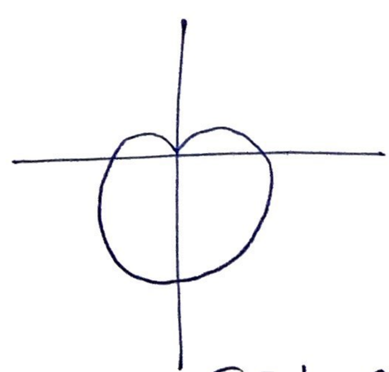


$r = 1 - \cos\theta$

- sym about the polar axis



$r = 1 + \sin\theta$



$r = 1 - \sin\theta$

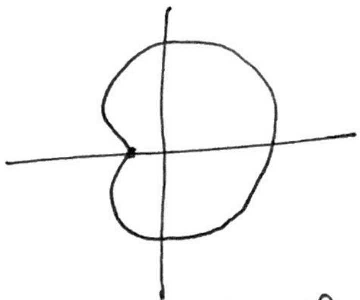
- Sym about $\theta = \pi/2$

variations $r = 2 + 2\sin\theta$, $3 + 3\sin\theta$

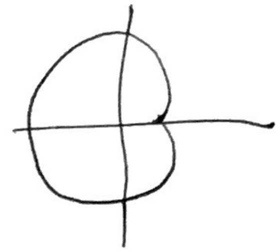
- Limaçon (snail) (Dimpled limacon)

$$r(\theta) = a \pm b \begin{Bmatrix} \cos(\theta) \\ \sin(\theta) \end{Bmatrix}$$

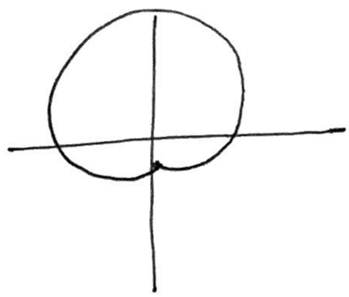
$\boxed{a > b}$



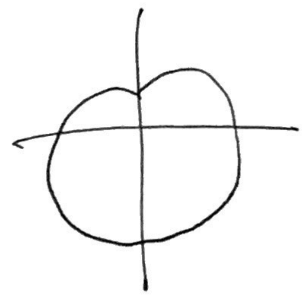
$r = a + b\cos\theta$



$r = a - b\cos\theta$



$r = a + b\sin\theta$

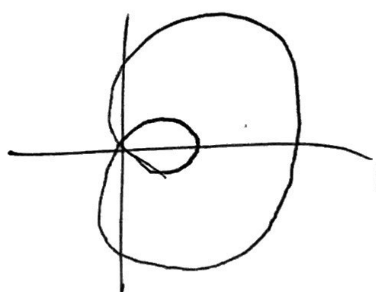


$r = a - b\sin\theta$

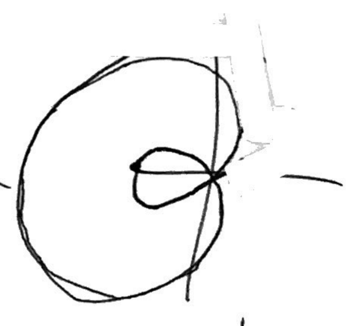
- Inner Loop Limacon

$$r(\theta) = a \pm b \begin{Bmatrix} \cos(\theta) \\ \sin(\theta) \end{Bmatrix}$$

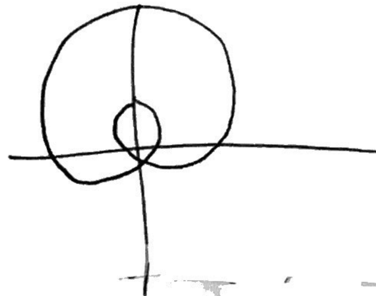
$\boxed{a < b}$



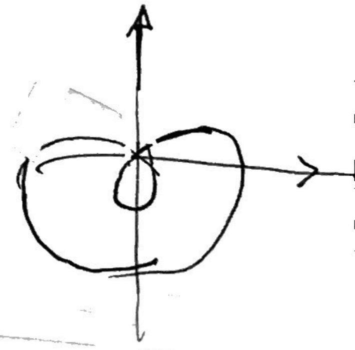
$r = a + b\cos\theta$



$r = a - b\cos\theta$



$r = a + b\sin\theta$

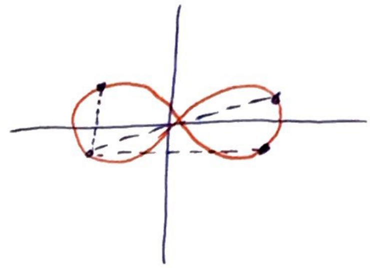


$r = a - b\sin\theta$

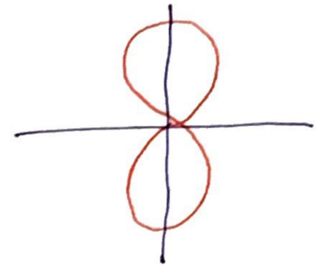
- lemniscates

$$r^2 = a^2 \begin{Bmatrix} \cos 2\theta \\ \sin 2\theta \end{Bmatrix}$$

all three symmetries

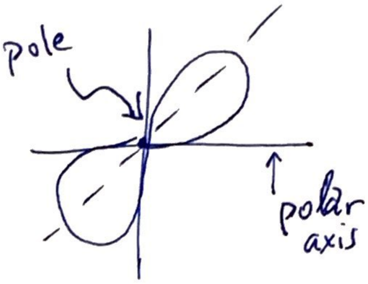


$r^2 = a^2 \cos 2\theta$

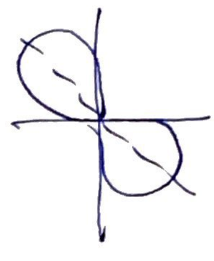


$r^2 = -a^2 \cos 2\theta$

only symmetry through the pole →



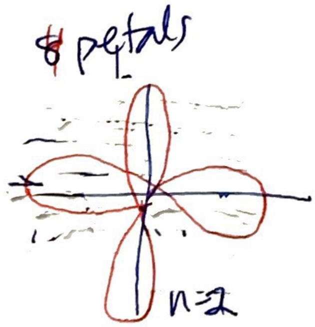
$r^2 = a^2 \sin 2\theta$



$r^2 = -a^2 \sin 2\theta$

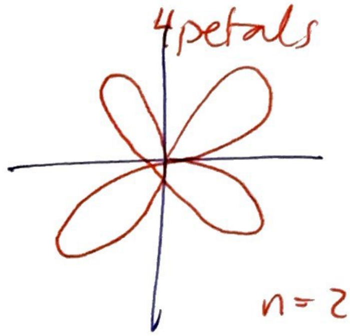
- roses

$$r = a \begin{Bmatrix} \cos(n\theta) \\ \sin(n\theta) \end{Bmatrix}$$



$r = a\cos(n\theta)$

$n = \text{even}$

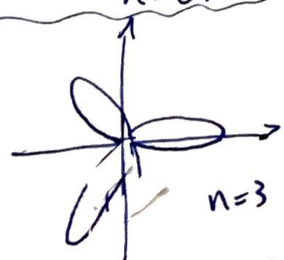


$r = a\sin(n\theta)$

$n = \text{even}$

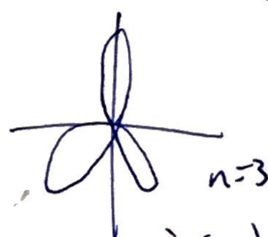
* $n = \text{odd}$
  n petals

* $n = \text{even}$
  $2n$ petals



$r = a\cos(n\theta)$

$n = \text{odd}$



$r = a\sin(n\theta)$

$n = \text{odd}$

(8)

- archimedes Spiral

$$\boxed{r = \theta}$$