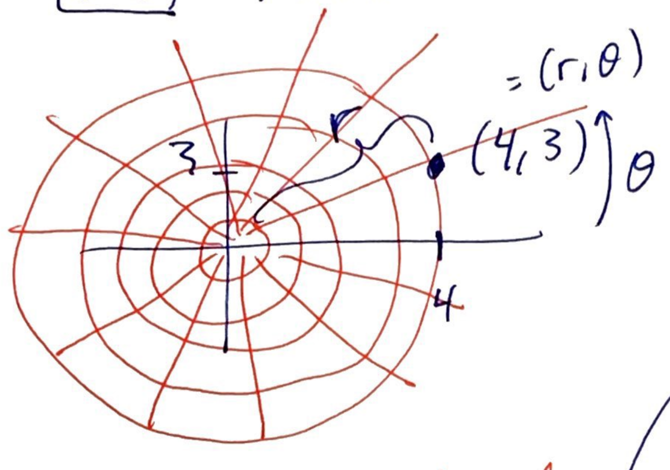
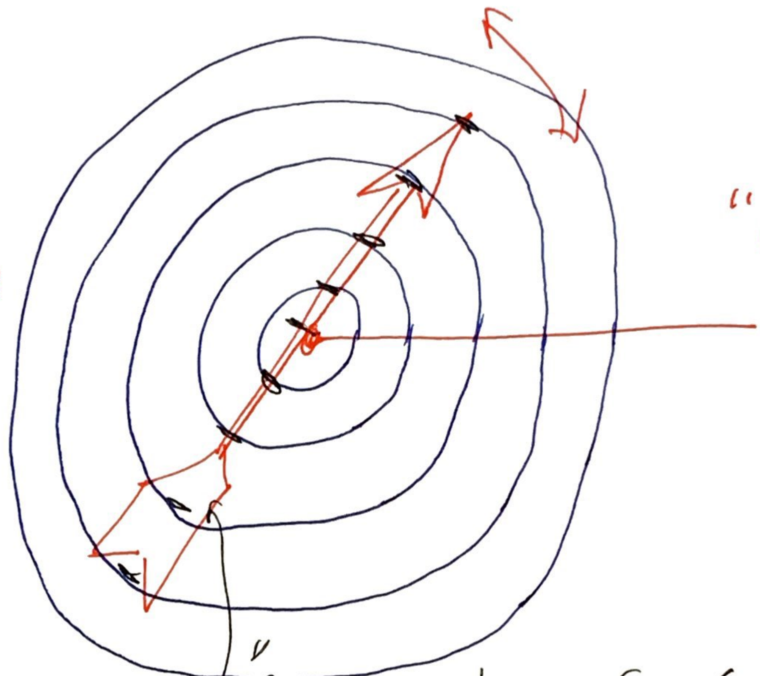
8.3 Polar Coordinates



polar (r, θ)

cartesian (x, y)



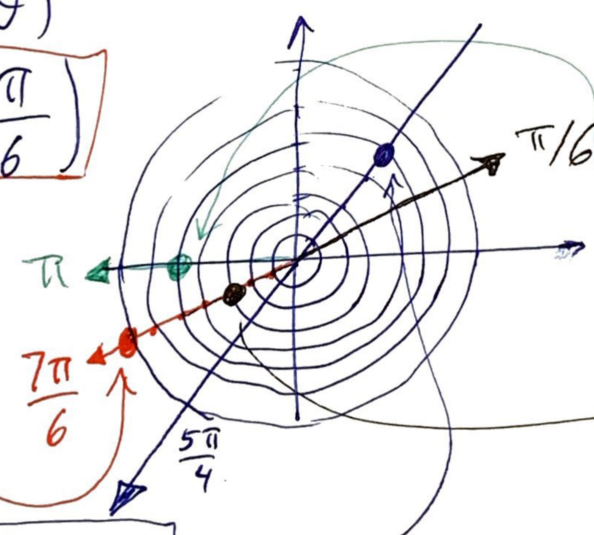
polar coordinates

"Spinner has an arrow on it"

* plot polar points...

(r, θ)

EX $\left(7, \frac{7\pi}{6}\right)$



$(5, \pi)$

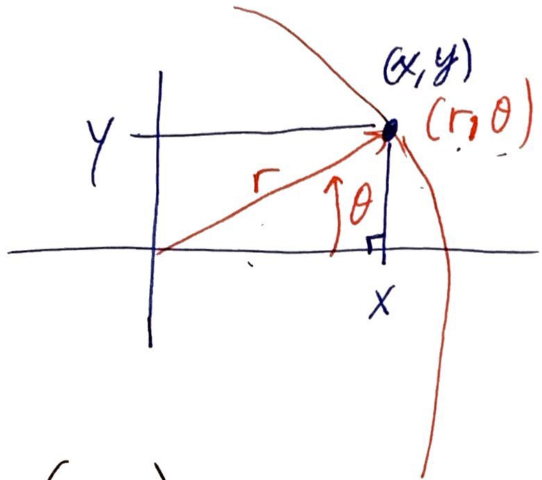
$\left(-3, \frac{\pi}{6}\right)$

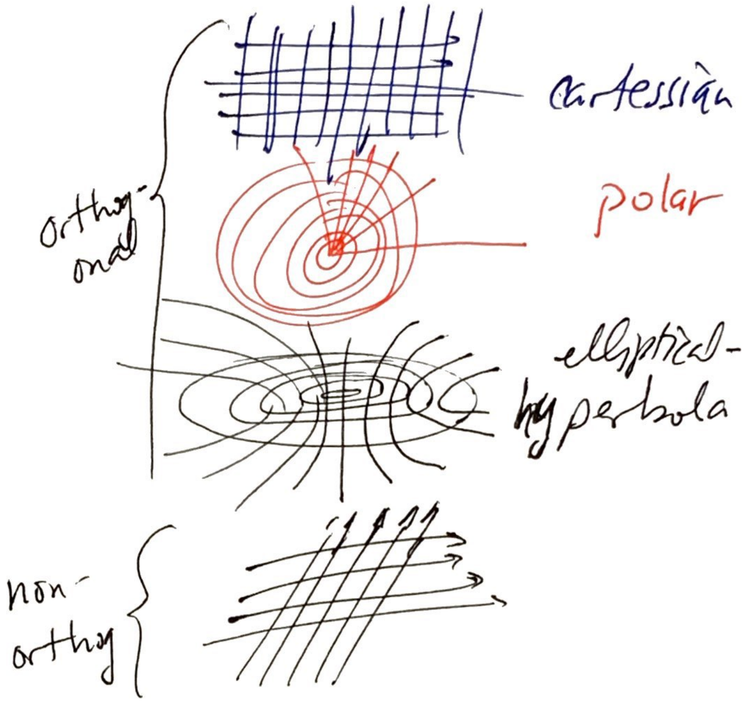
do angle 1st

$\left(-5, \frac{5\pi}{4}\right)$

Note that $\left(-5, \frac{5\pi}{4}\right)$ is also $\left(5, \frac{\pi}{4}\right)$

Convert between (r, θ) & (x, y)





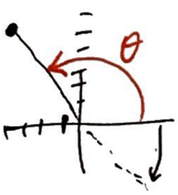
* Given (x, y) we can calculate (r, θ):

$$\boxed{\begin{aligned} r &= \sqrt{x^2 + y^2} \\ \theta &= \tan^{-1}(y/x) \end{aligned}}$$

* Given (r, θ) find (x, y)

$$\boxed{\begin{aligned} x &= r\cos(\theta) \\ y &= r\sin\theta \end{aligned}}$$

$\boxed{\text{EX}}$ $(-4, 6) = (x, y)$

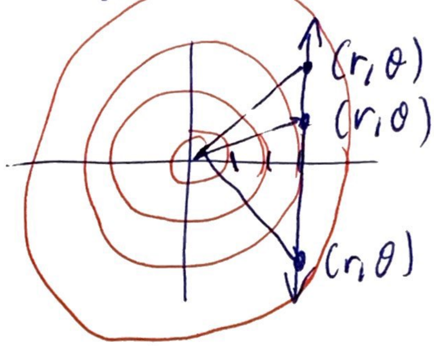


$$r = \sqrt{(-4)^2 + 6^2}$$
$$= \sqrt{16 + 36}$$
$$r = \sqrt{52}$$

$$\theta = \tan^{-1}\left(\frac{6}{-4}\right) + \pi$$

* Convert a cartesian eqn to a polar eqn.

EX $x = 3$



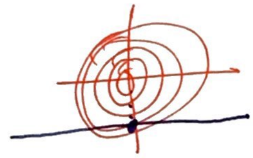
$$\boxed{x = r\cos\theta}$$

but $x=3$ $\left\{ \begin{array}{l} 3 = r\cos\theta \\ \boxed{r(\theta) = \dfrac{3}{\cos(\theta)}} \end{array} \right.$

or

$$\underline{\underline{r(\theta) = 3\sec(\theta)}}$$

EX $y = -4$



$y = r\sin\theta$

$$r(\theta) = \frac{-4}{\sin\theta} = \underline{\underline{-4\csc(\theta)}}$$

EX $x^2 - y^2 = x$ $\left\{ \text{use } \begin{array}{l} x = r\cos\theta \\ y = r\sin\theta \end{array} \right.$

$$r^2\cos^2\theta - r^2\sin^2\theta = r\cos\theta$$

$$r^{\cancel{2}}(\cos^2\theta - \sin^2(\theta)) = \cancel{r}\cos\theta$$

$$\boxed{r(\theta) = \frac{\cos(\theta)}{\cos(2\theta)}}$$ Hyperbola.

* go backwards

convert $\boxed{r = 4\cos\theta}$ to cartesian:

$x = r\cos\theta$

$y = r\sin\theta$

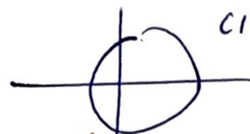
$$r = 4\left(\frac{x}{r}\right)$$

cross multiply

$$r^2 = 4x$$

but

$$\boxed{\underbrace{x^2+y^2}_{}= 4x}$$

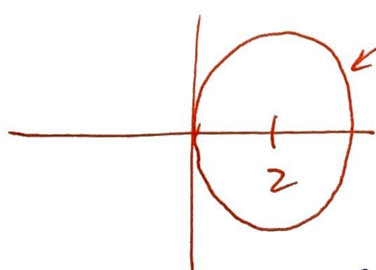


circle shifted

• note that we can complete the square ...

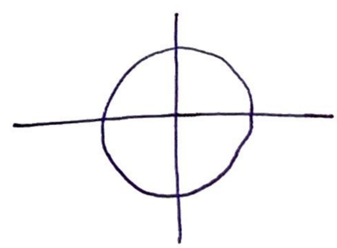
$$x^2 - 4x + 4 - 4 + y^2 = 0$$

$$\boxed{(x-2)^2 + y^2 = 4}$$



$\boxed{r = 4\cos\theta}$

Contrast this to $\boxed{r = 4}$



and $\boxed{r = 4\sin\theta}$