$\boxed{4.5}$ Log Properties

- $\boxed{\begin{aligned} \log(MN) &= \log(M) + \log(N) \\ \log\left(\frac{M}{N}\right) &= \log(M) - \log(N) \end{aligned}}$ } any base

BTW: NoNo

- $(x+y)^2 \neq x^2 + y^2$
- $\log(a+b) \neq \log(a) + \log(b)$
- $\frac{\cancel{a}+b}{\cancel{a}} = b$

$f = \log x$, $f^{-1} = a^x$

- $a^0 = 1 \quad \leftrightarrow \quad \boxed{\log_a(1) = 0}$
- $a^1 = a \quad \leftrightarrow \quad \boxed{\log_a(a) = 1}$
- $f(f^{-1}(x)) = x \quad \boxed{\log_a(a^x) = x}$
- $f^{-1}(f(x)) = x \quad \boxed{a^{\log_a(x)} = x}$
- $\boxed{\log_a(M^r) = r \log_a(M)}$
- change of base $\boxed{\log_a(M) = \frac{\log_b(M)}{\log_b(a)}}$

* Condense many add/subtract log terms into a single term:

Ex $\ln(7) + \ln(x) + \ln(y) = \ln(7 \cdot x \cdot y)$

Ex $3\log(x) - 2\log(x-1) + \log(5)$

$= \log(x^3) - \log(x-1)^2 + \log(5)$

$= \boxed{\log\left(\frac{x^3 \cdot (5)}{(x-1)^2}\right)}$

* break up complicated logs into a sum or difference of logs.

Ex $\ln\left(\frac{a^2}{b^3 c^5}\right)$

$= +\ln(a^2) - \ln(b^3 c^5)$

$= +\ln(a^2) - \{\ln(b^3) + \ln(c^5)\}$

$= +\ln(a^2) - \ln(b^3) - \ln(c^5)$

$= \boxed{2\ln(a) - 3\ln(b) - 5\ln(c)}$

Ex $\log\left(\sqrt{x^3 y^{-4}}\right)$

$= \log(x^3 y^{-4})^{1/2}$

$= \frac{1}{2}\log(x^3 y^{-4})$

$= \frac{1}{2}\{\log(x^3) + \log(y^{-4})\}$

$= \frac{1}{2}\{3\log(x) + (-4)\log(y)\}$

$= \boxed{\frac{3}{2}\log(x) - 2\log(y)}$

* change of base

Convert $\log_7(15)$ to $\log_e$ (aka ln)

$$= \boxed{\frac{\ln(15)}{\ln(7)}} \approx 1.39$$

Evaluate w/o a calculator...

$$6\log_8(2) + \frac{\log_8(64)}{3\log_8(4)}$$

$$= \log_8(2^6) + \frac{\log_8(64)}{\log_8(4^3)}$$

$$= \log_8(64) + \frac{\log_8(64)}{\log_8(64)}$$

$$= 2 + \frac{2}{2} = \boxed{3}$$

- $\log_8(64) = x$

$8^x = 64$

$\boxed{x = 2}$

- $\cancel{\log_8(4) = z}$

$\cancel{8^z = 4}$