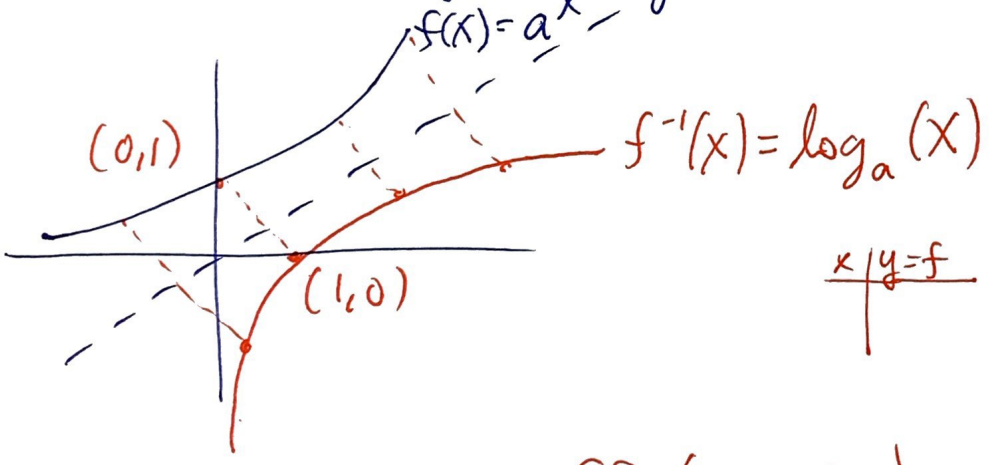
Math 104 | 4.3 | Logarithms

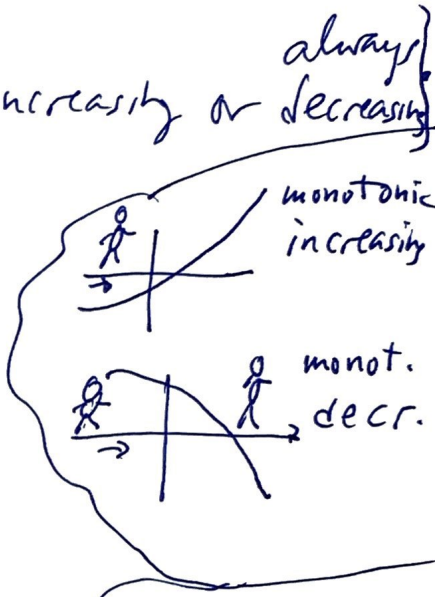
Just as $f(x)=\sqrt{x}$ has an inverse function of $f^{-1}(x)=x^2$, we know that since

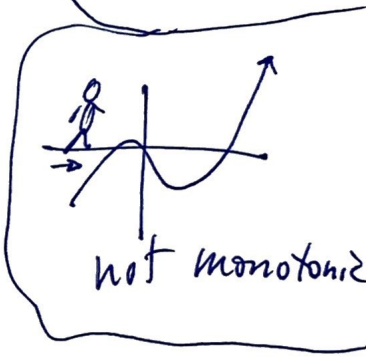
$f(x)=a^x$ is <u>monotonic</u> {always increasing or always decreasing}

thus passing the horizontal line test, we know that there is an inverse function:

$$\begin{cases} f(x)=a^x \\ f^{-1}(x)=\log_a(x) \end{cases}$$







x	y=f

y	$x=f^{-1}(\ )$

* Basic Properties $\begin{cases} f(f^{-1}(x))=x \\ f^{-1}(f(x))=x \end{cases}$

For us $f(f^{-1})=a^{f^{-1}}=a^{\log_a(x)}$.

$$\boxed{x=a^{\log_a(x)}}$$

Likewise $f^{-1}(f(x)) = x$

$\log_a(f) = x$

$$\log_a(a^x) = x$$

*Equivalent Problems

$f(x) = \sqrt{x} = 4$ is the same as

$f^{-1}(f) = (\sqrt{x})^2 = f^{-1}(4)$

$x = 4^2$

$\sqrt{x} = 4$ is equivalent to $x = 4^2$

($\sqrt{x}$ = $f(x)$; 4^2 = $f^{-1}(4)$)

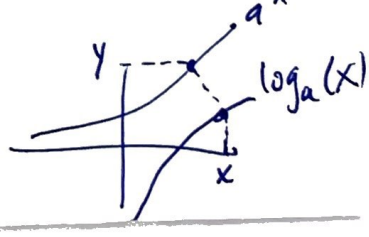
Likewise

$a^x = b$ is equivalent to $\log_a(b) = x$

EX write $4^x = y$ as a logarithm: $\log_4(y) = x$

EX $m^4 = 103$: it's equivalent problem is

$$\log_m(103) = 4$$



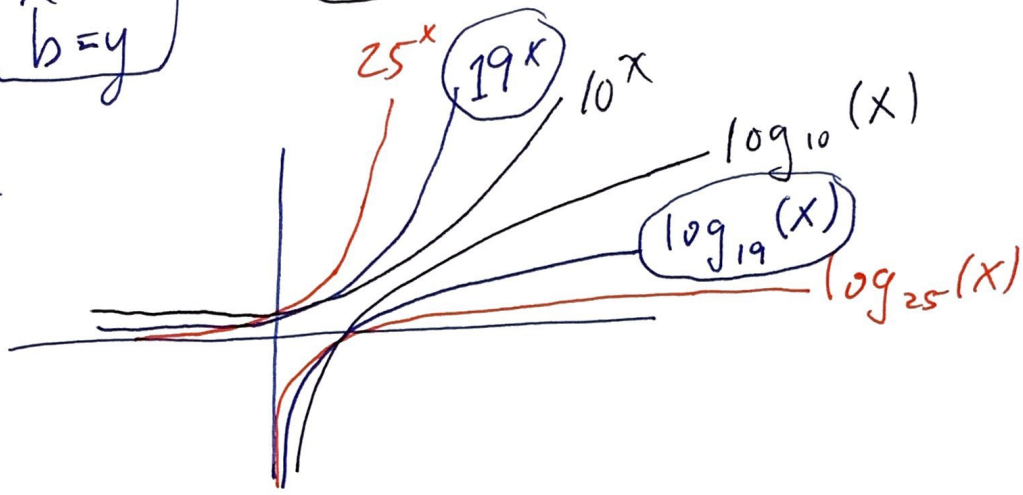
EX Write $19^x = y$ as an equivalent problem.

use $a^x = b \iff \log_a(b) = x$

match $a = 19$, $x = x$, $b = y$

$\log_{19}(y) = x$

* graphs



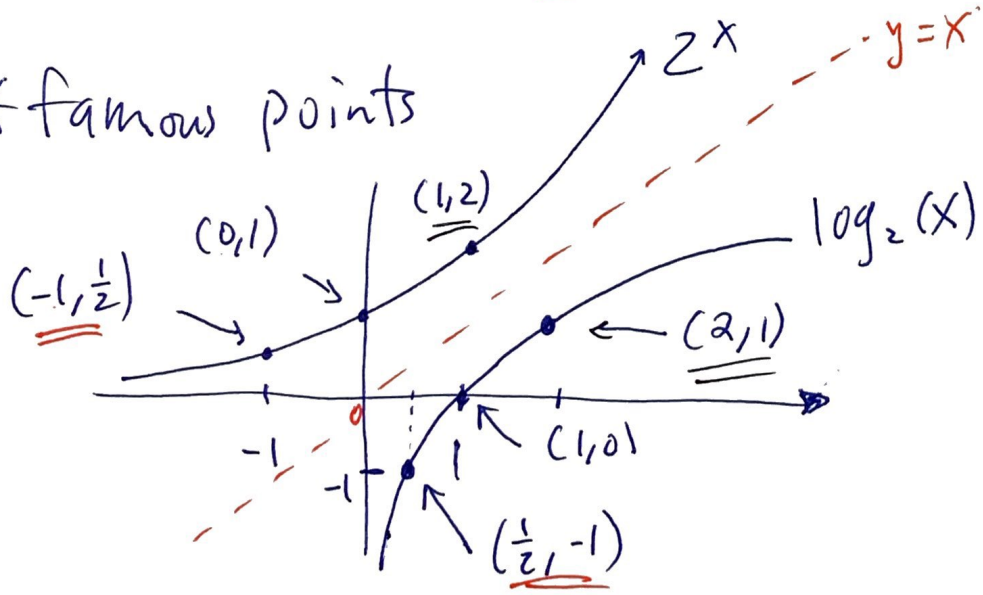
we don't bother writing the 10 for $\log_{10}(x)$

we just use $\log(x) \equiv \log_{10}(x)$

Likewise for e^x we use $\ln(x)$

so $\ln(x) \equiv \log_e(x)$

* famous points



EX Find the equivalent problem:

$y^x = 39/100$

$\log_y(39/100) = x$

$a^x = b \Leftrightarrow \log_a(b) = x$

* Lets go backwards...

EX Convert $\log_5(x) = 2$ to an exponent:

$5^2 = x$

* Solving log values (simple):

EX evaluate (w/o calculator) $\log_{10}(100)$.

(i) assign $\log_{10}(100)$ a variable: $\log_{10}(100) = x$

(ii) write the equivalent problem: $10^x = 100$

(iii) Solve the equivalat problem: $x = 2$

(iv) Final answer: $\log_{10}(100) = 2$

EX Evaluate $\log_3(81)$:

(i) $\log_3(81) = x$ (ii) $3^x = 81$ (iii) $x = 4$

(iv) $\log_3(81) = 4$

EX Solve for x: $\log_2(3x+4) = 4$

Hint: write the equiv. problem.

(i) $2^4 = 3x+4$

(ii) Solve for x: $16 = 3x+4$

$16-4 = 3x$

$12 = 3x$

$\boxed{x=4}$

EX Solve $\log_5(x) = 2$:

(i) $5^2 = x$ (ii) $\boxed{x=25}$

EX Evaluate $\log(100^8)$:

(i) $\log(100^8) = x$

(ii) $10^x = 100^8$

(iii) $10^x = (10^2)^8$

$10^x = 10^{16}$

(iv) $\boxed{x=16}$

EX $\log(0.001)$:

(i) $\log(0.001) = x$

(ii) $10^x = 0.001$

$\boxed{x=-3}$

* calculator

EX evaluate $\log(0.001)$

"0.001 [log]" we get [-3]

EX $\log(10{,}000)$: "10000 [log]" → 4

$f^{-1}(f(x)) = x$

$\log_{10}(10^x) = x$ → $\log_{10}(10^4) = 4$

* natural log:

EX evaluate $\ln(15)$:

"15 [ln]" → 2.708

EX $\ln(\sqrt{2})$ "2 [√x] [ln]" → 0.347