

3.5 Dividing Polynomials

Review $\frac{37}{3}$.

$$\begin{array}{r} 12 \text{ r } 1 \\ 3 \overline{) 37} \\ \underline{-3} \\ 07 \\ \underline{-6} \\ 1 \end{array}$$

Ans: $\frac{37}{3} = 12 + \frac{1}{3}$

*Now we do the same for polynomials

Ex $\frac{2x^2 - 9x - 5}{x - 5}$

$$\begin{array}{r} 2x + 1 \text{ r } 0 \\ x - 5 \overline{) 2x^2 - 9x - 5} \\ \underline{-(2x^2 - 10x)} \\ x - 5 \\ \underline{-(x - 5)} \\ 0 \end{array}$$

Ans:

$$\frac{2x^2 - 9x - 5}{x - 5} = 2x + 1$$

BTW: $2x^2 - 9x - 5 = \underline{(2x + 1)(x - 5)}$

we have factored this poly
since $r = 0$

②

$$\begin{array}{r}
 x^2 + 5x + 25 \quad r: -1 \\
 x-5 \overline{) x^3 + 0x^2 + 0x - 126} \\
 \underline{-(x^3 - 5x^2)} \quad \downarrow \\
 5x^2 + 0x \\
 \underline{-(5x^2 - 25x)} \quad \downarrow \\
 25x - 126 \\
 \underline{-(25x - 125)} \\
 -1
 \end{array}$$

Ans: $\frac{x^3 - 126}{x - 5} = x^2 + 5x + 25 + \frac{-1}{x - 5}$

$$\frac{x^4 - 3x^2 + 5x - 11}{x^2 - 1}$$

$$\begin{array}{r}
 x^2 - 2 \quad r. \quad 5x - 13 \\
 x^2 + 0x - 1 \quad \bigg) \quad x^4 + 0x^3 - 3x^2 + 5x - 11 \\
 \underline{-(x^4 + 0x^3 - x^2)} \quad \downarrow \quad \downarrow \\
 -2x^2 + 5x - 11 \\
 \underline{-(-2x^2 + 0x + 2)} \\
 5x - 13
 \end{array}$$

$$\frac{x^4 - 3x^2 + 5x - 11}{x^2 - 1} = x^2 - 2 + \frac{5x - 13}{x^2 - 1}$$

* when the denominator is a linear binomial, we can use a short cut method call synthetic division ③

EX $\frac{3x^3 - 2x^2 + x - 4}{x + 3}$

$$\begin{array}{r}
 3x^2 - 11x + 34 \\
 x+3 \overline{) 3x^3 - 2x^2 + x - 4} \\
 \underline{-(3x^3 + 9x^2)} \\
 -11x^2 + x \\
 \underline{-(-11x^2 - 33x)} \\
 34x - 4 \\
 \underline{-(34x + 102)} \\
 -106
 \end{array}$$

$$\frac{3x^3 - 2x^2 + x - 4}{x + 3} = 3x^2 - 11x + 34 - \frac{106}{x+3}$$

or $3x^3 - 2x^2 + x - 4 = (3x^2 - 11x + 34)(x + 3) - 106$

* synthetic div. (denominator must be linear)

change the sign $(x+3)$

$$\begin{array}{ccccccc}
 & x^3 & x^2 & x^1 & x^0 & & \\
 -3 & 3 & -2 & 1 & -4 & & \\
 & \downarrow & \downarrow & \downarrow & \downarrow & & \\
 & 3 & -11 & 34 & -106 & & \\
 & x^2 & x^1 & x^0 & r & &
 \end{array}$$

← reduced poly

← remainder

$$\frac{3x^3 - 2x^2 + x - 4}{x + 3} = 3x^2 - 11x + 34 - \frac{106}{x+3}$$

(4)

$$\underline{3x^3 - 2x^2 + x - 4}$$

$$= x (\underline{3x^2 - 2x + 1}) - 4$$

$$= x (x(3x - 2) + 1) - 4$$

$$x + 3 = 0$$

$$x = -3$$

$$f(-3) = -3 (-3 (\underbrace{3 \cdot (-3)}_{-9} - 2) + 1) - 4$$

$$-3 (\underbrace{-9 - 2}_{-11})$$

$$-3 (\underbrace{-11}_{34} + 1)$$

$$-3 (34) - 4$$

not finished with 3.5 (but almost)