Chapter 3: Complex Numbers & Polynomials

3.1 Complex Number: who wudda thunk that the dunce in Egypt who drew ire from his math teacher when he asked "What number squared is -1?"

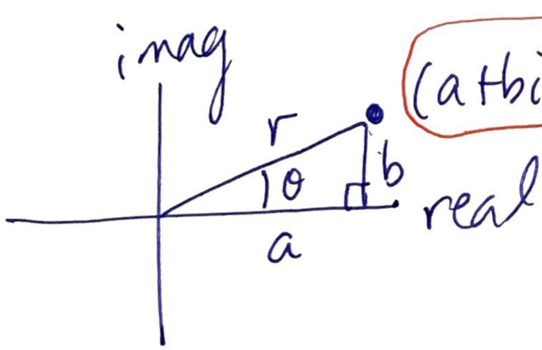
We call such a number an "imaginary number"

$\boxed{i^2 \equiv -1}$ some texts $\boxed{i \equiv \sqrt{-1}}$

A number with a real & imaginary part is called a "complex number"

$$\underbrace{a}_{\text{real}} + \underbrace{\underbrace{b}_{\text{real}}\, i}_{\text{imaginary}} \quad \leftarrow \text{complex}$$

$\boxed{\text{EX}}$ $2+3i$, $-7+\sqrt{2}\,i$, $\pi - \sqrt{-3} = \pi - \sqrt{3}\,i$



$(a+bi) = (r, \theta)$

$(r\cos\theta, r\sin\theta)$

Quadratic Formula

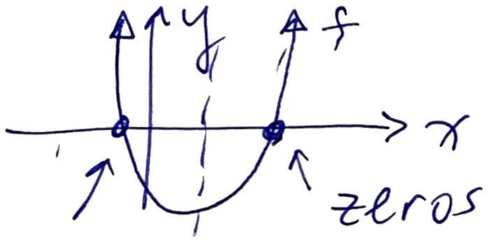
$$\begin{cases} a\cdot x^2+bx+c=0 \\ x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a} \end{cases}$$

Nomenclature:

ax^2+bx+c	Quadratic expression
$ax^2+bx+c=0$	Quad Eqn
$ax^2+bx+c\geq 0$	Quad Inequality
$f(x)=ax^2+bx+c$	Quadratic function
$x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$	Quad. formula

$\boxed{\text{Ex}}$ Find the ~~zeros~~ of this Quadratic function

$$f(x)=2x^2-4x-10$$



zeros a.k.a. "roots"

• set $f(x)=0$

$$2x^2-4x-10=0$$

$\div 2$

$$x^2-2x-5=0$$

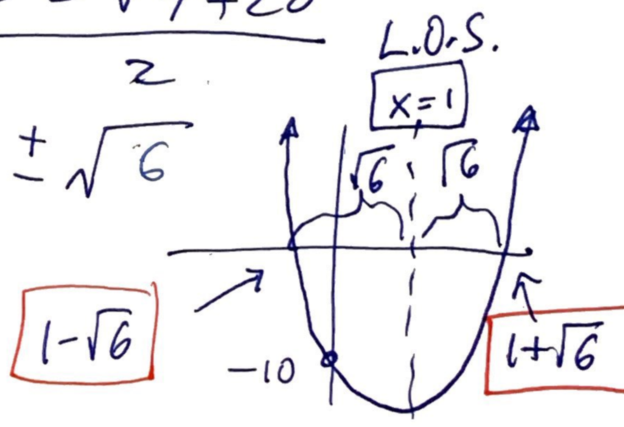
make assignments $a=1,\ b=-2,\ c=-5$

use formula $x=\dfrac{-(-2)\pm\sqrt{(-2)^2-4(1)(-5)}}{2(1)}$

$$x=\frac{2\pm\sqrt{4+20}}{2}$$

$$x=1\pm\sqrt{6}$$

$\sqrt{24}$
$=\sqrt{8\cdot 3}$
$=2\sqrt{2\cdot 3}=2\sqrt{6}$



$\boxed{\text{EX}}$

Find the zeros of this Quadratic function

$$f(x) = 2x^2 - 4x + 10$$ change to (+)

$$0 = 2x^2 - 4x + 10$$ mirror image & $\div 2$

$$x^2 - 2x + 5 = 0$$

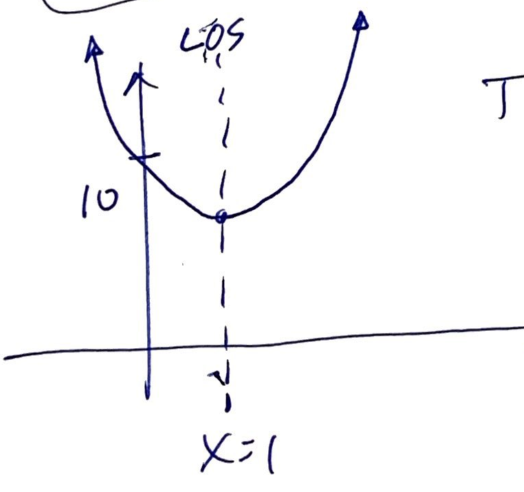
$a = 1,\ b = -2,\ c = 5$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(5)}}{2(1)}$$

$$x = \frac{2 \pm \sqrt{-16}}{2}$$ $\sqrt{-16} = 4i$

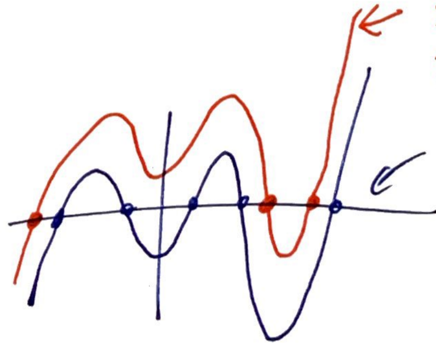
$$\boxed{x = 1 \pm 2i}$$ complex non-real

Graphically



This does not intersect the x-axis. It's zeros are $\therefore$ complex

* quintic $x^5 + 13x^4 - x^2 + 1$



3 real & 2 complex zeros

all real zeros

$\therefore$ "therefore"

* adding complex numbers: when we add complex numbers we add the real parts & the imaginary parts seperately:

$\boxed{EX}$ $(2+3i)+(-1+4i)$

$a+bi$

$= (2+(-1)) + (3i+4i)$

$= \boxed{1+7i}$

* multiply (Foil but treat the imaginary part like a radical)

$\boxed{EX}$ $(2+3i)(-1+4i)$

$=(2)(-1)+2\cdot 4i-3i+12i^2$ $\quad i^2=-1$

$=-2+8i-3i-12$

$=\boxed{-14+5i}$

$(2+\sqrt{3})(-4+\sqrt{5})$

$=2\cdot 4-4\sqrt{3}+\sqrt{3}\sqrt{5}+2\sqrt{5}$

$=8-4\sqrt{3}+\sqrt{15}+2\sqrt{5}$

$\boxed{EX}$ $(-3+\sqrt{6}\,i)(3-\sqrt{8}\,i)$

$= -9+3\sqrt{8}\,i+3\sqrt{6}\,i-\sqrt{6}\sqrt{8}\,i^2$ $\quad -1$

$= -9+6\sqrt{2}\,i+3\sqrt{6}\,i+\sqrt{48}$

$= -9+4\sqrt{3}+(6\sqrt{2}+3\sqrt{6})i$

$\sqrt{48}$
$3\cdot 16$
$=4\sqrt{3}$

$\boxed{EX}$ $(-1+2i)(-2+3i)$

$= (-1)(-2)+(-1)(3i)+(2i)(-2)+(2i)(3i)$

$= 2-3i-4i+6i^2$

$= 2-7i-6$ $= \boxed{-4-7i}$

* divide complex numbers (similar to rationalizing radical denominators

$\boxed{EX}$ $\dfrac{-1+2i}{-2+3i}$

complex conjugates

$= \left(\dfrac{-1+2i}{-2+3i}\right)\left(\dfrac{-2-3i}{-2-3i}\right)$

$= \dfrac{2-4i+3i-6i^2}{(-2)^2-(3i)^2}$

$= \dfrac{2-i+6}{4+9}$

$= \dfrac{8-i}{13}$

$= \boxed{\dfrac{8}{13}-\dfrac{1}{13}i}$

$\dfrac{1+\sqrt{6}}{1+\sqrt{3}} \cdot \left(\dfrac{1-\sqrt{3}}{1-\sqrt{3}}\right)$

$= \dfrac{1+\sqrt{6}-\sqrt{3}-\sqrt{18}}{1^2+\cancel{\sqrt{3}}-\cancel{\sqrt{3}}-\sqrt{3}^2}$

$= \dfrac{1+\sqrt{6}-\sqrt{3}-2\sqrt{3}}{1-3}$

$= \dfrac{1+\sqrt{6}-3\sqrt{3}}{-2}$

$(a+b)(a-b)=a^2-b^2$

EX $$\frac{(2+i)(4-2i)}{(1+i)}$$

$$= \frac{8+\cancel{4i}-2i^2-\cancel{4i}}{1+i}$$

$$= \frac{8+2}{1+i}$$

$$= \frac{10}{1+i}\cdot\left(\frac{1-i}{1-i}\right)$$

$$= \frac{10-10i}{1^2-i^2}$$

$$= \frac{10-10i}{1+1}$$

$$= \boxed{5-5i}$$

* powers of i :

$i^0 = 1$

$i^1 = i$

$i^2 = -1$

$i^3 = -i$

$i^4 = i^3 \cdot i = -i \cdot i = 1$

$i^5 = i^4 \cdot i = 1 \cdot i = i$

repeats

- So Find i^7

$= i\,i\,i\,i\,i\,i\,i$

$= (-1)(-1)(-1)i$

$= \boxed{-i}$

- Find i^{63}

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Use modulus function "remainder"

$$\begin{array}{r} 15\,r\,3 \\ 4\,\overline{)\,63} \\ -4\downarrow \\ \hline 23 \\ -20 \\ \hline 3 \end{array}$$

remainder

$i^0 = 1$

$i^1 = i$

$i^2 = -1$

$i^3 = -i$

So $i^{63} = \underbrace{(i^4)^{15}}_{=1} \cdot \underline{\underline{i^3}} = 1 \cdot (-i) = -i$

$\boxed{i^{63} = -i}$

$\boxed{\text{EX}}$ i^{92}

$$\begin{array}{r} 23\,r\,0 \\ 4\,\overline{)\,92} \\ -8 \\ \hline 12 \end{array}$$

so $\boxed{i^{92} = 1}$

$\boxed{\text{EX}}$ i^{167}

$$\begin{array}{r} 41\,r\,3 \\ 4\,\overline{)\,167} \\ 16 \\ \hline 7 \\ -4 \\ \hline 3 \end{array}$$

$\boxed{i^{167} = -i}$

Break 10 min