III.6 (Cont.) Binomial expansion

$$\boxed{(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k}$$

EX

$n=3$ $\quad (x+y)^3 = \sum_{k=0}^{3} \binom{3}{k} x^{3-k} y^k$

$$= \underbrace{\binom{3}{0} x^{3-0} y^0}_{k=0} + \underbrace{\binom{3}{1} x^{3-1} y^1}_{k=1} + \underbrace{\binom{3}{2} x^{3-2} y^2}_{k=2} + \underbrace{\binom{3}{3} x^{3-3} y^3}_{k=3}$$

$$\boxed{\binom{n}{0} = 1, \quad \binom{n}{n} = 1, \quad \binom{n}{n-1} = n, \quad \binom{n}{1} = n}$$

$$= 1x^3y^0 + 3x^2y^1 + 3x^1y^2 + 1\cdot x^0 y^3$$

$$\boxed{(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3}$$

If needed $\quad \binom{n}{k} = \dfrac{n!}{(n-k)!\,k!}$

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⊛ the r+1 term of the binomial expansion $(x+y)^n = \dots\dots$

$$a_{r+1} = \binom{n}{r} x^{n-r} y^{r}$$

the 1^{st} term has $r = 0$

r^{th} term

$\boxed{EX}$ the 11^{th} term, starting counting at 1 of $(x+y)^{17}$ is what?

$$\binom{17}{10} x^{17-10} y^{10}$$

$r = 10$

$$= \frac{17!}{(17-10)!\,10!} x^7 y^{10}$$

$$= \frac{17 \cdot \overset{4}{\cancel{16}} \cdot \overset{5}{\cancel{15}} \cdot \overset{2}{\cancel{14}} \cdot 13 \cdot \overset{6}{\cancel{12}} \cdot 11 \cdot \cancel{10!}}{(\cancel{7} \cdot \underset{3}{\cancel{6}} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot 1)\,\cancel{10!}} x^7 y^{10}$$

$$= \frac{17 \cdot 4 \cdot 13 \cdot \overset{2}{\cancel{6}} \cdot 11}{\cancel{3}} x^7 y^{10}$$

$$22 \times 52$$

$$\times 17$$

$$= \boxed{19{,}448\, x^7 y^{10}}$$

11^{th} term start counting @ 1

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End of 11.6