$\boxed{11.6}$ Binomial Thm

* A binomial is an expression with two terms in it.

$(a+b)$, or $(3+x)$, (y^2-y), ...

* We consider now a binomial raised to ~~the~~ n^{th} power. These terms arise freq'tly in STEM fields

$\boxed{\text{Ex}}$ $(a+b)^0 = 1$

$(a+b)^1 = a+b$

$(a+b)^2 = a^2+2ab+b^2$

$(a+b)^3 = (a+b)(a^2+2ab+b^2)$

$= a^3 + 2a^2b + ab^2 + ba^2 + 2ab^2 + b^3$

$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$(a+b)^4 = (a+b)(a^3+3a^2b+3ab^2+b^3)$

$= a^4 + 3a^3b + 3a^2b^2 + ab^3$
$+ ba^3 + 3a^2b^2 + 3ab^3 + b^4$

$n=4$ $\quad (a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

getting tired yet? There is a better way ...

*Pascal's triangle

1

1 1

n=2 → 1 2 1

n=3 → 1 3 3 1

n=4 → 1 4 6 4 1

n=5 → 1 5 10 10 5 1

n=6 1 6 15 20 15 6 1

* Compare these numbers to the numerical coefficients for $(a+b)^n$ – they match!

EX State the expansion of $(a+b)^6$:

$$(a+b)^6 = a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + b^6$$

* For the $(a+b)^N$ general expansion we have:

$$\boxed{(a+b)^N = \sum_{k=0}^{N} \frac{N!}{(N-k)!\,k!} a^{N-k} b^k} = \sum_{k=0}^{N} \binom{N}{k} a^{N-k} b^k$$

$$\boxed{N! \equiv N\cdot(N-1)\cdot(N-2)\cdots(3)(2)(1)}$$ N factorial

EX $6! = 6\cdot5\cdot4\cdot3\cdot2\cdot1 = 720$

3

- The term $\boxed{\dfrac{N!}{(N-k)!\,k!} \text{ is abbreviated as } \dbinom{N}{k}}$

"$\binom{N}{k}$" is called different names: "Combination", "binomial coefficient" to name two.

* Learning how to count:

In counting, $\binom{N}{k}$ is written as ${}_N C_k$ and is the number of ways to extract k items from a collection of N items, ordering is not important.

$\boxed{Ex}$ We need to send to the office 3 ppl from our collection of 19 students.

The number of ways to do this is ${}_{19}C_3$ or $\binom{19}{3}$

- This button is on your calculator.

19 $\boxed{2^{nd}}$ $\boxed{nC_r}$ 3 $\boxed{=}$

- Manually

don't ask your calculator to perform larger factorials!

use this trick instead.

$$ {}_{19}C_3 = \binom{19}{3} = \frac{19!}{(19-3)!\,.\,3!} = \frac{19!}{16!\,3!} = \frac{19\cdot 18\cdot 17\cdot \cancel{16!}}{\cancel{16!}\;3\cdot 2\cdot 1} = \frac{19\cdot \cancel{18}^{3}\cdot 17}{\cancel{6}} $$

$= \boxed{969 \text{ ways to choose 3 ppl from a group of 19}}$ To be cont.