

11.6 Binomial Thm

1

* A binomial is an expression with two terms in it.

$$(a+b), \text{ or } (3+x), (y^2-y), \dots$$

* We consider now a binomial raised to the n^{th} power. These terms arise frequently in STEM fields

[EX] $(a+b)^0 = 1$

$$(a+b)^1 = a+b$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a+b)^3 = (a+b)(a^2 + 2ab + b^2)$$

$$= a^3 + \underline{2a^2b} + \underline{ab^2} + \underline{ba^2} + \underline{2ab^2} + b^3$$

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a+b)^4 = (a+b)(a^3 + 3a^2b + 3ab^2 + b^3)$$

$$= a^4 + \underline{3a^3b} + \underline{3a^2b^2} + \underline{ab^3}$$

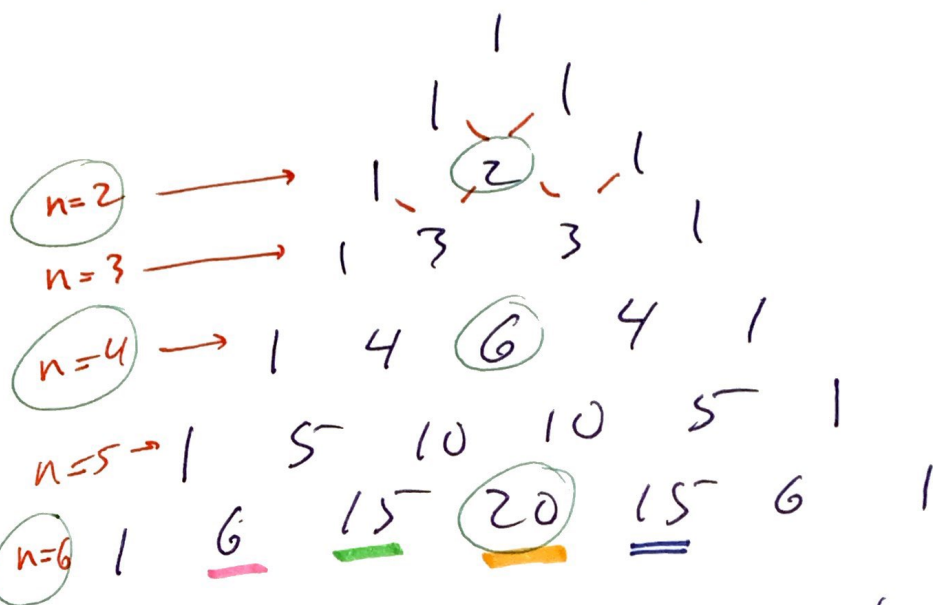
$$+ \underline{ba^3} + \underline{3a^2b^2} + \underline{3ab^3} + b^4$$

$n=4$ $(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

getting tired yet? There is a better way...

* Pascal's triangle

(2)



* Compare these numbers to the numerical coefficients for $(a+b)^n$ - they match!

EX

State the expansion of $(a+b)^6$:

$$(a+b)^6 = a^6 + \underline{6}a^5b + \underline{15}a^4b^2 + \underline{20}a^3b^3 + \underline{15}a^2b^4 + 6ab^5 + b^6$$

* For the $(a+b)^N$ general expansion we have:

$$(a+b)^N = \sum_{k=0}^N \frac{N!}{(N-k)!k!} a^{N-k} b^k = \sum_{k=0}^N \binom{N}{k} a^{N-k} b^k$$

$$N! \equiv N \cdot (N-1) \cdot (N-2) \cdots (3)(2)(1)$$

N factorial

EX

$$6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720$$

• The term $\frac{N!}{(N-k)! k!}$ is abbreviated as $\binom{N}{k}$ (3)

" $\binom{N}{k}$ " is called different names: "Combination",
"binomial coefficient" to name two.

* Learning how to count:

In counting, $\binom{N}{k}$ is written as ${}_N C_k$ and is

the number of ways to extract k items
from a collection of N items, ordering is
not important.

Ex We need to send to the office 3 ppl
from our collection of 19 students.

The number of ways to do this is ${}_{19} C_3$
or $\binom{19}{3}$

• This button is on your calculator.

19 $\boxed{2^{nd}}$ $\boxed{nCr}$ 3 $\boxed{=}$

• Manually

$${}_{19} C_3 = \binom{19}{3} = \frac{19!}{(19-3)! 3!} = \frac{19!}{16! 3!} = \frac{19 \cdot 18 \cdot 17 \cdot \cancel{16!}}{\cancel{16!} 3 \cdot 2 \cdot 1} = \frac{19 \cdot 18 \cdot 17}{6}$$

= 969 ways to choose 3 ppl from a group of 19 To be cont.