11.3 Geometric Sequences

We now look at a sequence where the adjacent terms are ratios of each other: The Geometric Seq.

Ex $1, 3, 9, 27, 81$ (n=1, n=2, n=3, n=4, n=5)

$\times 3 \quad \times 3 \quad \times 3$ ← all adjacent terms have a ratio of 3

the general n^{th} term:

$$a_n = 3^n$$

$a_1 = 3$
$a_2 = 9$ No!

try $a_n = 3^{n-1}$

$a_1 = 3^{1-1} = 3^0 = 1$

$a_2 = 3^{2-1} = 3^1 = 3$

$a_3 = 3^{3-1} = 3^2 = 9$

So now the 11^{th} term is

$$a_{11} = 3^{11-1} = 3^{10} = 59{,}049$$

* In general a geometric sequence has a general term of

$$a_n = a_1 r^{n-1}$$

r = common ratio

EX Find the common ratio

$$\left\{-2, \frac{1}{2}, -\frac{1}{8}, \frac{1}{32}, -\frac{1}{128}, \ldots\right\}$$

$*\left(-\frac{1}{4}\right)$ $*\left(-\frac{1}{4}\right)$ $*\left(-\frac{1}{4}\right)$

$r = -\frac{1}{4}$, $a_1 = -2$

$$a_n = -2\left(-\frac{1}{4}\right)^{n-1}$$

test: $a_1 = -2\left(-\frac{1}{4}\right)^{1-1} = -2 \cdot 1 = \underline{\underline{-2}}$

$a_2 = -2\left(-\frac{1}{4}\right)^{2-1} = -2\left(-\frac{1}{4}\right) = \frac{1}{2}$

$\vdots$

* C.S.I. problems:

EX write the 1st three terms of a geometric sequence if $a_7 = 64$ and $a_{10} = 512$

$a_n = a_1 r^{n-1}$

$n=7$: $a_7 = a_1 r^6$ $\Rightarrow$ $64 = a_1 r^6$

$n=10$: $a_{10} = a_1 r^9$ $\Rightarrow$ $512 = a_1 r^9$

non-linear system

$a_1 = ?$, $r = ?$

To Solve ÷ the two to get r:

$$\frac{64}{512} = \frac{a_1 r^6}{a_1 r^9}$$

$\Rightarrow \frac{1}{8} = r^{6-9}$

$\Rightarrow \frac{1}{8} = r^{-3}$

$\Rightarrow 8 = r^3$

$\Rightarrow \boxed{r = 2}$

To get a_1 just substitute $r=2$ into either $n=7$ or $n=10$ expression:

$$64 = a_1(2)^6 \Rightarrow a_1 = \frac{64}{64} = 1$$

So $a_n = 1 \cdot 2^{n-1}$

$$\boxed{a_n = 2^{n-1}} \quad = \frac{2^n}{2}$$

1st three terms: $\{2^{1-1}, 2^{2-1}, 2^{3-1}, \ldots\}$

or $\{1, 2, 4, \ldots\}$

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* recursive terms

$\boxed{a_1 = \#\ a_n = a_{n-1} \cdot r}$

EX what is the recursive term for the sequence

$\{14, 56, 224, 896, \ldots$

$\frac{56}{14} = 4$ *4 *4

$\boxed{a_n = a_{n-1} \cdot 4}$ $\boxed{a_1 = 14}$

11.3 is finished

* Thanksgiving weekend
  - → 11.2, 11.3 → re work Test 4 & Test 3
  - → odd in the Stewart Algebra Review for E.C.