11.2 Arithmetic Sequences

when we see that the pattern in a sequence is a fixed difference, we have an arithmetic sequence

Ex

$\{5, 11, 17, 23, 29, \ldots\}$

+6 +6 +6 +6 ← arithmetic sequence.

6 is called the <u>common difference</u> "d"

recursive algorithm: $\boxed{a_n = a_{n-1} + d}$

$n=2$: $a_2 = a_{2-1} + d$

or

$a_2 = a_1 + d$

$a_3 = a_2 + d \rightarrow a_3 = (a_1 + d) + d = a_1 + 2d$

$a_4 = a_3 + d \rightarrow a_4 = (a_1 + 2d) + d = a_1 + 3d$

$a_5 = a_4 + d \rightarrow a_5 = (a_1 + 3d) + d = a_1 + 4d$

$\vdots$

$a_n = a_{n-1} + d$

$$\boxed{a_n = a_1 + d(n-1)}$$

explicit (general) term

So $\{\overset{n=1}{5}, \overset{n=2}{11}, \overset{n=3}{17}, \overset{n=4}{23}, 29, \ldots \underbrace{5 + 6(n-1)}_{a_n}, \ldots\}$

$5 + 6(5-1)$

* non-arithmetic seq.

$\boxed{\text{EX}}$ Analyze

$$\{4, 16, 64, 256, 1024, \dots\}$$

- How do I get from 4 to 16? $+12$, $*4$
- How do I get from 16 to 64? $+48$, $*4$
- How do I get from 64 to 256? $\underline{\underline{*4}}$ $+192$

Q: Is this arithmetic? No we are not adding a common difference.

{ B.T.W.: this sequence is a <u>geometric sequence</u> (11.3) since we are multiplying by a constant ratio, 4 }

$\boxed{\text{EX}}$ $\{11.4, 9.3, 7.2, 5.1, 3.0, \dots\}$

-2.1 -2.1 -2.1

- arithmetic
- $d = -2.1$
- general term

$$a_n = a_1 + (n-1)d$$
$$a_n = 11.4 + (n-1)(-2.1)$$
$$\boxed{a_n = 11.4 - 2.1(n-1)}$$

- recursive form

$$a_n = a_{n-1} + d$$
$$\boxed{a_n = a_{n-1} - 2.1}$$

* C.S.I.

EX Write the 1st five terms of an arithmetic seq if $a_1 = 17$ and $a_7 = -31$

use $\boxed{a_n = a_1 + d(n-1)}$

$n=7$ $\quad a_7 = a_1 + d(7-1)$

$a_7 = 17 + d(6)$

but $-31 = 17 + d \cdot 6$ solve for d: $d = \frac{-31-17}{6} = -8$

So the general term, $\boxed{a_n = 17 - 8(n-1)}$

Now lets answer the question

$\{17 - 8\cdot(1-1),\ 17 - 8\cdot(2-1),\ 17 - 8\cdot(3-1),\ 17 - 8\cdot(4-1),\ 17 - 8\cdot(5-1), \ldots\}$

$\boxed{\{17, 9, 1, -7, -15, \ldots\}}$ ← answer

$-8 \quad -8 \quad -8 \quad -8 \quad \ldots$

3 write the 1st four terms of a sequence (arithmetic) if $a_{13} = -60$ and $a_{33} = -160$

$a_n = a_1 + (n-1)d$

$n=13: \; a_{13} = a_1 + 12 \cdot d$

$n=33: \; a_{33} = a_1 + 32 \cdot d$

} Solve the system for a_1 & d

or $\Rightarrow \begin{cases} -60 = a_1 + 12d \\ -160 = a_1 + 32d \end{cases}$ } 2 eqns & 2 unknowns

Lets use inverse matrix:

$$\Rightarrow \underbrace{\begin{pmatrix} 1 & 12 \\ 1 & 32 \end{pmatrix}}_{A} \begin{pmatrix} a_1 \\ d \end{pmatrix} = \begin{pmatrix} -60 \\ -160 \end{pmatrix}$$

for, and only for, a 2X2 we exchange the diagonal elements, then change the sign on the off diagonal elements, then ÷ by the determinant = product of diag − prod. of off diag.

$$A^{-1} = \frac{\begin{pmatrix} 32 & -12 \\ -1 & 1 \end{pmatrix}}{1 \cdot 32 - 1 \cdot 12} = \frac{1}{20} \begin{pmatrix} 32 & -12 \\ -1 & 1 \end{pmatrix}$$

Now use A^{-1}:

$$\begin{pmatrix} a_1 \\ d \end{pmatrix} = \frac{1}{20} \begin{pmatrix} 32 & -12 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} -16 \\ -160 \end{pmatrix} = \frac{1}{20} \begin{pmatrix} -16 \cdot 32 + 160 \cdot 12 \\ -1 \cdot (-16) + (-160) \cdot 1 \end{pmatrix}$$

$$\begin{pmatrix} a_1 \\ d \end{pmatrix} = \frac{1}{20} \begin{pmatrix} 1408 \\ -144 \end{pmatrix} \Rightarrow \boxed{\begin{matrix} a_1 = 70.4 \\ d = -7.2 \end{matrix}}$$

now write the 1st 4 terms..

* Finally, recursive formula...

EX Use the given recursive formula to write the first four terms: $\begin{cases} a_1 = 39 \\ a_n = a_{n-1} - 3 \end{cases}$ ← "d = −3"

$\boxed{\{39, \underset{-3}{36}, \underset{-3}{33}, \underset{-3}{30}, 27, \ldots\}}$

EX write a recursive relation if $\{-15, -7, 1, \ldots\}$ $+8$ $+8$ $d = 8$

$\boxed{a_1 = -15,\ a_n = a_{n-1} + 8}$

seed

explicit general term

Alternatively: $\boxed{a_n = -15 + 8(n-1)}$

$\boxed{\text{11.2 is finnished}}$