

11.2 Arithmetic Sequences

①

When we see that the pattern in a sequence is a fixed difference, we have an arithmetic sequence

ex

$$\{ 5, 11, 17, 23, 29, \dots \}$$

$+6 +6 +6 +6$ ← arithmetic sequence.

6 is called the common difference "d"

recursive algorithm:

$$a_n = a_{n-1} + d$$

n=2: $a_2 = a_{2-1} + d$

or

$$\begin{aligned} a_2 &= a_1 + d \\ a_3 &= a_2 + d \\ a_4 &= a_3 + d \\ a_5 &= a_4 + d \\ &\vdots \\ a_n &= a_{n-1} + d \end{aligned}$$

$$a_3 = (a_1 + d) + d = a_1 + 2d$$

$$a_4 = (a_1 + 2d) + d = a_1 + 3d$$

$$a_5 = (a_1 + 3d) + d = a_1 + 4d$$

$$a_n = a_1 + d(n-1)$$

explicit (general) term

$$S_0 \{ \overset{n=1}{5}, \overset{n=2}{11}, \overset{n=3}{17}, \overset{n=4}{23}, 29, \dots, \underbrace{5 + 6(n-1)}_{a_n}, \dots \}$$

* non-arithmetic seq.

EX Analyze

{4, 16, 64, 256, 1024, ...}



- How do I get from 4 to 16? $\begin{matrix} \nearrow +12 \\ \searrow \times 4 \end{matrix}$
- How do I get from 16 to 64? $\begin{matrix} \nearrow +48 \\ \searrow \times 4 \end{matrix}$
- How do I get from 64 to 256? $\begin{matrix} \nearrow +192 \\ \searrow \times 4 \end{matrix}$

Q: Is this arithmetic? No we are not adding a common difference.

{ B.T.W.: this sequence is a geometric sequence since we are multiplying by a constant ratio, 4 } - 11.3



EX {11.4, 9.3, 7.2, 5.1, 3.0, ...}



• recursive form

$$a_n = a_{n-1} + d$$

$$a_n = a_{n-1} - 2.1$$

- arithmetic
- $d = -2.1$
- general term
- $a_n = a_1 + (n-1)d$
- $a_n = 11.4 + (n-1)(-2.1)$
- $a_n = 11.4 - 2.1(n-1)$

* C.S.I.

EX Write the 1st five terms of an arithmetic seq if $a_1 = 17$ and $a_7 = -31$

use $a_n = a_1 + d(n-1)$

$n=7 \quad a_7 = a_1 + d(7-1)$

$a_7 = 17 + d(6)$

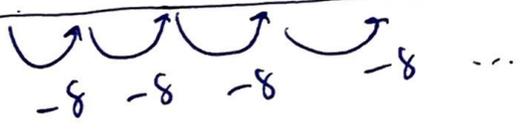
but \downarrow
 $-31 = 17 + d \cdot 6$ solve for d : $d = \frac{-31-17}{6} = -8$

So the general term, $a_n = 17 - 8(n-1)$

Now lets answer the question

$\{ 17 - 8 \cdot (1-1), 17 - 8 \cdot (2-1), 17 - 8 \cdot (3-1), 17 - 8 \cdot (4-1), 17 - 8 \cdot (5-1), \dots \}$

$\{ 17, 9, 1, -7, -15, \dots \}$ ← answer



3 Write the 1st four terms of a sequence (arithmetic) if $a_{13} = -60$ and $a_{33} = -160$

$$a_n = a_1 + (n-1)d$$

$$\left. \begin{array}{l} \rightarrow n=13: a_{13} = a_1 + 12 \cdot d \\ \rightarrow n=33: a_{33} = a_1 + 32 \cdot d \end{array} \right\}$$

Solve the system for a_1 & d

or $\Rightarrow \left\{ \begin{array}{l} -60 = a_1 + 12d \\ -160 = a_1 + 32d \end{array} \right\}$ 2 eqns & 2 unknowns

Lets use inverse matrix:

$$\Rightarrow \underbrace{\begin{pmatrix} 1 & 12 \\ 1 & 32 \end{pmatrix}}_A \begin{pmatrix} a_1 \\ d \end{pmatrix} = \begin{pmatrix} -60 \\ -160 \end{pmatrix}$$

for, and only for, a 2×2 we exchange the diagonal elements, then change the sign on the off diagonal elements, then \div by the determinant = product of diag - prod. of off diag.

$$A^{-1} = \frac{\begin{pmatrix} 32 & -12 \\ -1 & 1 \end{pmatrix}}{1 \cdot 32 - 1 \cdot 12} = \frac{1}{20} \begin{pmatrix} 32 & -12 \\ -1 & 1 \end{pmatrix}$$

Now use IA^{-1} :

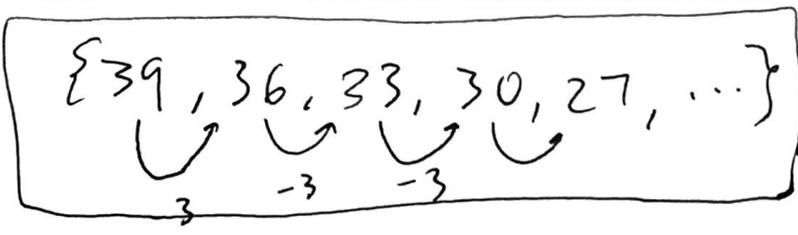
$$\begin{pmatrix} a_1 \\ d \end{pmatrix} = \frac{1}{20} \begin{pmatrix} 32 & -12 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} -160 \\ -160 \end{pmatrix} = \frac{1}{20} \begin{pmatrix} -16 \cdot 32 + 160 \cdot 12 \\ -1 \cdot (-160) + (-160) \cdot 1 \end{pmatrix}$$

$$\begin{pmatrix} a_1 \\ d \end{pmatrix} = \frac{1}{20} \begin{pmatrix} 1408 \\ -144 \end{pmatrix} \Rightarrow \begin{array}{|l} a_1 = 70.4 \\ d = -7.2 \end{array}$$

now write the 1st 4 terms...

* Finally, recursive formula...

EX use the given recursive formula to write the first four terms: $\begin{cases} a_1 = 39 \\ a_n = a_{n-1} - 3 \end{cases}$ ← "d = -3"



EX write a recursive relation if $\{-15, -7, 1, \dots\}$ with arrows indicating a common difference of +8 between consecutive terms.

$a_1 = -15, a_n = a_{n-1} + 8$

seed

explicit general term

Alternatively = $a_n = -15 + 8(n-1)$

11.2 is finished