$\boxed{\text{Chapter 11}}$ Sequences & Series

Goal is to approximate a function by using an ∞ series:

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots \qquad |x| < 1$$

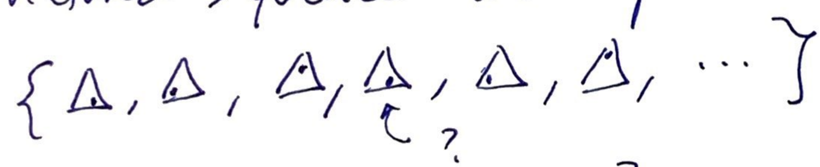
$\hookrightarrow -1 < x < 1$

$$e^x = 1 + x + \frac{x^2}{6} + \frac{x^3}{24} + \cdots$$

$\boxed{11.1}$ Sequences

A sequence is a set of numbers (or objects).

A structured sequence has a pattern.

$\boxed{\text{ex}}$ 

$\boxed{\text{ex}}$ $\left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \cdots\right\}$ $\frac{1}{5}$

* general term: a_n

$$\{\overset{n=1}{a_1}, \overset{n=2}{a_2}, \overset{n=3}{a_3}, \ldots, \overset{n^{th}\text{ term}}{a_n}, \cdots\}$$

In the example above

$a_1 = 1 \qquad a_4 = \frac{1}{4}$

$a_2 = \frac{1}{2} \qquad \vdots$

$a_3 = \frac{1}{3} \qquad \boxed{a_n = \frac{1}{n}}$ general term

Q: what is the 41^{st} term in the sequence

$a_{41} = \frac{1}{41}$

$\boxed{\text{EX}}$ write the 1^{st} 5 terms of the sequence whoes general term is $a_n = \frac{n^2}{2n+1}$

$a_1 = \frac{1^2}{2\cdot1+1} = \frac{1}{3}$

$a_2 = \frac{2^2}{2\cdot2+1} = \frac{4}{5}$

$a_3 = \frac{3^2}{2\cdot3+1} = \frac{9}{7}$

$a_4 = \frac{4^2}{2\cdot4+1} = \frac{16}{9}$

$a_5 = \frac{5^2}{2\cdot5+1} = \frac{25}{11}$

answer: $\left\{ \frac{1}{3}, \frac{4}{5}, \frac{9}{7}, \frac{16}{9}, \frac{25}{11}, \ldots, \frac{n^2}{2n+1}, \ldots \right\}$

$\boxed{\text{EX}}$ Given the sequence find the general term:

$\{ \underset{n=1}{4} \quad \underset{n=2}{7} \quad \underset{n=3}{12} \quad \underset{n=4}{19} \quad \underset{n=5}{28}, \ldots \}$

0^{th} Q: how do we convert n into the value in the sequence?

1^{st} Q: What is the next term?

2^{nd} Q: what do we do to get from $\frac{1}{4}$ to $\frac{1}{7}$ $(+3)$ then from $7 \xrightarrow{+5} 12$, then $12 \xrightarrow{+7} 19$, ...

A_2: we are adding a consecutive odd number to the denominator...

$\boxed{\text{EX}}$ Write the 1st 4 terms of the sequence whose general term is $a_n = \dfrac{a_{n-1} + 2n}{a_{n-1} - 1}$ previous term

and whose 1st term, a_1, is -4

$a_1 = -4$

$$a_2 = \frac{-4 + 2\cdot 2}{-4 - 1} = \frac{0}{-5} = 0$$

$$a_3 = \frac{0 + 2\cdot 3}{0 - 1} = -6$$

$$a_4 = \frac{-6 + 2\cdot 4}{-6 - 1} = \frac{2}{-7} = -\frac{2}{7}$$

$$a_5 = \frac{-\frac{2}{7} + 2\cdot 5}{-\frac{2}{7} - 1} = -\frac{68}{9}$$

$$\boxed{\left\{-4, 0, -6, -\frac{2}{7}, -\frac{68}{9}, \ldots, \frac{a_{n-1} + 2n}{a_{n-1} - 1}, \ldots\right\}}$$

Back to the unsolved example, but we need to focus on the denominator

denominator

$\{4, 7, 12, 19, 28, 39\}$

a_1, $a_2 = a_1 + 3$, $a_3 = a_2 + 5$, $a_4 = a_3 + 7$

$a_1 = 4$

$$\boxed{a_n = a_{n-1} + 2n + 1}$$

A_1: The next term

$\{4, 7, 12, 19, 28 \ldots\}$

$n =$ 1 2 3 4 5

+3, 5, 7, 9, 11

next term is $28 + 11 = \boxed{39}$

we increase by $2n+1$

$2n+1$ {odd num}

{$2n$ even}

A_0:

$a_n = 2n+1$

Test: $a_1 = 2 \cdot 1 + 1 = 3$ No! lets ans this later...

We need to examine a different type of sequence to Solve this.

\# recursive sequences:

$a_1 = 6$

$a_2 = a_1 + 5 = 6 + 5 = 11$

$a_3 = a_2 + 5 = 11 + 5 = 16$

$\boxed{a_n = a_{n-1} + 5}$

general term

$\boxed{\text{seed term}}$

when the general term involves a previous term we have a $\boxed{\text{recursive}}$ sequence

* factorial

$$n! \equiv n \cdot (n-1) \cdot (n-2) \cdots \cdot 3 \cdot 2 \cdot 1$$

ex: $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$

simplify:

$$\frac{(n+1)!}{n!} = \frac{(n+1)\cancel{n!}}{\cancel{n!}} = \underline{\underline{n+1}}$$