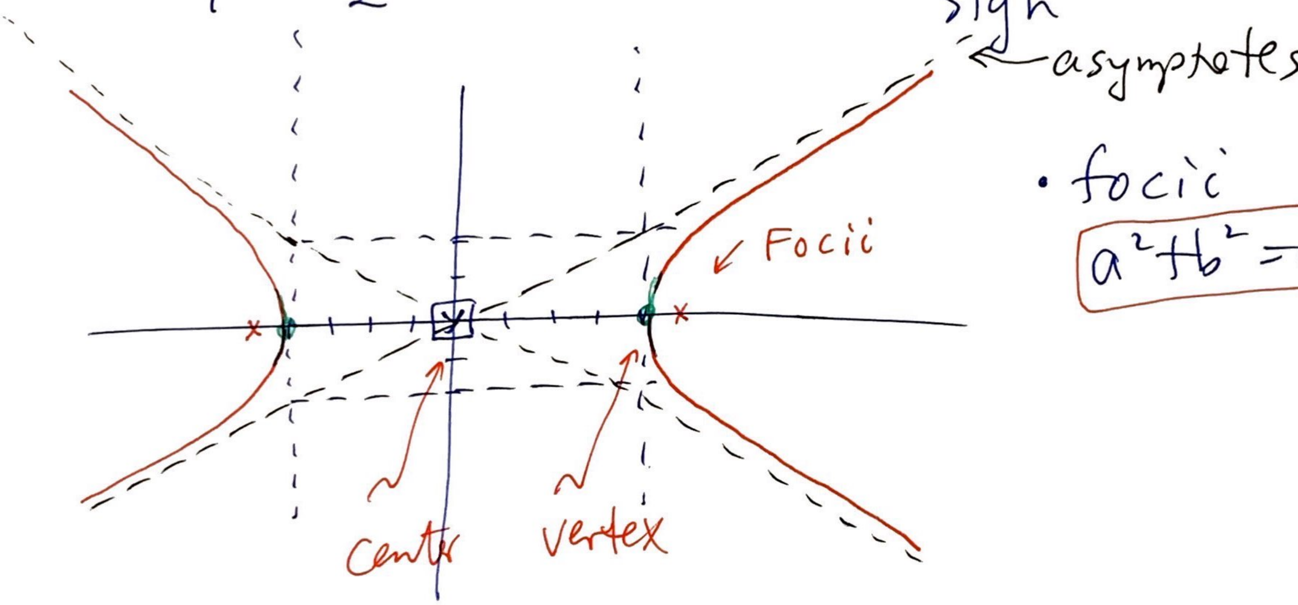
$\boxed{10.2}$ Hyperbolas

these are just like ellipses but they live outside the box. There focii are outside the box and the have asymptotes that pass through the opposite diagonals of the box.

Opens in x-axis

$\boxed{EX}$ $+\frac{x^2}{4^2} - \frac{y^2}{2^2} = 1$ we now have a negative sign



- focii

$\boxed{a^2+b^2=c^2}$

C/W Mod 4 (Cont.)

$\boxed{\#10}$ Sketch the hyperbola. Label the vertices & focii

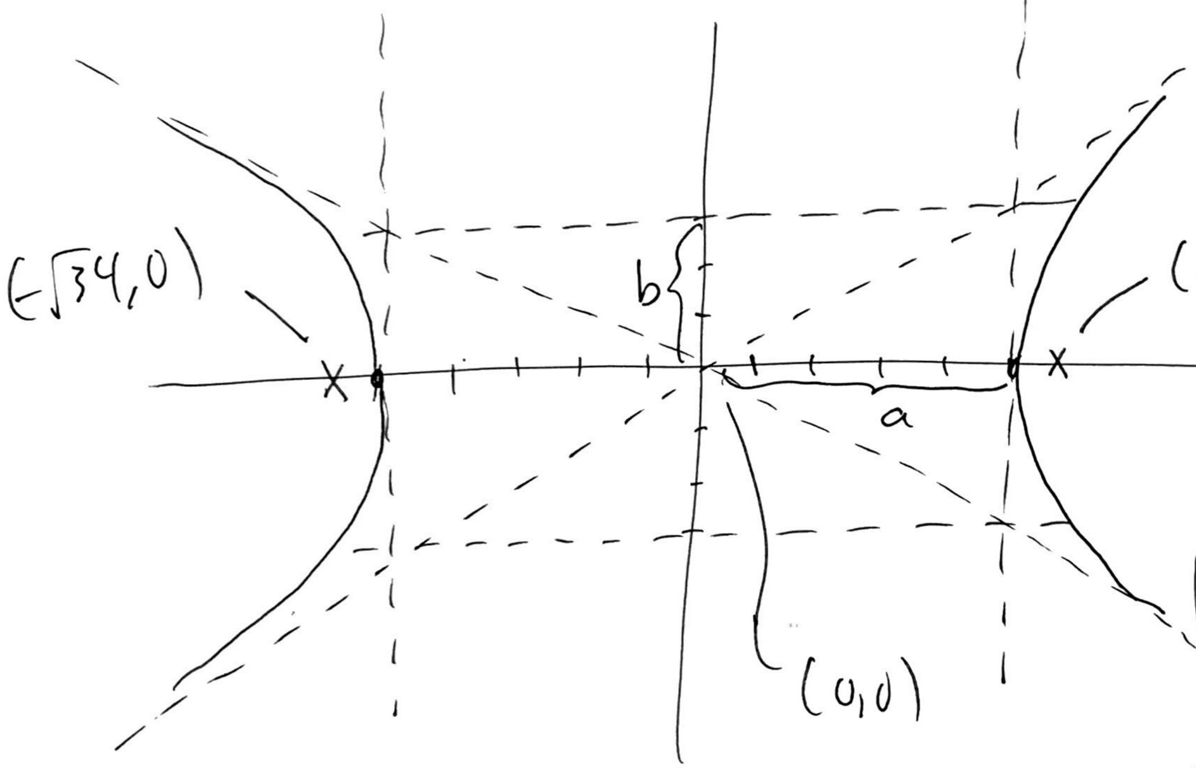
Also give the eqn of the asymptotes.

"a" is always the number under the x^2 term

$$\frac{x^2}{5^2} - \frac{y^2}{3^2} = 1$$

- $a = \underline{5}$ $b = \underline{3}$ $c = \sqrt{a^2+b^2} = \underline{\sqrt{34}}$
- center : $(0, 0)$

- opens: L/R or UP/Dn



rise

$$\boxed{y = \pm\frac{b}{a}x}$$

run

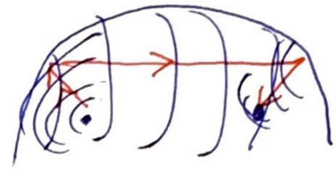
$$y = \pm\frac{3}{5}x$$

eqn of asymptote

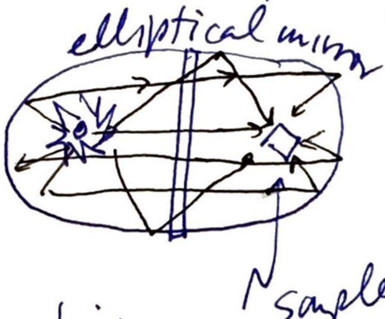
Application "Conic Cuts"

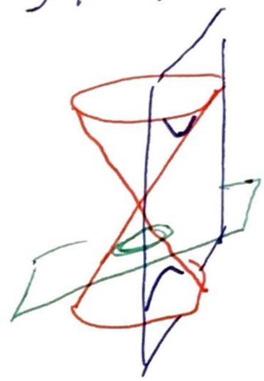
* Ellipses

applications : whisper gallery



: oven :

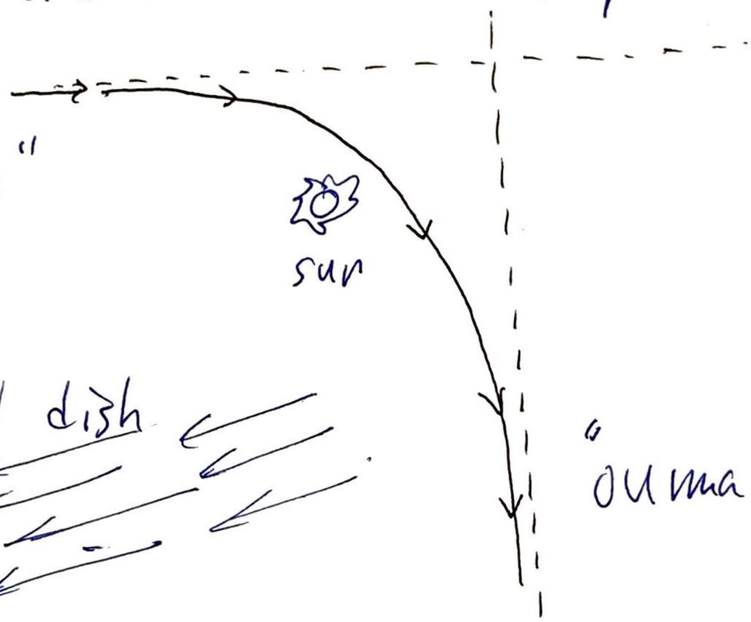




- planetary orbits → Kepler

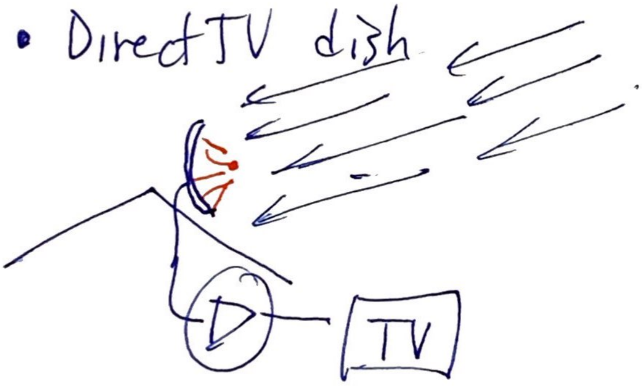
* Hyperbolas

- "orbits"



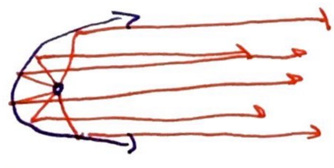
"oumauma"

- DirectTV dish



* parabolas

- "orbits" (not very frequent.
- Headlamps



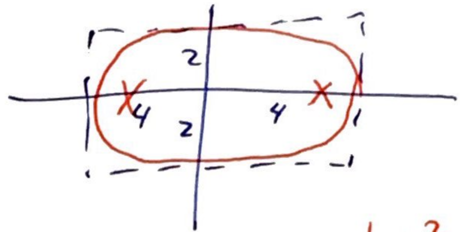
parallel beam

- sideline mic (ditto)

10.2 Hyperbolas (cont.)

- ellipses

$$\frac{x^2}{4^2}+\frac{y^2}{2^2}=1$$

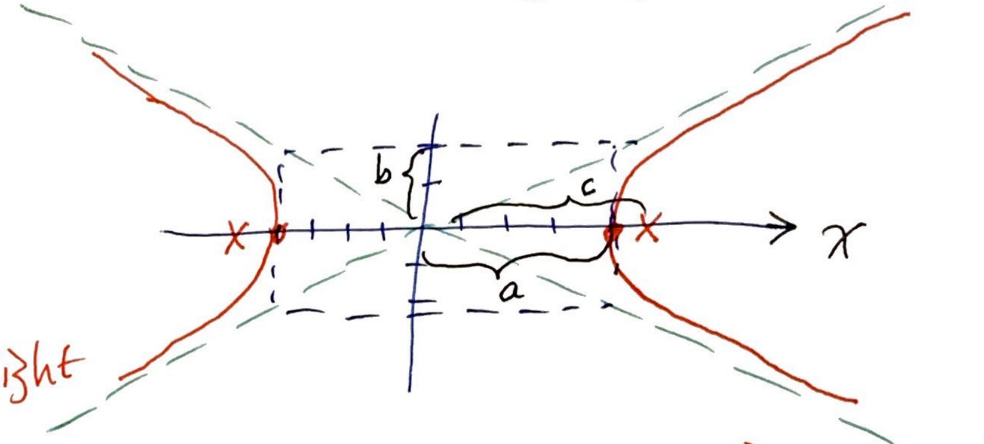


$c^2=|a^2-b^2|$

- hyperbola

$$\frac{x^2}{4^2}-\frac{y^2}{2^2}=1$$

⊕ we open L/Right



$c^2=a^2+b^2$

- eqns of the asymptotes:

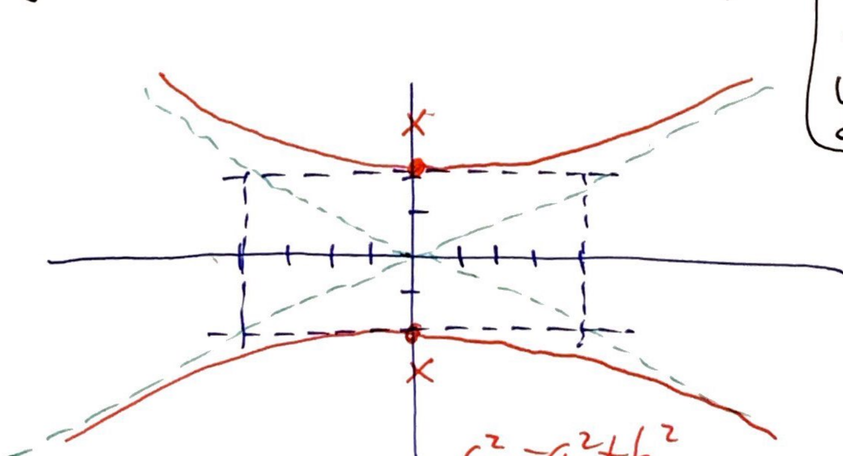
slope
$$\begin{cases} m_+ = \dfrac{\text{rise}}{\text{run}} = \dfrac{b}{a} = \dfrac{2}{4} \\ m_- = -\dfrac{2}{4} \end{cases}$$

eqn: $y = mx + b^{\nearrow 0} = \dfrac{b}{a}x = \dfrac{2}{4}x \Rightarrow$ $\boxed{\begin{array}{l} y=\frac{1}{2}x \\ y=-\frac{1}{2}x \end{array}}$

- hyperbola

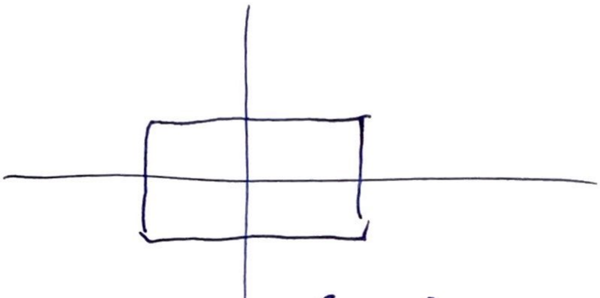
$$-\frac{x^2}{4^2}+\frac{y^2}{2^2}=1$$

⊕ opens up/down

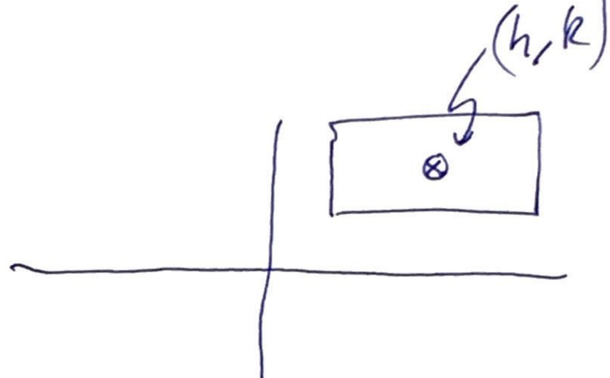


$c^2=a^2+b^2$

* off - center



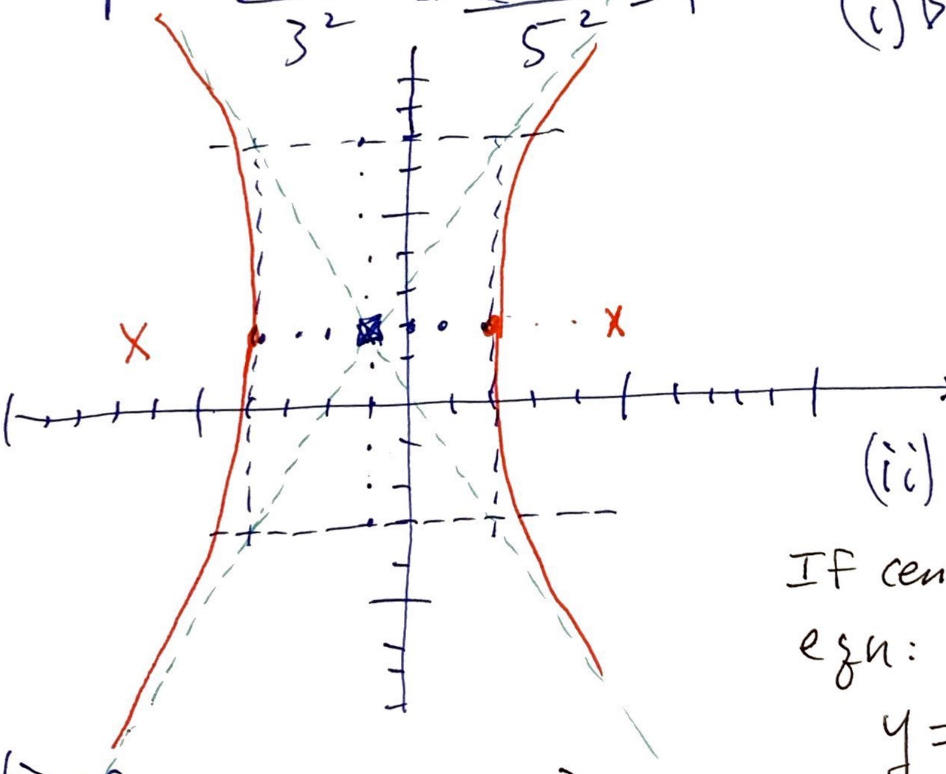
$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$$



$$\boxed{\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1}$$

$\boxed{\text{EX}}$ Graph $\dfrac{(x+1)^2}{3^2}-\dfrac{(y-2)^2}{5^2}=1$

(i) BOX
- center $(-1,2)$
- sides @ ± 3
- top/bottom @ ± 5



(ii) Asymptotes

IF center @ $(0,0)$

eqn:

$$y=\pm\frac{5}{3}x$$

Now shift the line:

$$\boxed{(y-2)=\pm\frac{5}{3}(x+1)}$$

(iii) vertices

opens L/R b/c $\oplus\dfrac{(x+1)^2}{3^2}$

(iv) focii

$$c^2=a^2+b^2$$

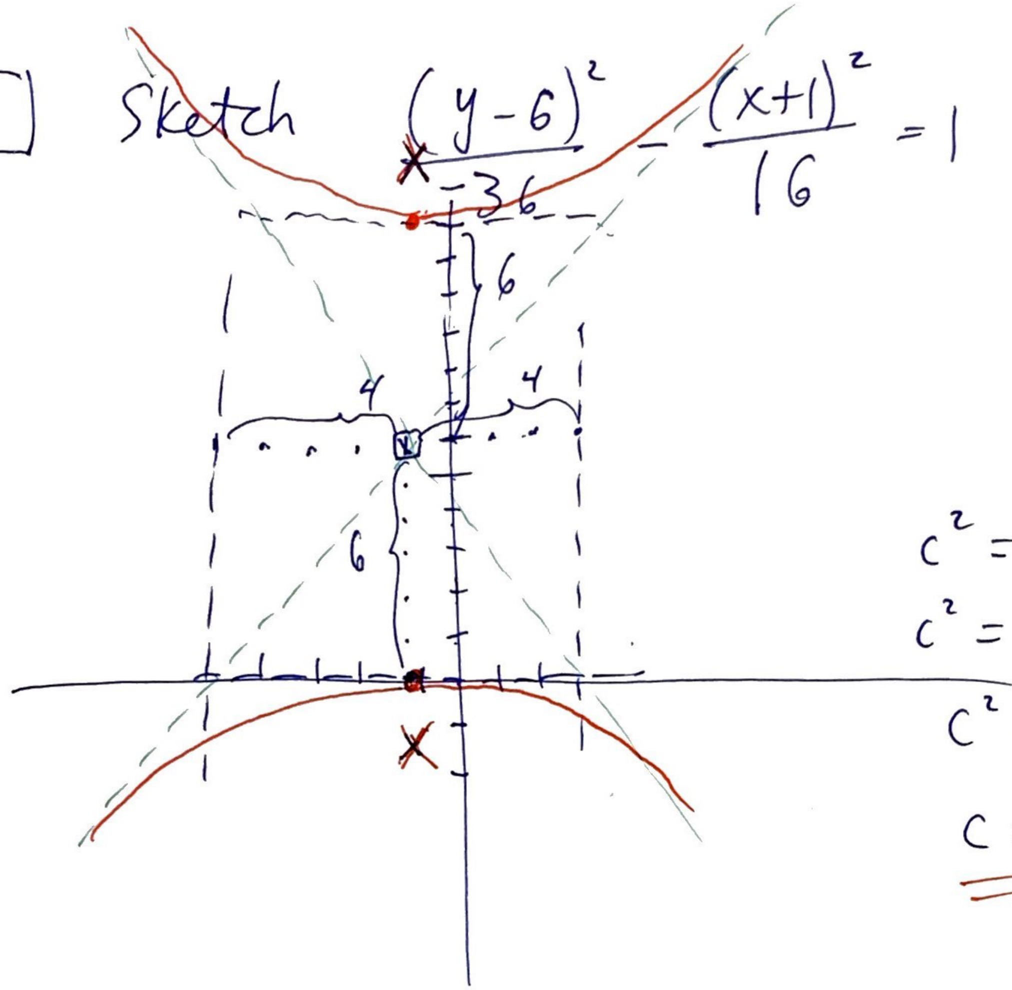
$$c^2=3^2+5^2$$

So $c=\pm\sqrt{34}\approx\pm\underline{5.9}$ from the center (not origin)

Math 104 CW Mod V Name________

$\boxed{1}$ Sketch $\dfrac{(y-6)^2}{36} - \dfrac{(x+1)^2}{16} = 1$

(i) • center $(-1,6)$
(ii) • box
(iii) • vertices up/Dn
(v) • focii
(iv) • eqns of asymtote



$c^2 = a^2 + b^2$

$c^2 = 16 + 36$

$c^2 = 52$

$c = \pm\sqrt{52} \approx \pm 7.5$

• vertices: $(-1,6) \pm (0,6) = (-1,0)$ & $(-1,12)$

• focii: $(-1,6) \pm (0,\sqrt{52}) = (-1, 6-\sqrt{52})$ and $(-1, 6+\sqrt{52})$

• asymptotes: $y = \pm \frac{6}{4}x$ if @ $(0,0)$

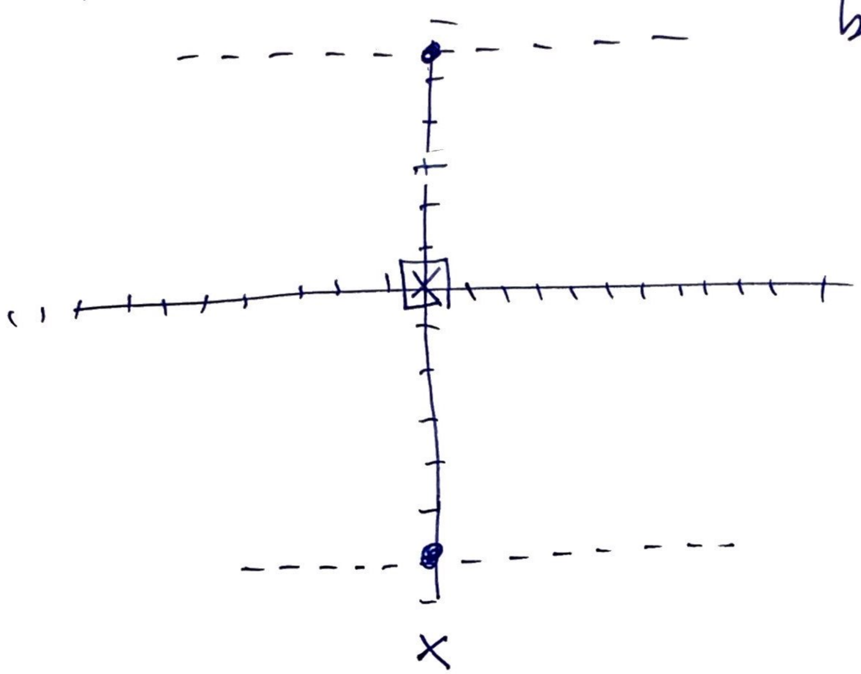
$$\boxed{(y-6) = \pm \frac{6}{4}(x+1)}$$

4

* C.S.I.

EX: vertices @ (0,6) and (0,−6) and one foci @ (0,−8) What is the eqn

*graph given info 1st



half way through between the vertices is the center so center is @ $\begin{pmatrix} x=0 \\ y=0 \end{pmatrix}$

$\frac{1}{2}$

$\frac{1}{2}$

* display a hyperbola's std. form, then populate it

$$\pm \frac{(x-h)^2}{a^2} \mp \frac{(y-k)^2}{b^2} = 1$$

$h \to 0$, $k \to 0$, $a^2 \to ?$, $b^2 \to 6^2$

$$-\frac{x^2}{a^2} + \frac{y^2}{6^2} = 1$$

* ~~Fill in missing info~~:

$$\boxed{-\frac{x^2}{28} + \frac{y^2}{36} = 1}$$

$c^2 = a^2 + b^2$

$(8)^2 = a^2 + 6^2$

$a^2 = 8^2 - 6^2$

$= 64 - 36$

$= 28$

$\boxed{a = \sqrt{28}}$