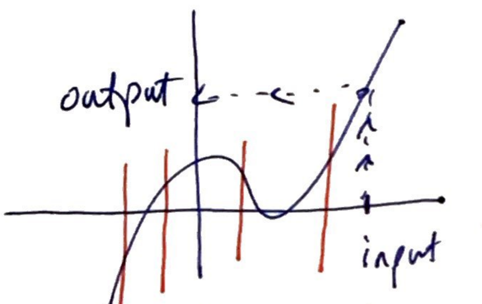
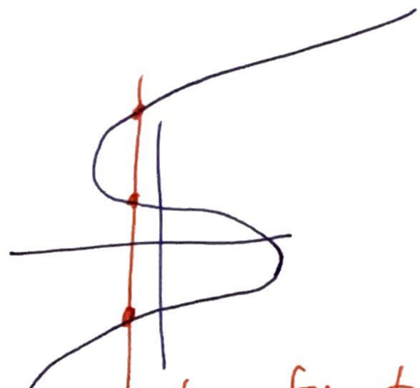
1.7 Inverse Functions

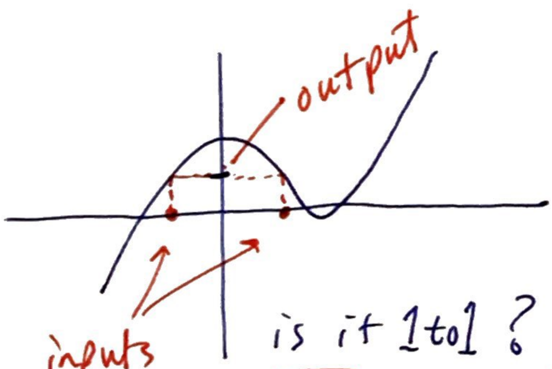
- A <u>function</u> is a relation that has a single output for each valid input.
- A <u>one-to-one</u> function is a function that, for each output only one input yeilded that output.



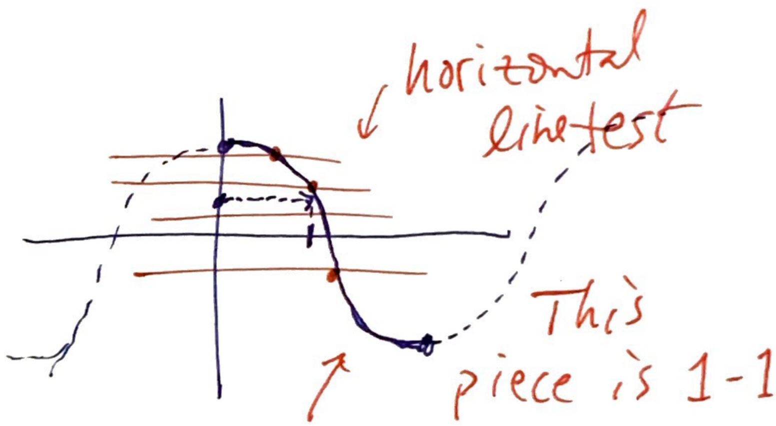
A function



not a function



$\boxed{\text{No}}$ $\exists$ 2 inputs that yeild the output show



$f(x) = Cos(x)$

↑ principple cosine.

* All 1-to-1 functions have an inverse function.

- An <u>inverse function</u> is a function that reverses the input and output of a given 1-1 function.

- $f(x) = y = \sin(x)$ • Solve for x; $x = \sin^{-1}(y)$

but to graph we like $y = \sin^{-1}(x)$ {swap x for y}

- $f^{-1}(x) = \sin^{-1}(x)$ is ≠ "inverse" <u>not reciprocal</u> $\cancel{\frac{1}{f}}$ $2^{-1} = \frac{1}{2}$

*procedures for finding finding an inverse function

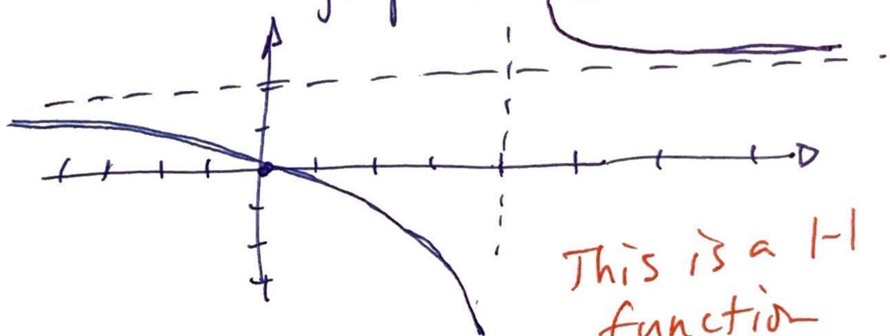
$\boxed{\text{EX}}$ $f(x) = \dfrac{2x}{x-4}$ {in a later section we learn to graph these}

future section

V.A : $x = 4$

H.A : $y = 2$

zero @ $x = 0$



This is a 1-1 function

So we know f has an inverse function

1. Replace $f(x)$ with y: $y = \dfrac{2x}{x-4}$
2. Swap x for y: $x = \dfrac{2y}{y-4}$
3. Solve for y:

$$x(y-4) = 2y$$
$$xy - x4 = 2y$$
$$xy - 2y = x4$$
$$y(x-2) = 4x$$
$$y = \frac{4x}{x-2}$$

4. Replace y with $f^{-1}(x)$: $\boxed{f^{-1}(x) = \dfrac{4x}{x-2}}$

Math104 $\boxed{\text{CW \#1 Week 2}}$

#1 Find f^{-1} if

$$f(x) = x^2 - 7$$
$$y = x^2 - 7$$
$$x = y^2 - 7$$
$$y^2 = x + 7$$
$$y = \sqrt{x+7}$$

$\boxed{f^{-1} = \sqrt{x+7}}$

<u>$f^{-1}(x) = \sqrt{x+7}$</u>

* What do we have inverse function for?

- Middle School : Solve for x : $\frac{x}{2} = 7 \quad * 2$

$x = 7 \cdot 2$

$\boxed{x = 14}$

- Pre-Calculus: Solve for x : $\frac{x}{2} = 7$

Hmmm $f(x) = \frac{x}{2}$ and this is then the problem of $\boxed{\text{when is } f(x) = 7}$

- And since $\frac{x}{2}$ is 1-1 we know f^{-1} exists and we know the 4 steps to find $f^{-1}(x)$.

1. $f = \frac{x}{2}$
2. $y = \frac{x}{2}$
3. $x = \frac{y}{2}$, $y = 2x$
4. $\boxed{f^{-1}(x) = 2x}$

- Now apply f^{-1} to both sides of the problem

$$f(x) = 7$$

$$\underbrace{f^{-1}(f(x))} = f^{-1}(7)$$

$$x = f^{-1}(7)$$

$$x = 2 \cdot 7$$

$$\boxed{x = 14}$$

purpose and property of f^{-1}:

$f^{-1}(f(x)) = x$ ← $f^{-1}\left(\frac{x}{2}\right) = 2 \cdot \frac{x}{2} = x$

Another example: $f(x) = \sin(x)$

$$\boxed{\text{Solve for } x: \ -\frac{\sqrt{3}}{2} = \sin(x)}$$

- note that if $f(x) = \sin x$ we seek the value of x for when $\boxed{f(x) = -\frac{\sqrt{3}}{2}}$
- note that $f^{-1}(x) = \sin^{-1}(x)$
- apply $f^{-1}(x)$ to both sides of $f(x) = -\frac{\sqrt{3}}{2}$

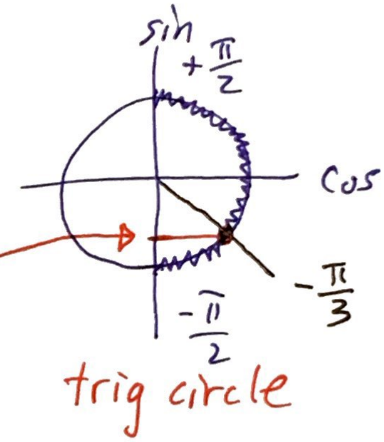
$$f^{-1}(f(x)) = f^{-1}\left(-\frac{\sqrt{3}}{2}\right)$$

- but we know $f^{-1}(f(x)) = x$ so the LHS:

$$x = f^{-1}\left(-\frac{\sqrt{3}}{2}\right)$$

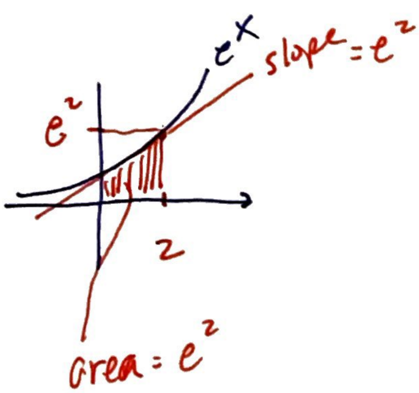
$$\boxed{x = \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)}$$

$$\boxed{\boxed{x = -\pi/3}}$$



List of some function, inverse function pairs

function	Inverse function
$f(x) = x$	$f^{-1}(x) = x$
$f(x) = \frac{x}{2}$	$f^{-1}(x) = 2x$
$f(x) = \sqrt{x}$	$f^{-1}(x) = x^2$
$f(x) = x^2$	$f^{-1}(x) = \sqrt{x}$
$f(x) = \cos(x)$	$f^{-1}(x) = \cos^{-1}(x) = \arccos(x)$
$f(x) = e^x$	$f^{-1}(x) = \ln(x)$
$f(x) = \ln(x)$	$f^{-1}(x) = e^x$
$f(x) = \frac{2x}{x-4}$	$f^{-1}(x) = \frac{4x}{x-2}$
⋮	⋮



$\boxed{EX}$ Solve for x

inverse function.

$$e^{3x+1} = 10$$

apply "$\ln(x)$" to both sides

$$\underline{\ln(e^{3x+1})} = \ln(10)$$

$$3x+1 = \ln(10) \longrightarrow 3x = \ln(10) - 1$$

$$\boxed{x = \frac{\ln(10)-1}{3}} \quad \div 3$$

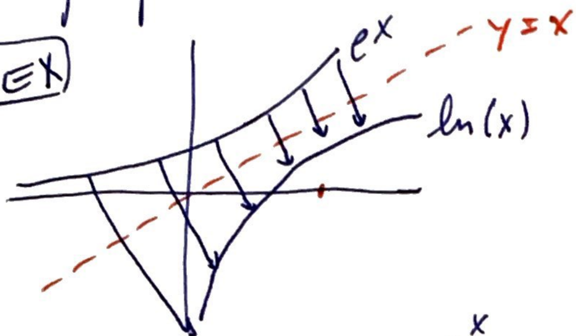
$\approx$

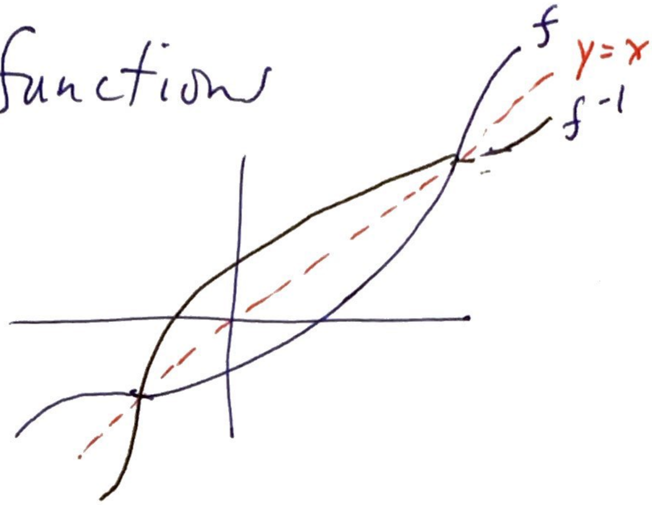
$f'(f(x)) = x$

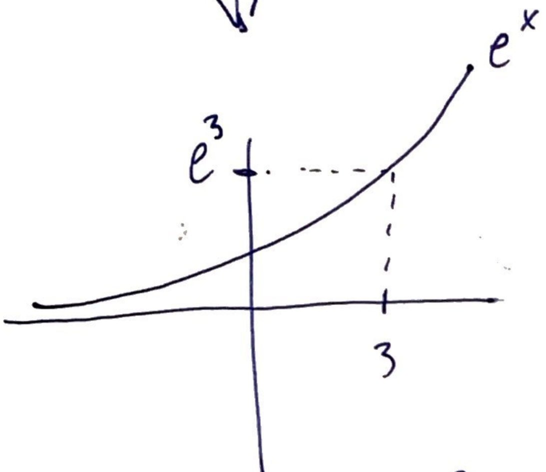
$10\ \boxed{\ln}\ \boxed{-}\ 1\ \boxed{=}\ \boxed{\div}\ \boxed{3}\ \boxed{=}$

* graphs of inverse functions

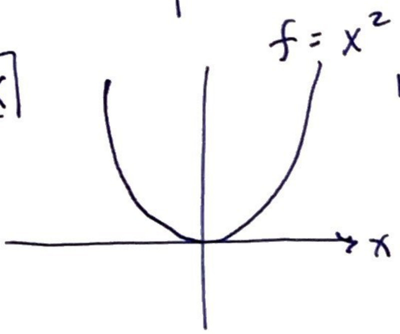
EX





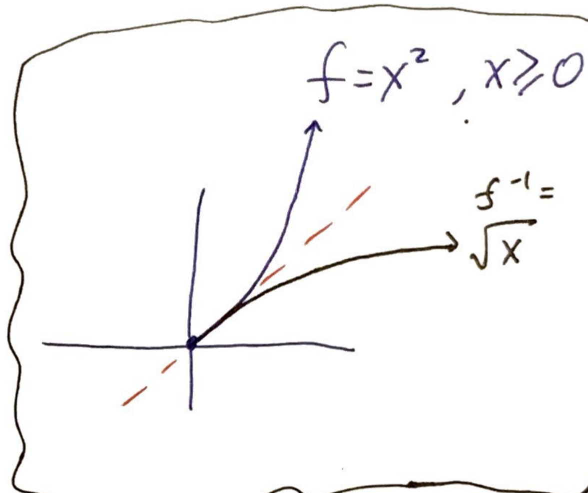


EX



not 1-1

focus on only a piece



* numerical relationships

x	0	1	2	3	4	5
$f(x)$	8	0	7	4	2	6

Q1: function? yes. all inputs have only one output

Q2: 1-1? yes No duplicate outputs.

Q3: write down f^{-1} ("swap x & y")

x	8	0	7	4	2	6
$f^{-1}(x)$	0	1	2	3	4	5