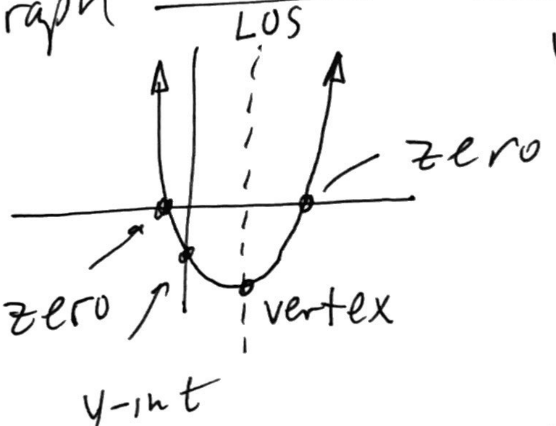
$\boxed{3.2}$ Quadratic Functions

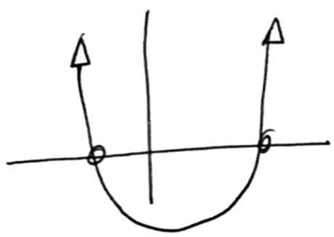
• $f(x) = ax^2 + bx + c$

∘ graph

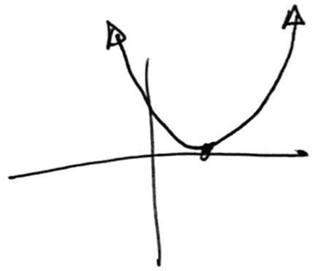


$y = ax^2 + bx + c$

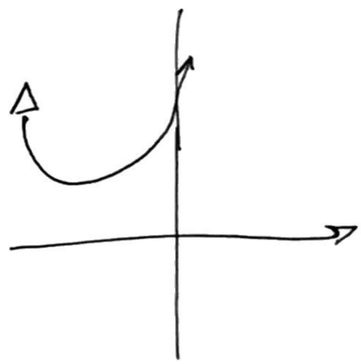
∘ zeros : $y = 0$ or $\{f(x) = 0\}$



2-zeros



1-zero (double)



no-zeros (complex conjugates)

$\boxed{EX}$ Find the zeros of

$f(x) = 2x^2 - 6x + 5$

$2x^2 - 6x + 5 = 0$

2, 1   5, 1

This is a "prime quadratic"

So... $x = \dfrac{6 \pm \sqrt{36 - 4 \cdot 2 \cdot 5}}{2 \cdot 2}$

$x = \dfrac{6 \pm \sqrt{-40}}{4}$

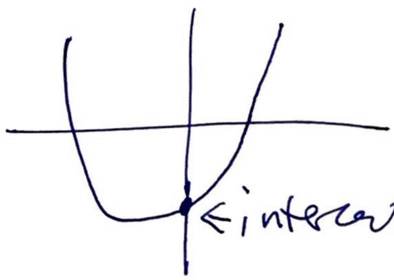
$x = \dfrac{6}{4} \pm \dfrac{2\sqrt{10}}{4} i$

$\boxed{x = \dfrac{3}{2} \pm \dfrac{\sqrt{10}}{2} i}$

* y-intercept of a quadratic function ②

$\boxed{Ex}$ $f(x) = 5x^2 - 8x + 5$ general form

where does it's graph intercept the y-axis?



$f(0) = 5 \cdot 0^2 - 8 \cdot 0 + 5$

$f(0) = 5$

y-int $\boxed{(x,y) = (0,5)}$

$\boxed{Ex}$ $f(x) = (x-2)(x+4)$

$f(0) = (0-2)(0+4)$

$= (-2)(4)$

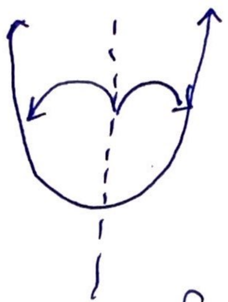
$= -8$

$\boxed{(x,y) = (0,-8)}$

alt. you could foil:

$f(x) = x^2 + 2x - 8$

* Line of symmetry



Line of symmetry

$$x = -\frac{b}{2a}$$

$$f(x) = ax^2 + bx + c$$

zeros

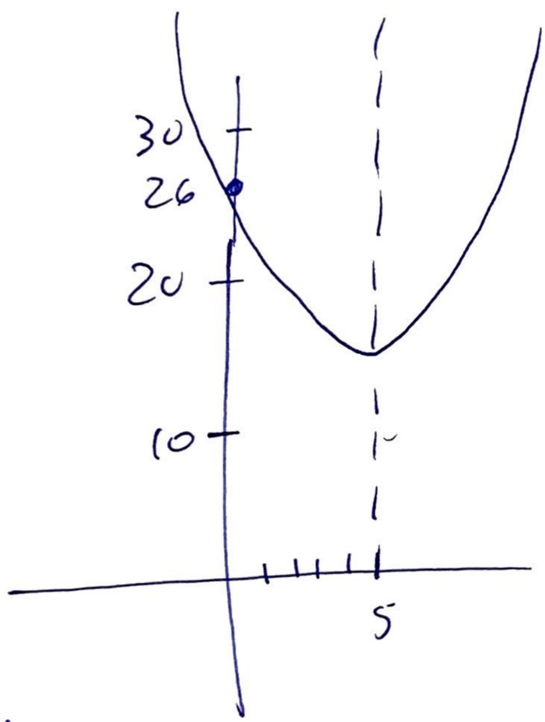
$$0 = ax^2 + bx + c$$

quad. formula

$$x = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

L.O.S.

$\boxed{\text{EX}}$ Find the L. of. Sym for $f(x) = x^2 - 10x + 26$



$a = 1,\ b = -10,\ c = 26$

$$x = -\frac{b}{2a}$$

$$x = \frac{-(-10)}{2(1)}$$

$$x = \frac{10}{2}$$

$$\boxed{x = 5}$$

* vertex

$$\boxed{(x,y) = \left(-\frac{b}{2a},\ f\left(-\frac{b}{2a}\right)\right)}$$

f @ L.O.S.

L. of. Sym

$\boxed{\text{EX}}$ Find the vertex of the function:

$f(x) = 2x^2 - 10x + 4$

• L.O.S.: $x = \frac{-b}{2a} = \frac{-(-10)}{2(2)} = \frac{10}{4} = \boxed{\frac{5}{2}}$

• so... $y = f(\text{LOS}) = 2\left(\frac{5}{2}\right)^2 - 10\left(\frac{5}{2}\right) + 4$

$= 2 \cdot \frac{25}{\cancel{4}_2} - 25 + 4$

$= \frac{25}{2} - 25 + 4$

$= \frac{25}{2} - 21$

$= \frac{25}{2} - \frac{42}{2}$

$= -\frac{17}{2}$

vertex: $\boxed{(x,y) = \left(\frac{5}{2}, -\frac{17}{2}\right)}$

$\boxed{\text{EX}}$ $f(x) = x(x-5)$

$= x^2 - 5x$

• $a = 1,\ b = -5,\ c = 0$

$x_{\text{los}} = -\frac{(-5)}{2(1)} = \frac{5}{2}$

$f\left(\frac{5}{2}\right) = \left(\frac{5}{2}\right)\left(\frac{5}{2} - 5\right)$

$= \frac{5}{2} \cdot \left(-\frac{5}{2}\right)$

$= -\frac{25}{4}$

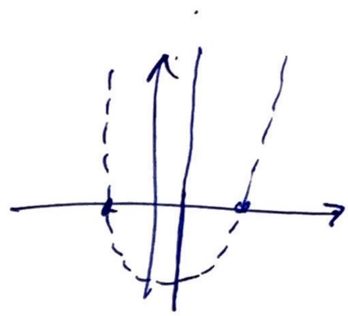
$\boxed{(x,y) = \left(\frac{5}{2}, -\frac{25}{4}\right)}$

*alt method for functions with real zeros

EX $f(x) = (x-2)(3x+5)$

zeros : $x-2=0$ or $3x+5=0$

$\boxed{x=2}$ or $\boxed{x=-\frac{5}{3}}$



LOS is half way between the zeros.

• mid point $x_{\text{mid point}} = \dfrac{x_1+x_2}{2}$

$= \dfrac{2+\frac{-5}{3}}{2}$

$= 1 - \dfrac{5}{6}$

$\boxed{x = \dfrac{1}{6}}$ L.O.S.

Check: $f(x) = (x-2)(3x+5)$

$= 3x^2+5x-6x-10$

$f = 3x^2 - x - 10$

$a=3,\ b=-1,\ c=-10$

LOS: $x = \dfrac{-b}{2a} = \dfrac{-(-1)}{2(3)} = \boxed{\dfrac{1}{6}}$

* Tell me all about it

upside down!

yint

$\boxed{EX}$ $f(x) = -4x^2 + 6x - 1$

- LOS: $x = -\frac{b}{2a} = \frac{-(6)}{2(-4)} = \boxed{\frac{3}{4}}$

- vertex $f\left(\frac{3}{4}\right) = -4\left(\frac{3}{4}\right)^2 + 6\left(\frac{3}{4}\right) - 1$

$= -\frac{9}{4} + \frac{18}{4} - \frac{4}{4} \rightarrow y = \frac{5}{4}$

$\boxed{\left(\frac{3}{4}, \frac{5}{4}\right)}$

- y-int: -1 $\quad \boxed{(0,-1)}$

- zeros: $0 = -4x^2 + 6x - 1$

or $\quad 4x^2 - 6x + 1 = 0$

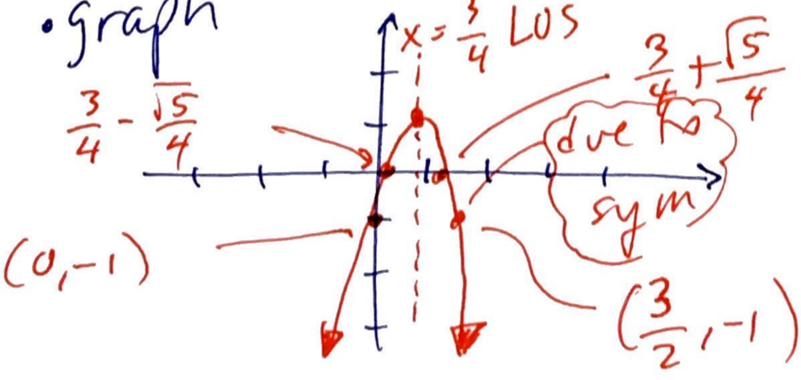
$a = 4, b = -6, c = 1$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(4)(1)}}{2(4)}$$

$$x = \frac{6 \pm \sqrt{36 - 16}}{8} = \frac{6 \pm \sqrt{20}}{8} = \frac{6 \pm 2\sqrt{5}}{8}$$

$\boxed{x = \frac{3}{4} \pm \frac{\sqrt{5}}{4}}$

- graph



- std form

$f(x) = a(x-h)^2 + k$

$\boxed{f(x) = -4\left(x - \frac{3}{4}\right)^2 + \frac{5}{4}}$

* Finding the equation of a quadratic given some information:

Ex vertex: $(h,k) = (-2,-1)$
passes through $(x,y) = (-4,3)$

Form: $f(x) = a(x-h)^2 + k$ ← std. form

$y = a(x-(-2))^2 + (-1)$

$\boxed{y = a(x+2)^2 - 1}$

Point: $3 = a(-4+2)^2 - 1$

Solve: $3 = a \cdot 4 - 1$

$3+1 = a \cdot 4$

$4 = a \cdot 4 \rightarrow a = 1$

Final: $y = 1 \cdot (x+2)^2 - 1$

$y =$

$\boxed{f(x) = (x+2)^2 - 1}$ std. form

$f(x) = x^2 + 4x + 3$ gen form