

1.4 function Composition

①

In this section we deal with two functions and the multiple ways of their interactions

$$\boxed{\text{ex}} \begin{cases} f(x) = 2x^2 + 1 \\ g(x) = x \end{cases}$$

$$(i) f \cdot g = (2x^2 + 1)(x) = \underline{\underline{2x^3 + x}}$$

$$(ii) f/g = \boxed{\frac{2x^2 + 1}{x}}$$

$$(iii) g/f = \boxed{\frac{x}{2x^2 + 1}}$$

* Composition : one function swallows the other.

notation $f \circ g$ "eff composed with gee"

$$\boxed{f \circ g \equiv f(g)}$$

$$\boxed{\text{ex}} \text{ let } f(x) = \sqrt{x-2}$$

Find $(f \circ g)(x)$

$$= f(g(x))$$

$$= \sqrt{g(x) - 2}$$

$$= \sqrt{\frac{x}{3} - 2}$$

$$\text{let } g(x) = x/3$$

$$\begin{cases} f(\) = \sqrt{\ } - 2 \\ g(\) = (\) / 3 \end{cases}$$

$$\rightarrow \boxed{(f \circ g)(x) = \sqrt{\frac{x}{3} - 2}}$$

1.4 cont.

EX Find $(g \circ f)(x)$

$$= g(f(x))$$

$$= (f(x))^2 + 3$$

$$= (\sqrt{x} + 2)^2 + 3$$

$$= \sqrt{x}^2 + 2\sqrt{x} \cdot 2 + 2^2 + 3$$

$$g \circ f = \boxed{x + 4\sqrt{x} + 7}$$

$$\begin{cases} f(x) = \sqrt{x} + 2 \\ g(x) = x^2 + 3 \end{cases}$$

$$\begin{cases} f(\) = \sqrt{\ } + 2 \\ g(\) = (\)^2 + 3 \end{cases}$$

(2)

* Decomposition

Write a function as a composition of two functions

EX $h(x) = \left(\frac{1}{2x-3}\right)^2$

$$= f\left(\frac{1}{2x-3}\right)$$

$$= f(g)$$

$$= f \circ g$$

$$\begin{aligned} f &= (\)^2 \\ g &= \frac{1}{2x-3} \end{aligned}$$

$$\boxed{\begin{aligned} f(x) &= x^2 \\ g(x) &= \frac{1}{2x-3} \end{aligned}}$$

Also possible

$$f(x) = \frac{1}{x^2}$$

$$g(x) = 2x-3$$

* numerical (tabular)

x	0	1	2	3	4	5	6	7	8	9
f(x)	7	6	5	8	4	0	2	1	9	3
g(x)	9	5	6	2	1	8	7	3	4	0

[EX] Find $(f \circ g)(8)$

$$= f(g(8))$$

$$= f(4)$$

$$= \boxed{4}$$

[EX] $g(f(5))$

$$= g(0)$$

$$= \boxed{9}$$

[EX] $(f \circ f)(1)$

$$= f(f(1))$$

$$= f(6)$$

$$= \boxed{2}$$

$(f \circ g)(x) \neq (g \circ f)(x)$

* $f(x) = 4x + 8$
 $g(x) = 7 - x$ } find $f(g(0))$

$$= f(7 - 0)$$

$$= f(7)$$

$$= 4 \cdot 7 + 8$$

$$= \boxed{36}$$

find $g(f(0))$

$$= g(4 \cdot 0 + 8)$$

$$= g(8)$$

$$= 7 - 8$$

$$= \boxed{-1}$$