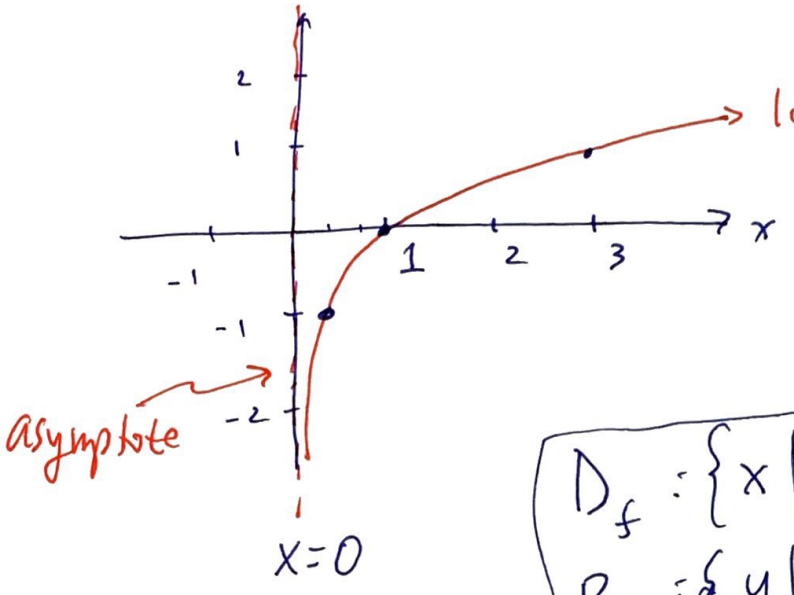
4.4 Graphs of Logs

$f(x) = \log_3(x)$: $\left(\frac{1}{3}, -1\right)$, $(1, 0)$, $(3, 1)$



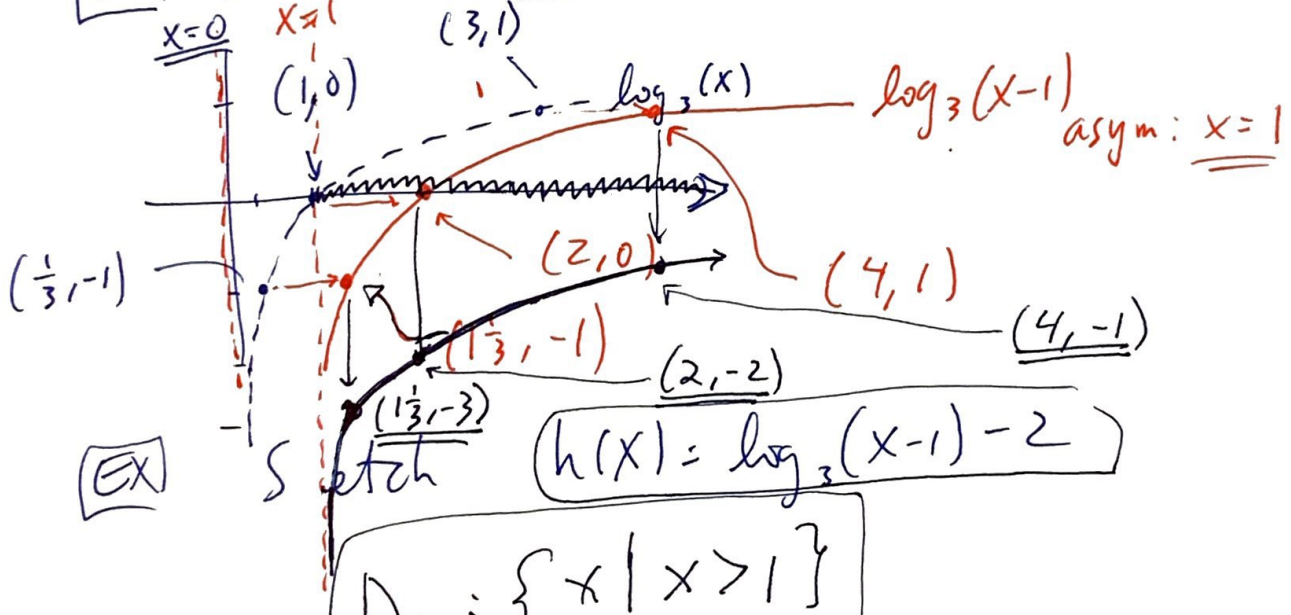
$\log_a(x)$: $\left(\frac{1}{a}, -1\right)$, $(1, 0)$, $(a, 1)$

$D_f : \{x \mid x > 0\}$

$R_f : \{y \mid y \in \mathbb{R}\}$

* Transformation :

EX Sketch $g(x) = \log_3(x-1)$



EX Sketch $h(x) = \log_3(x-1) - 2$

$D_h : \{x \mid x > 1\}$

$R_h : \{y \mid y \in \mathbb{R}\}$

* All logarithms have a domain such that the argument is > 0

EX $f(x) = \log_2(12-3x) - 3$

$12-3x > 0$ (i)

$-3x > -12$ (ii)

$x < \frac{-12}{-3}$

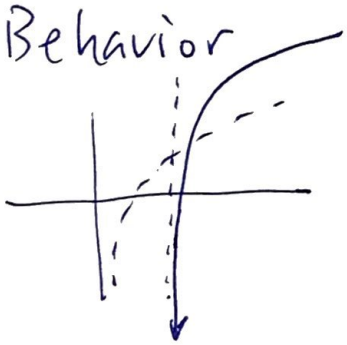
$x < 4$ (iii)

$D_f : \{x \mid x < 4\}$

* The vertical asymptote $0 = 12-3x$

$x = 4$

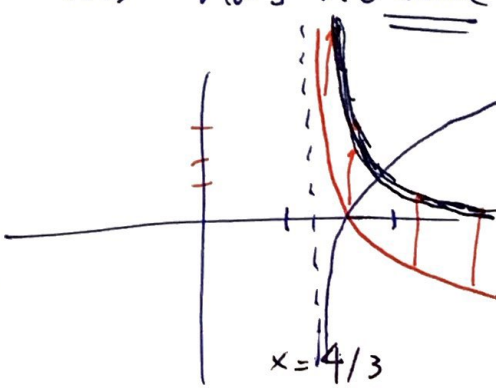
* End Behavior



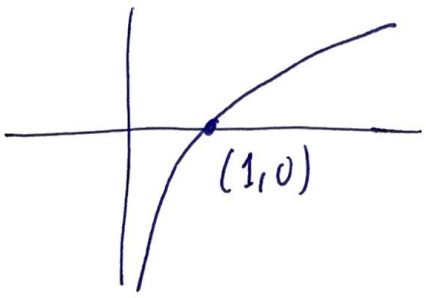
- as x increase $\log_a$ also increases w/o bound
- as x decreases the graph approaches the asymptote but otherwise has no bound

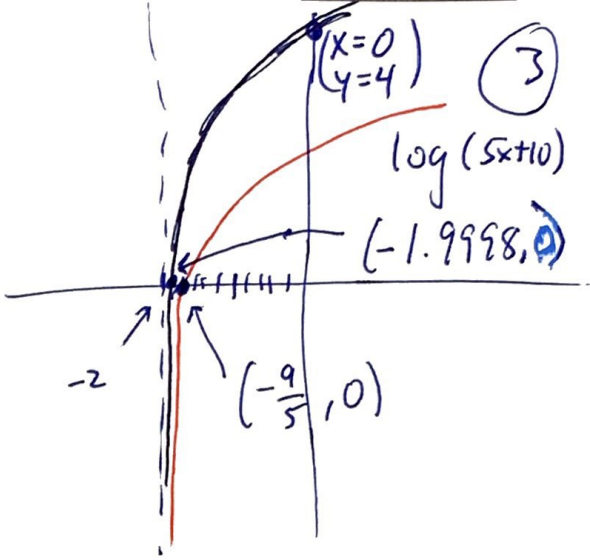
EX $h(x) = -\log(3x-4) + 3$ (horiz. shift, vertical boost)

- asymptote $0 = 3x-4 \rightarrow x = 4/3$
- the (-) sign flips the log upside down
- then the graph is raised 3 units.



* intercepts





EX $f(x) = \log(5x+10) + 3$

(i) asymptote $0 = 5x+10$

$x = -2$

(ii) x-intercept occurs when the argument is 1. So set the argument to 1

$1 = 5x+10$

$-9 = 5x$

$x = -9/5$ for $\log(5x+10)$

but what about for $\log(5x+10)+3$

The intercept changes: So set the function to zero and solve...

$f(x) = \log(5x+10)+3$

$0 = \log(5x+10)+3$

$-3 = \log(5x+10)$

$10^{-3} = 5x+10$

$-10+10^{-3} = 5x$

$x = \frac{10^{-3}-10}{5} = \frac{0.001-10}{5} = \frac{-9.999}{5} \approx -1.9998$

$(-1.9998, 0)$

(iii) For the y-intercept set x to 0

$$f(0) = \log(5\cdot 0+10)+3$$
$$= \log(10)+3$$
$$= 1+3$$
$$= 4 \rightarrow \boxed{(0,4)\ \text{y-int.}}$$

$$\boxed{D_f: \{x \mid x > -2\} \quad R_f: \{y \mid y \in \mathbb{R}\}}$$

* Solving log equations with Desmos:

EX Solve for x: (#52)

$$\log(2x-3)+2 = -\log(2x-3)+5$$
$$2\log(2x-3) = 5-2$$
$$\log(2x-3) = 3/2 \iff 10^{3/2} = 2x-3$$
$$\boxed{x = \frac{10^{1.5}+3}{2}} \text{ exact}$$

EX #53 Solve for x:

$$\ln(x-2) = -\ln(x+1)$$

- $f = \ln(x-2)$
- $g = -\ln(x+1)$
- zoom in on the intersection
- read of x value where the two functions intersect