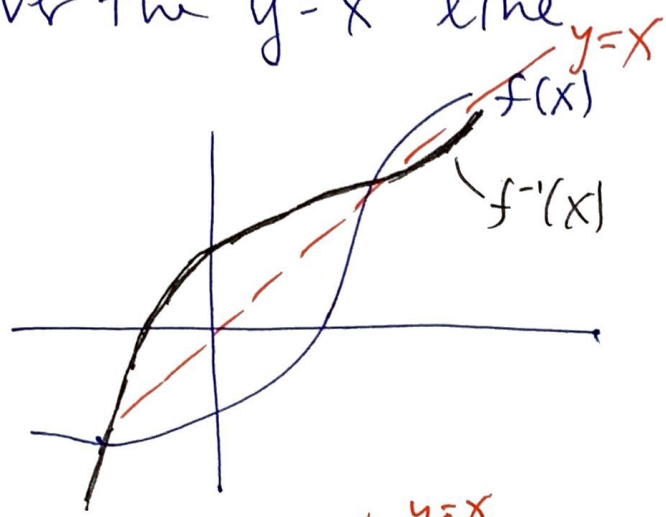
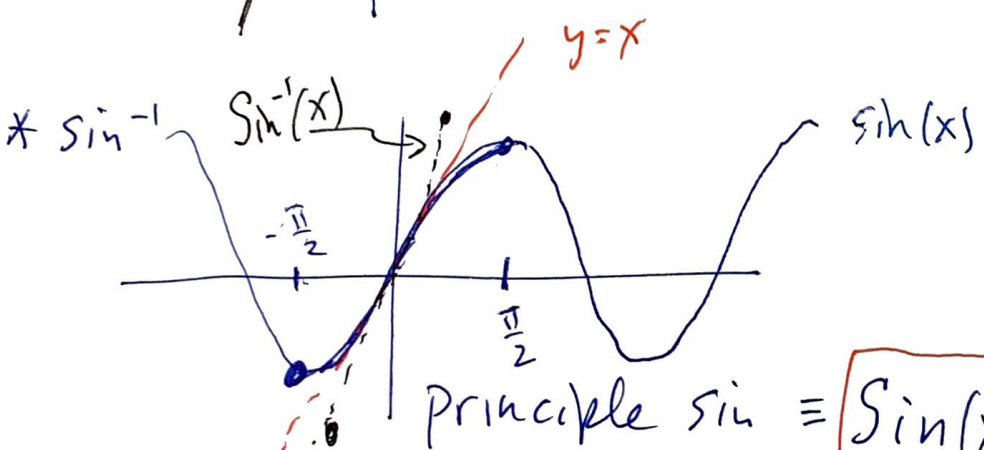
6.3 Inv. Trig Functions

Recall that if $f(x)$ is graphed, then $f^{-1}(x)$, it's inverse is $f(x)$'s graph reflected over the $y=x$ line

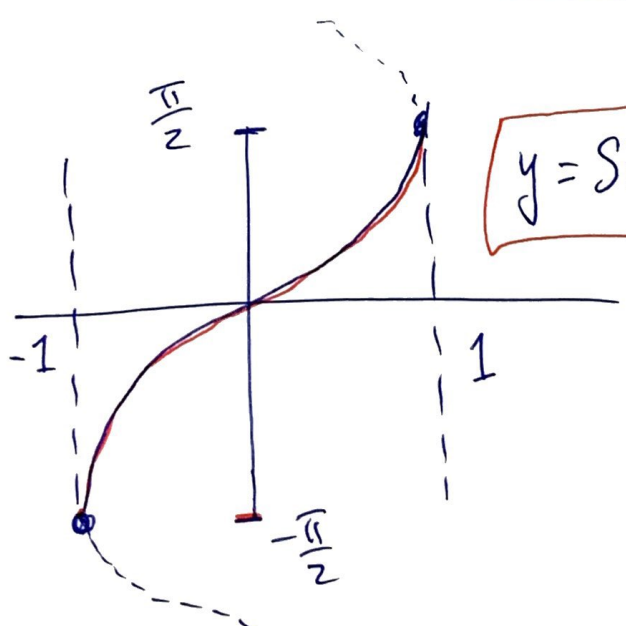


need f to be a function & a 1-1 function



Principle sin $\equiv$ $\boxed{Sin(x)}$ $\rightarrow$ $D:[-\frac{\pi}{2},\frac{\pi}{2}]$ $R:[-1,1]$

by itself

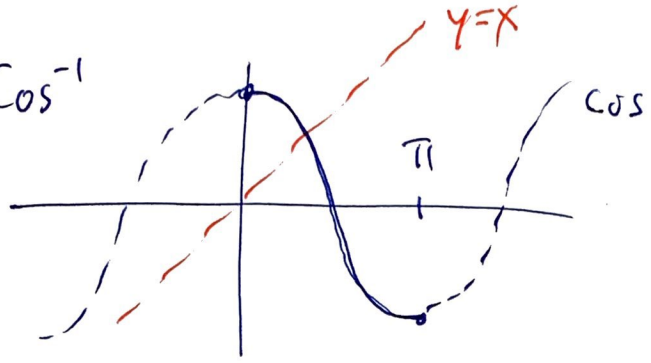


$\boxed{y=Sin^{-1}(x)}$

$D_{Sin^{-1}(x)}: \{x \mid x \in [-1,1]\}$

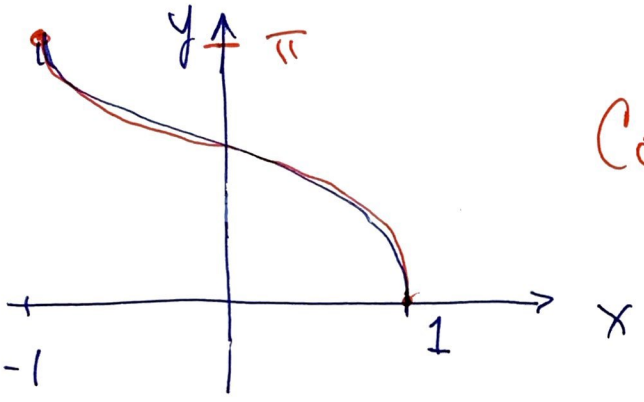
$R_{Sin^{-1}(x)}: \{y \mid y \in [-\frac{\pi}{2},\frac{\pi}{2}]\}$

* $\cos^{-1}$



$\boxed{\cos(x)}$ $D:[0,\pi]$

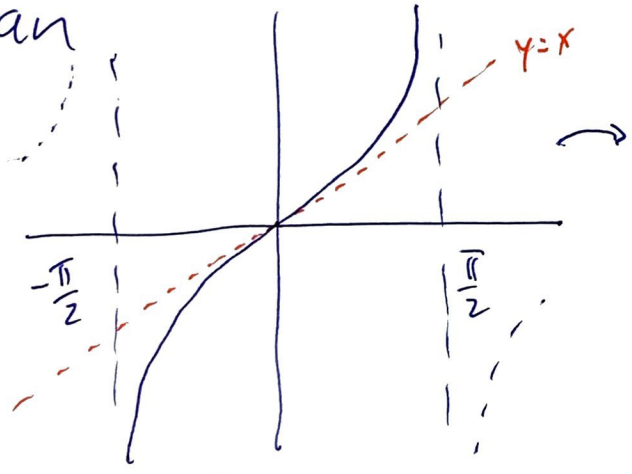
$R:[-1,1]$

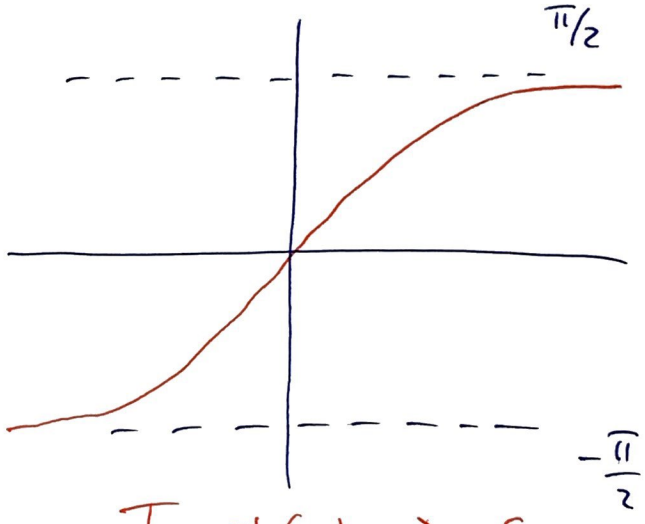


$\cos^{-1}(x)$: $D:[-1,1]$

$R:\{y \mid y \in [0,\pi]\}$

* tan





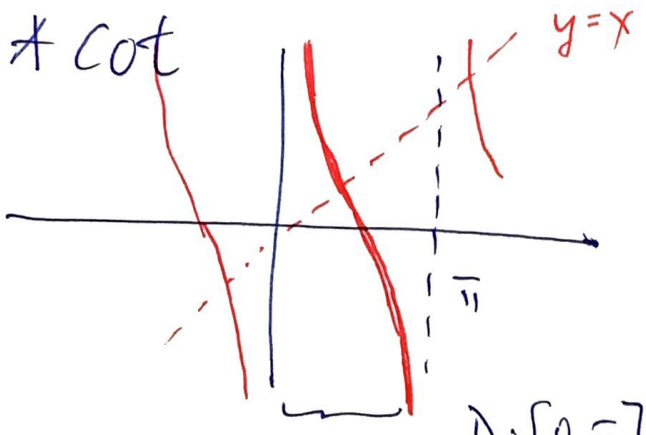
$\tan(x)$ $D:[-\frac{\pi}{2},\frac{\pi}{2}]$

$R:[-\infty,\infty]$

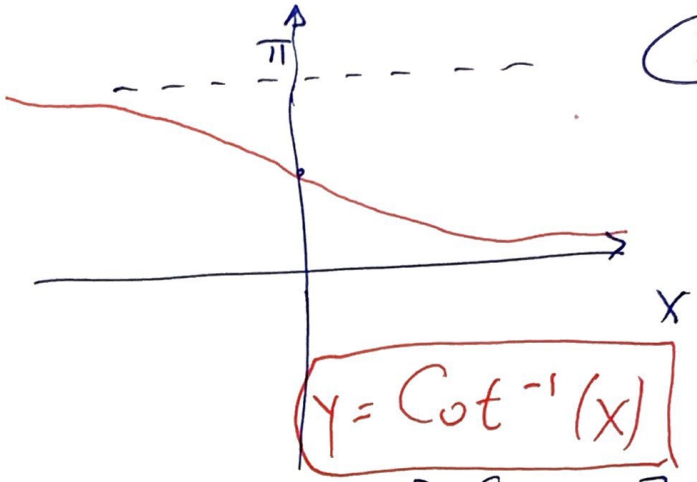
$\tan^{-1}(x)$ $D:[-\infty,\infty]$

$R:[-\frac{\pi}{2},\frac{\pi}{2}]$

* cot



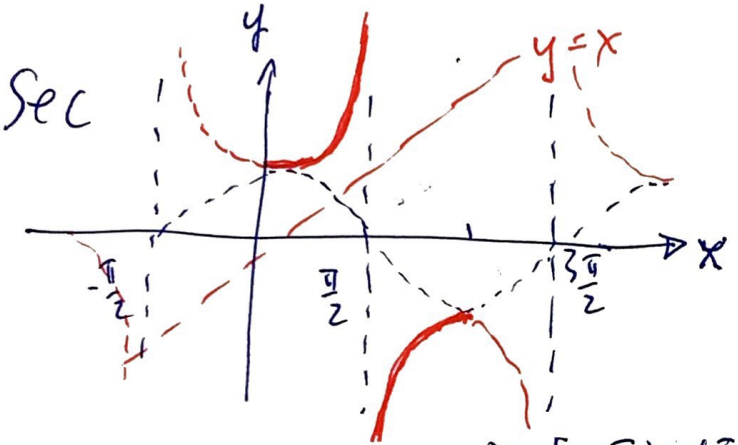
D: $[0, \pi]$
R: $[-\infty, \infty]$



$y = \cot^{-1}(x)$

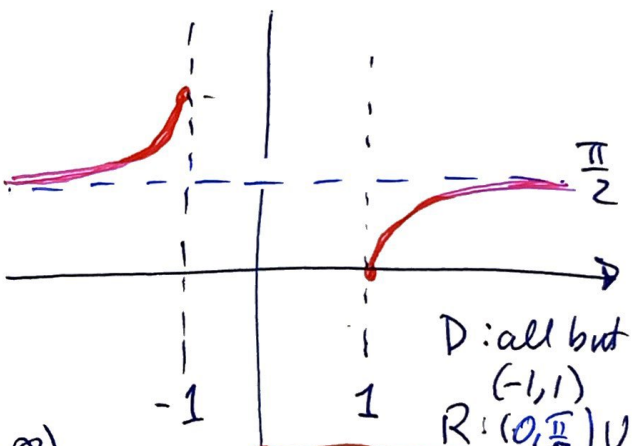
D: $[-\infty, \infty]$
R: $[0, \pi]$

* Sec



$y = \sec(x)$

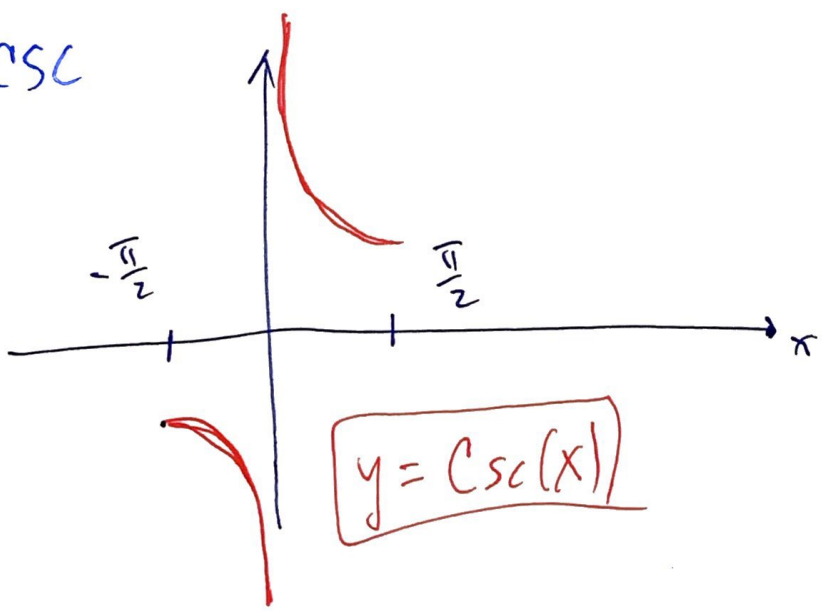
D: $[0, \frac{\pi}{2}) \cup (\frac{\pi}{2}, \pi]$
R: $[-\infty, -1] \cup [1, \infty)$



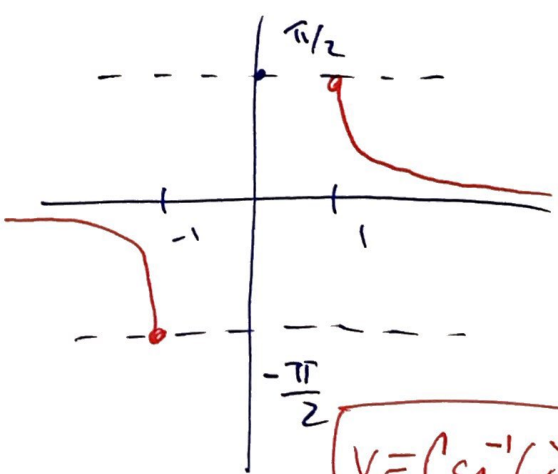
D: all but $(-1, 1)$
R: $(0, \frac{\pi}{2}) \cup (\frac{\pi}{2}, \pi]$

$y = \sec^{-1}(x)$

* csc



$y = \csc(x)$



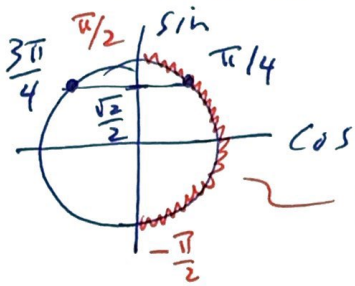
$y = \csc^{-1}(x)$

* work examples of inverse trig

EX $\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$

$= \boxed{\frac{\pi}{4}}$

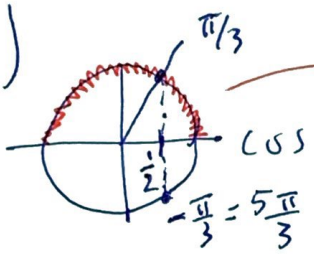
Remember that trig^{-1} are angles



$\sin^{-1}$ Range = Sin Dom.

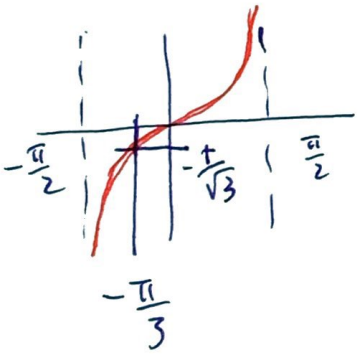
EX $\cos^{-1}\left(\frac{1}{2}\right)$

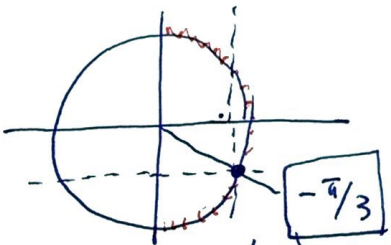
$= \boxed{\frac{\pi}{3}}$



$\cos^{-1}$ (Dom) = Cos Rang

EX $\tan^{-1}\left(\frac{-1}{\sqrt{3}}\right)$





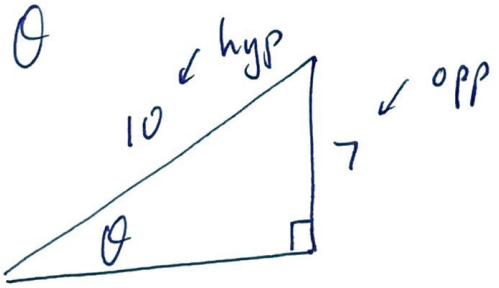
← Answer

let $\theta = \tan^{-1}\left(\frac{-1}{\sqrt{3}}\right)$

$\Rightarrow \tan\theta = \frac{-1}{\sqrt{3}}$ ÷ top & bot by 2

$\frac{\sin\theta}{\cos\theta} = \frac{-1}{\sqrt{3}} = \frac{-1/2}{\sqrt{3}/2}$

EX Find θ



$\sin\theta = \frac{opp}{hyp} = \frac{7}{10}$

$\theta = \sin^{-1}(7/10)$

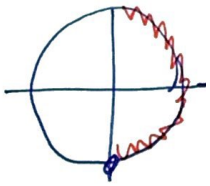
$\boxed{\theta = 44.4^\circ}$

* Trig eats Trig

EX $\sin^{-1}(\underbrace{\cos(\pi)}_{-1})$

$= \sin^{-1}(-1)$

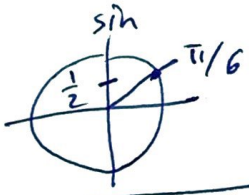
$= \boxed{-\pi/2}$

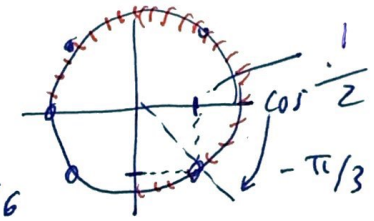


EX $\sin^{-1}\left(\cos\left(\frac{5\pi}{3}\right)\right)$

$= \sin^{-1}\left(\frac{1}{2}\right)$

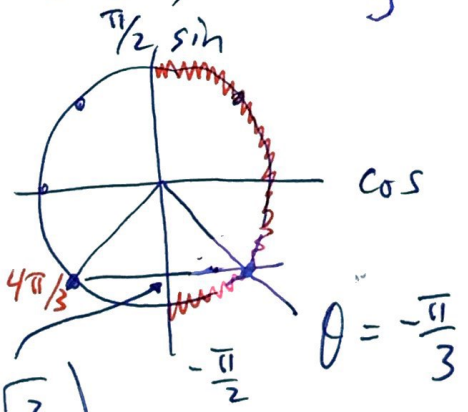
$= \boxed{\pi/6}$





Trig^{-1}(Trig)

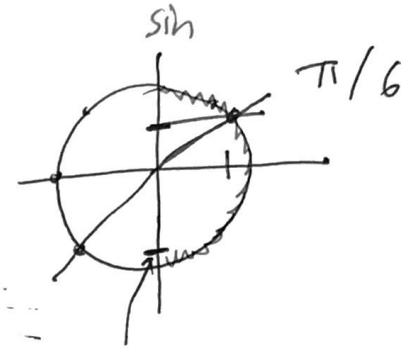
EX $\sin^{-1}\left(\sin\left(\frac{4\pi}{3}\right)\right) = \frac{4\pi}{3}$ yes?



$= \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

$= \boxed{-\frac{\pi}{3}}$

* Trig(Trig^{-1})



EX $\cos\left(\sin^{-1}\left(\frac{1}{2}\right)\right)$

$= \cos\left(\frac{\pi}{6}\right)$

$= \boxed{\frac{\sqrt{3}}{2}}$

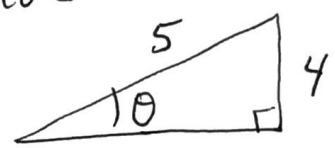
EX $\cos\left(\underbrace{\sin^{-1}\left(\frac{4}{5}\right)}_{\theta}\right)$

- method 1: Δ method

(i) θ

(ii) $\sin\theta = \frac{4}{5}$ $\frac{\text{opp}}{\text{hyp}}$

(iii)



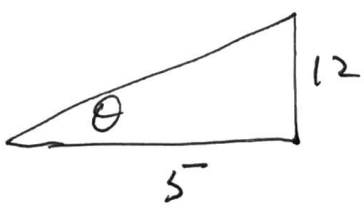
(iv) $\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{\sqrt{5^2-4^2}}{5} = \boxed{\frac{3}{5}}$

$\cos\left(\sin^{-1}\left(\frac{4}{5}\right)\right) = \frac{3}{5}$

- method 2: Identity method ((later))

EX $\sin\left(\underbrace{\tan^{-1}\left(\frac{12}{5}\right)}_{\theta}\right)$

$\tan\theta = \frac{12}{5}$



So $\sin(\theta) = \frac{\text{opp}}{\text{hyp}} = \frac{12}{\sqrt{12^2+5^2}} = \frac{12}{\sqrt{169}} = \boxed{\frac{12}{13}}$

EX

$$\tan\left(\underbrace{\sin^{-1}(x-1)}_{\theta}\right)$$

$$\sin\theta = \frac{x-1}{1} = \frac{\text{opp}}{\text{hyp}}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \boxed{\frac{(x-1)}{\sqrt{1-(x-1)^2}}}$$

Consider 6.3 completed

7.5 on Thursday

Practice 5: Solve for p.

$$\frac{b}{1}+\frac{d}{1}=\frac{f}{1}$$