7.5 Solving Trig Eqns

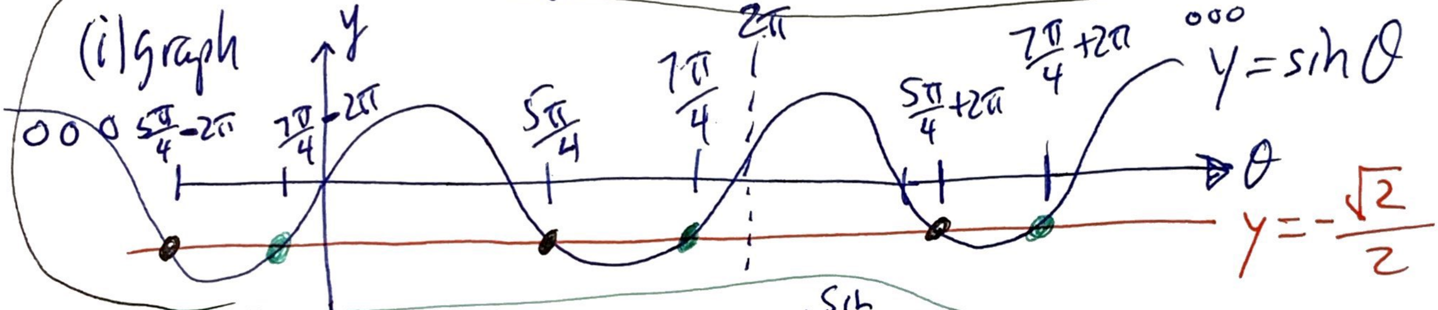
We now address solving equations (the essence of algebra)

EX Solve for x:
$\sin(6x)\cos(11x) - \cos(6x)\sin(11x) = -0.1$
{we will do this later}

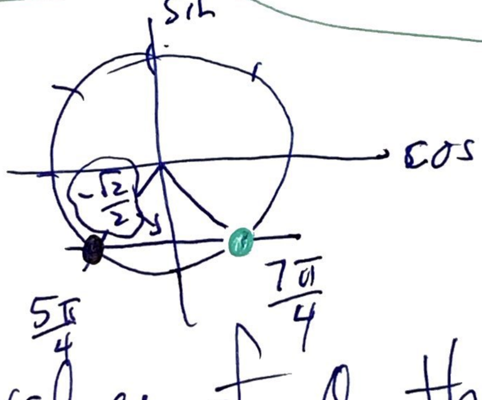
* start simple: consider $2\sin\theta = -\sqrt{2}$ what is θ?

$\div 2$ $\quad y = \sin\theta$ & when is $y = \frac{-\sqrt{2}}{2}$

(i) graph



(ii) we can also look @ the trig circle to assertain the values of θ that solve the eqn.



We have infinitely many answers. In fact we have 2 families of ∞ many answers.

$\boxed{\text{EX}}$ Cont.

$\sin\theta = -\frac{\sqrt{2}}{2}$

Familia uno: $\boxed{\theta = \frac{5\pi}{4} \pm 2n\pi}$

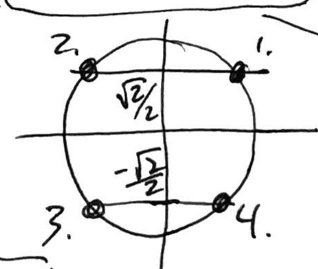
Familia dos: $\boxed{\theta = \frac{7\pi}{4} \pm 2n\pi}$

Q: What is the soln between $[0, 2\pi)$?

Ans: $\boxed{\theta = \frac{5\pi}{4} \text{ \& } \frac{7\pi}{4}}$

$\boxed{\text{EX}}$ Solve for x: $4\sin^2 x - 2 = 0$

- isolate $\sin^2 x$: $\sin^2 x = \frac{2}{4}$
- square root: $\sin x = \pm\sqrt{\frac{1}{2}}$ $\left(\frac{\sqrt{2}}{\sqrt{2}}\right)$
- rationalize denom: $\boxed{\sin x = \pm\frac{\sqrt{2}}{2}}$
- examine both branch (+) (−)



- Families:

$$\boxed{\begin{array}{l} 1.\ \frac{\pi}{4} \pm 2\pi n \\ 2.\ \frac{3\pi}{4} \pm 2\pi n \\ 3.\ \frac{5\pi}{4} \pm 2\pi n \\ 4.\ \frac{7\pi}{4} \pm 2\pi n \end{array}}$$

Index knowledge
even only: $2n$
odd only: $2n+1$

$\Rightarrow \frac{2n+1}{4}\pi$ all families

$\frac{\pi}{4} \pm \frac{\pi}{2}\cdot n$

$= \frac{\pi}{4} \pm \frac{2\pi n}{4}$

$= (2n+1)\frac{\pi}{4}$

- Between $[0, 2\pi)$ all soln only: $n=0$ $\boxed{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}}$

* Be on the lookout for quadratic like eqns.

EX $\csc^2 x - 4 = 0$

method 1

- isolate trig

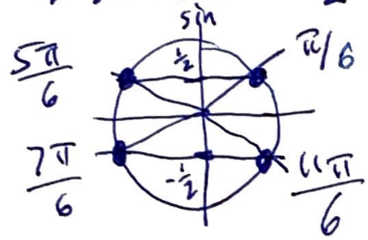
$\csc^2 x = 4$

- $\sqrt{\ }$

$\csc(x) = \pm 2$

- invert

$\sin(x) = \pm \frac{1}{2}$



$$\boxed{\begin{array}{l} \frac{\pi}{6} \pm 2\pi n \\ \frac{5\pi}{6} \pm 2\pi n \\ \frac{7\pi}{6} \pm 2\pi n \\ \frac{11\pi}{6} \pm 2\pi n \end{array}}$$

method 2

$(\csc x - 2)(\csc x + 2) = 0$

$\hookrightarrow \csc x = -2$

$\hookrightarrow \csc x = 2$

Focus between $[0, 2\pi)$

$$\boxed{\left\{ \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6} \right\}}$$

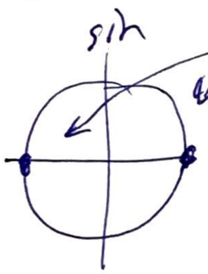
* Factoring

EX $\sec(x)\sin(x) - 2\sin(x) = 0$, Solve for x.

Factor

$\Rightarrow \sin(x) \cdot [\sec(x) - 2] = 0$

$\sin(x) = 0$



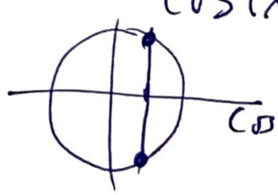
$\boxed{x = \pm n\pi}$

or

$\sec(x) - 2 = 0$ isolate

$\sec(x) = 2$ Flip

$\cos(x) = 1/2$



$$\boxed{\begin{array}{l} \mathrm{I}: \frac{\pi}{3} \pm 2\pi n \\ \mathrm{II}: \frac{5\pi}{3} \pm 2\pi n \end{array}}$$

$\boxed{EX}$ $2\cos^2(t) + \cos(t) = 1$

$u = \cos(t)$ substitution

$2u^2 + u - 1 = 0$

Factor

$(2u-1)(u+1) = 0$

$\hookrightarrow u = \frac{1}{2}$ $\quad \hookrightarrow u = -1$

unsubstitute

$\cos(t) = \frac{1}{2}$ $\quad \cos(t) = -1$

etc. $\quad$ etc.

$\boxed{EX}$ $2\sin(x)\cos(x) - \sin(x) + 2\cos(x) - 1 = 0$

$\sin(x)\left[2\cos(x) - 1\right] + (2\cos(x) - 1) = 0$

$(2\cos(x) - 1)(\sin(x) + 1) = 0$ ...

* more involved identities may be needed

$\boxed{EX}$ $\sin(3x)\cos(6x) - \cos(3x)\sin(6x) = -0.9$

$\cos(3x + 6x)$

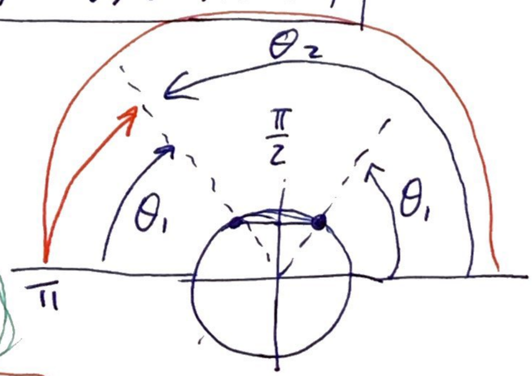
$$\boxed{\begin{aligned}\cos(A \pm B) &= \cos(A)\cos(B) \mp \sin(A)\sin(B)\\ \sin(A \pm B) &= \sin(A)\cos(B) \pm \cos(A)\sin(B)\end{aligned}}$$

$\sin(3x - 6x) = -0.9$

$\sin(-3x) = -0.9$

$-\sin(3x) = -0.9$

Solve $\rightarrow$ $\boxed{\sin(3x) = 0.9}$



$\theta_1 = 3x = \sin^{-1}(0.9)$

$\frac{\theta_1}{3} = \boxed{x = \frac{\sin^{-1}(0.9)}{3}}$

$\theta_2 = \pi - \theta_1$

$\boxed{\theta_2 = \pi - \sin^{-1}(0.9)}$

$\theta_2/3 = \boxed{x = \frac{\pi}{3} - \frac{\sin^{-1}(0.9)}{3}}$

* Numerical approach

$\boxed{\text{EX}}$ $\cos(6x) - \cos(3x) = 0$

Solve both analytically and then view on Desmos.

1^{st} use this identity

$$\cos 2\theta = 2\cos^2\theta - 1$$

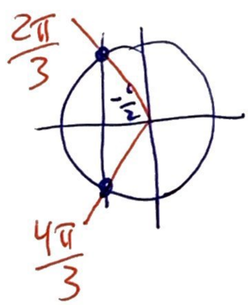
$$\Rightarrow \quad 2\cos^2(3x) - 1 - \cos(3x) = 0$$

or $\quad 2\cos^2(3x) - \cos(3x) - 1 = 0$

which factors

$$(2\cos(3x) + 1)(\cos(3x) - 1) = 0$$

$\cos(3x) = -\frac{1}{2}$



- $3x = \frac{2\pi}{3} + 2\pi n$

$$\boxed{x = \frac{2\pi}{9} + \frac{2\pi n}{3}}$$

- $3x = \frac{4\pi}{3} + 2\pi n$

$$\boxed{x = \frac{4\pi}{9} + \frac{2\pi n}{3}}$$

$\cos(3x) = 1$

$3x = 0, 2\pi, 4\pi, \ldots$

$3x = 2\pi n$

$x = \pm\frac{2\pi n}{3}$

$$\boxed{x = \frac{2\pi n}{3}}$$

$$\boxed{x = -\frac{2\pi n}{3}}$$

On Desmose ...
separate the eqn
So we see
$\cos(6x) = \cos(3x)$

In desmos boxes

$\boxed{y = \cos(6x)}$

$\boxed{y = \cos(3x)}$

Then click on the intersections and read out the x-values

and compare ...

$\boxed{\text{WARNING}}$: $\cos(6x) = \cos(3x)$
this only yeilds $6x = 3x \pm 2n\pi$
one set of family $\boxed{x = \pm 2\pi n/3}$

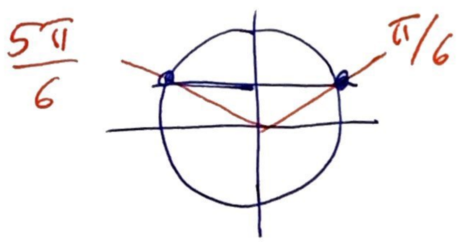
Read out yeilds: $x = \frac{2\pi}{9}, \frac{4\pi}{9}, \frac{2\pi}{3}, \frac{8\pi}{9}, \frac{4\pi}{3} \ldots$

EX Solve both numerically & analytically

$$6\sin^2 x - 5\sin x + 1 = 0 \quad \rightarrow \quad 6\sin^2(x) = 5\sin(x) - 1$$

$$(2\sin(x) - 1)(3\sin(x) - 1) = 0$$

$\sin(x) = \frac{1}{2}$

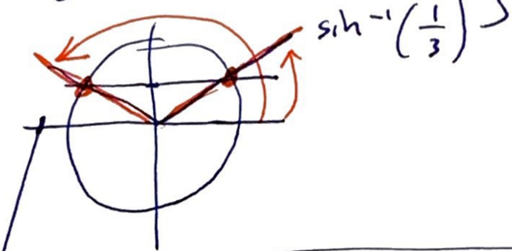


$$\boxed{x = \frac{\pi}{6} \pm 2\pi n}$$

$$\boxed{x = \frac{5\pi}{6} \pm 2\pi n}$$

$\sin(x) = \frac{1}{3}$

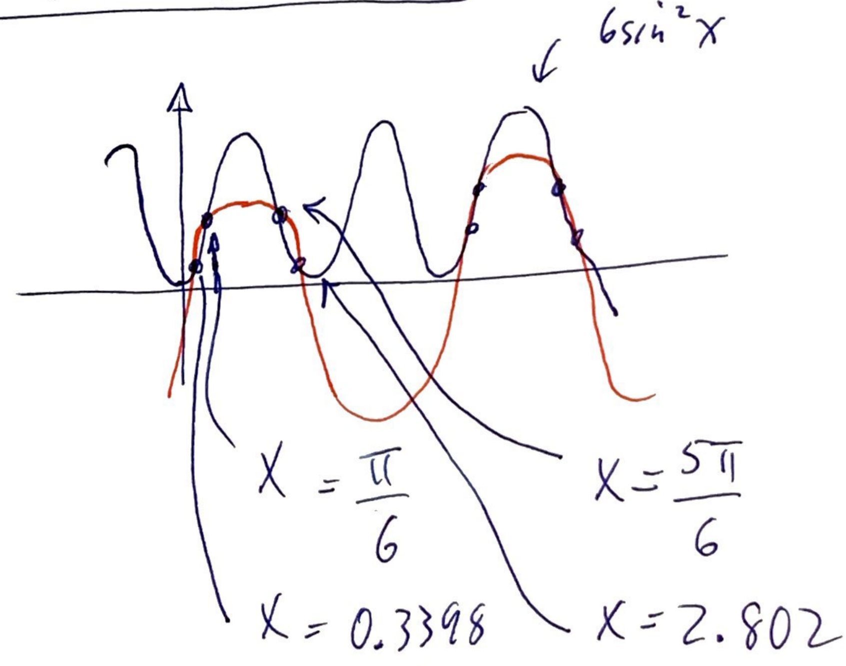
$$\boxed{x = \sin^{-1}\left(\frac{1}{3}\right) \pm 2\pi n}$$



$$\boxed{x = \pi - \sin^{-1}(1/3) \pm 2\pi n}$$

On Desmos

$$\boxed{y = 6\sin^2(x)}$$
$$\boxed{y = 5\sin(x) - 1}$$



Compare to analytical:
note that $\sin^{-1}(0.3333) = 0.3398$

and $\pi - \sin^{-1}(0.3333) = 2.8017$