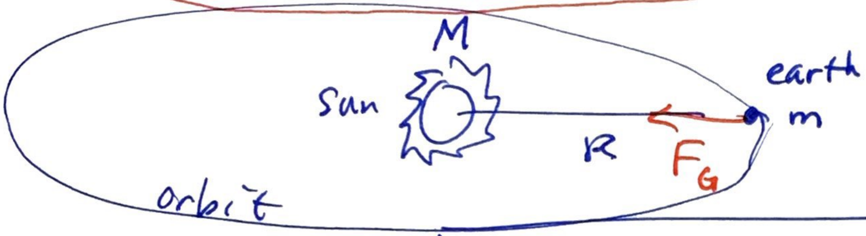
Gravitational Force

- Newton surmised that there is a force between the Moon and the Earth since the moon is not traveling in a straight line.



$$F_G \propto M, m, \frac{1}{R^2}$$

- Add const. of proportions...

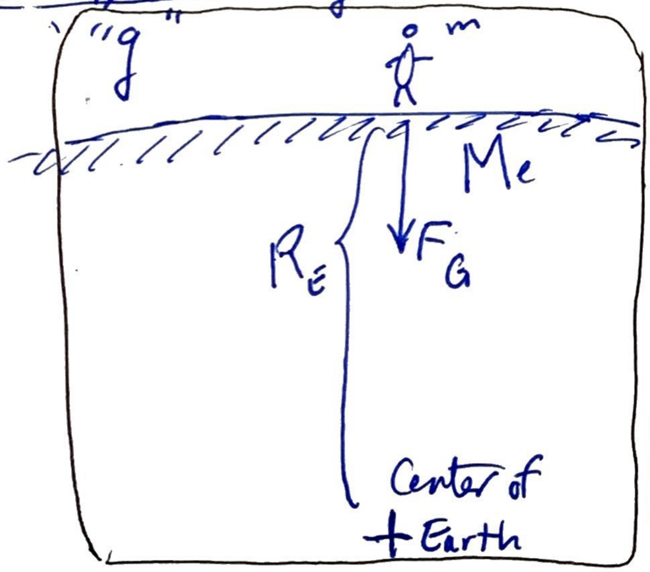
$$\boxed{F_G = G\frac{Mm}{R^2}}$$

"Fourth Law"

G = gravitational constant

$$G = 6.67 \times 10^{-11} \frac{Nm^2}{kg^2}$$

Q: what about little "g"?



$$F_G = G\frac{M_E\, m}{R_E^2}$$

$$F_G = \underbrace{\left(G\frac{M_E}{R_E^2}\right)}_{g} m$$

$$\boxed{F_G = g\, m}$$

- $\boxed{g = G\frac{M_E}{R_E^2}} = \left(6.67 \times 10^{-11} \frac{Nm^2}{kg^2}\right)\frac{(5.98 \times 10^{24}\, kg)}{(6.38 \times 10^{6}\, m)^2}$

M_{Earth}

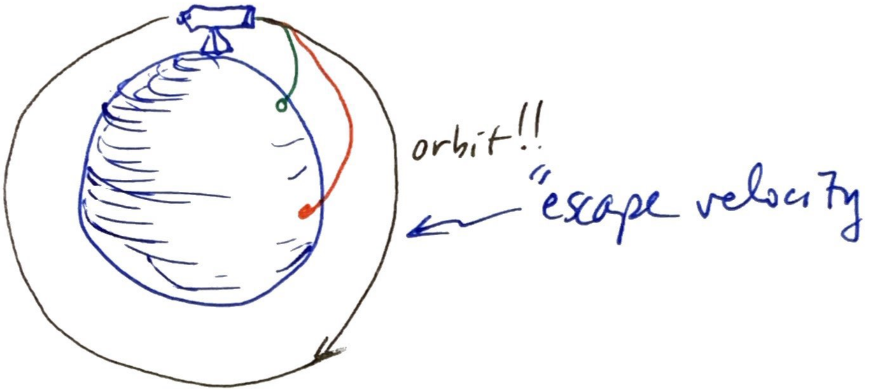
R_{Earth}

$= 9.799\ m/s^2$

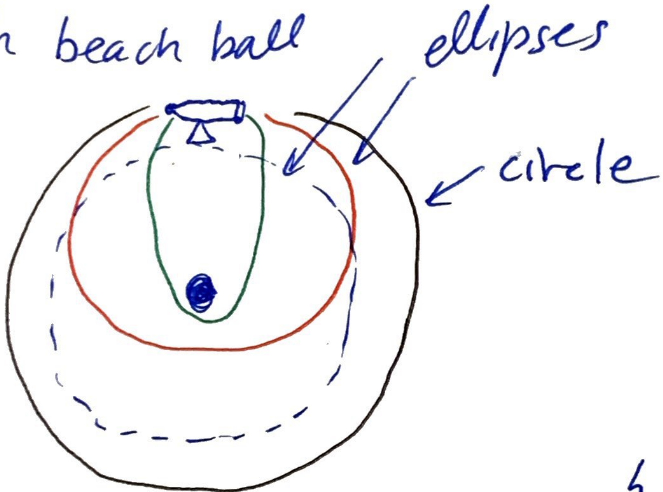
So $g_{Earth} = 9.8\ m/s^2$

Orbits

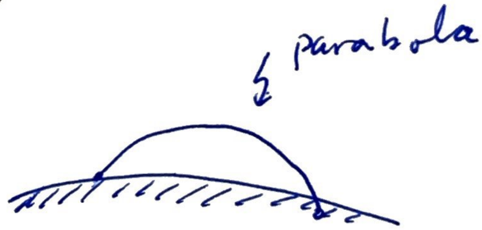
Newton surmised that if a cannonball is shot horizontally out of a cannon it could orbit the earth



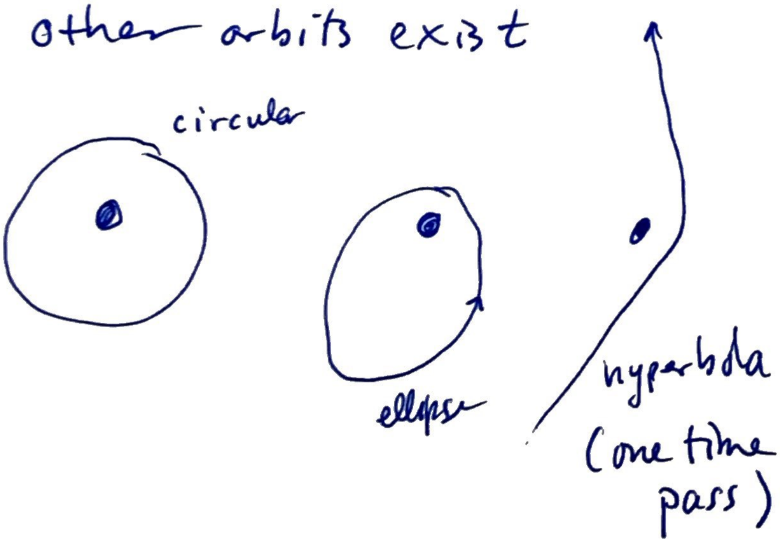
- shrink earth beach ball



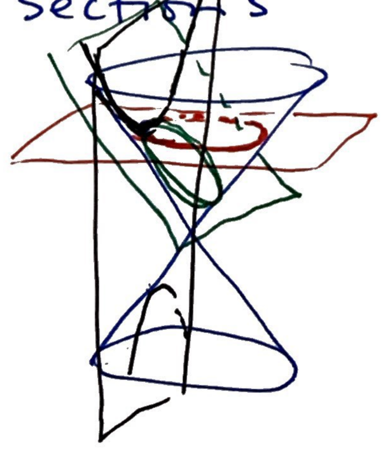
- local, near surface, the piece of the ellipse looks like a parabola.



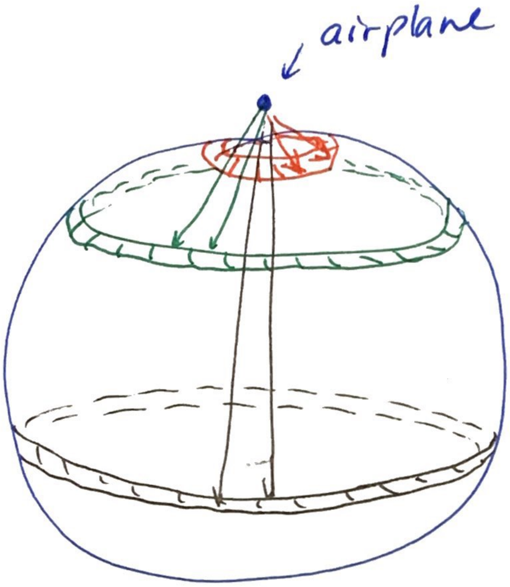
- other orbits exist



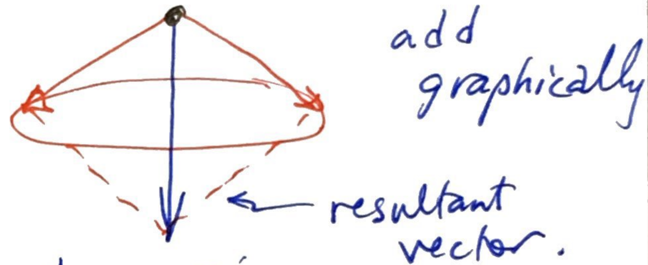
- all orbits are conic sections



⊛ Spherical Objects (Earth)

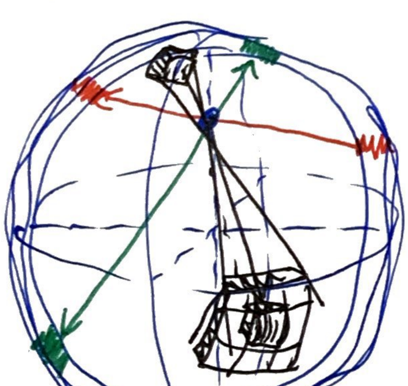


- EVERY piece of earth attracts the plane towards it.
- Due to symmetry the forces add to one pointing towards the center.



- In calculus we can add all these pieces and sum them up in a triple integral {see Wikipedia: "Shell Theorem" for the math}

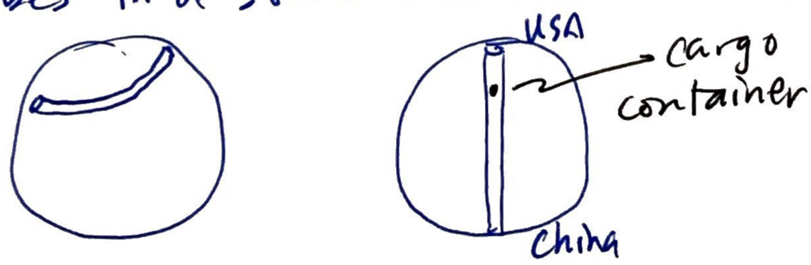
- Inside of a <u>spherical shell</u> of mass (Hollow Earth)



Due to the <u>inverse square law</u>, $\frac{1}{R^2}$, for each piece of mass on the shell, there is R^2 times mass on the opposite side of the shell that cancels out

⟹ $\boxed{\text{an astronaught floats in a hollow planet}}$

- Tubes in a solid Earth would provide "free" transportation



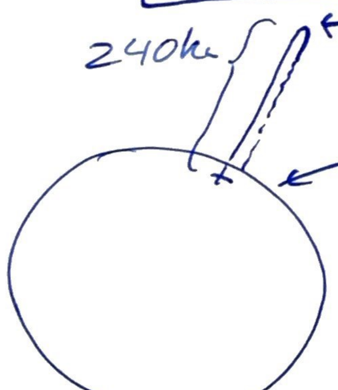
Drop in hole, it acc'lts to center then slow to a stop at the other end. Grab it!!

Ex the ISS is at 240 km above the surface of the Earth. What is "g" up on station? ④

$$g_{ISS} = G\frac{M_E}{(R_E + h)^2}$$

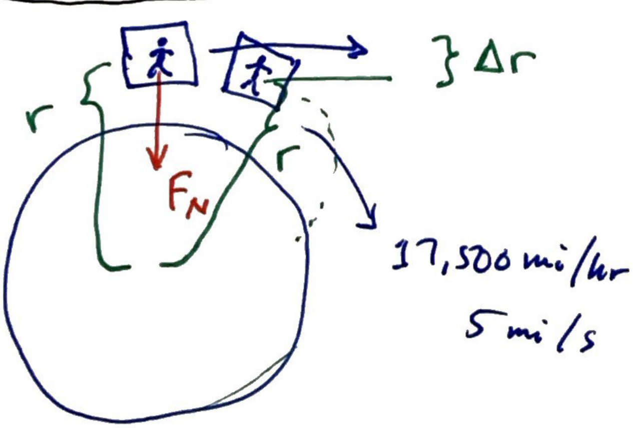
$$= \frac{\left(6.67\times10^{-11}\,\frac{Nm^2}{kg^2}\right)\left(5.98\times10^{24}\,kg\right)}{\left(6.38\times10^{6}\,m + 240{,}000\,m\right)^2}$$

$$= \boxed{9.10\ m/s^2}$$



weight = 70kg (9.1) = $\underline{\underline{637\ N}}$

weight = 70(9.8) = $\underline{\underline{686\ N}}$

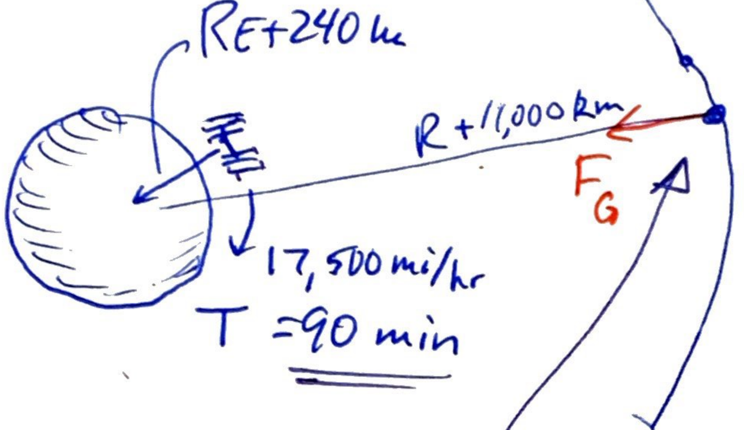


You feel weightlessness because you have motion parallel to surface, even though you fall towards the surface. That parallel speed keeps you from missing the Earth.

5

The further away we travel from Earth, the slower we need to travel to maintain constant orbital distance.

Q: what is this relation between v & r ?



speed keep me above the same spot on the earth. "geosynchronous orbit"

$T = 24$ hr.

$\sum F = m a_c$

$$\boxed{F_G = m_{sat}\, a_c}$$

$$G \frac{M_E \cancel{m}_{sat}}{(R_E + h_{sat})^2} = \cancel{m}_{sat} \frac{V^2}{(R_E + h_{sat})}$$

- let $R = R_E + h_{sat}$ {center of Earth-to-Satellite}

$$G\frac{M_E}{R^2} = \frac{V^2}{R}$$

$$\boxed{V = \sqrt{G M_E / R}}$$

Ex At the I.S.S., how fast does the station need to be traveling to keep orbit constant?

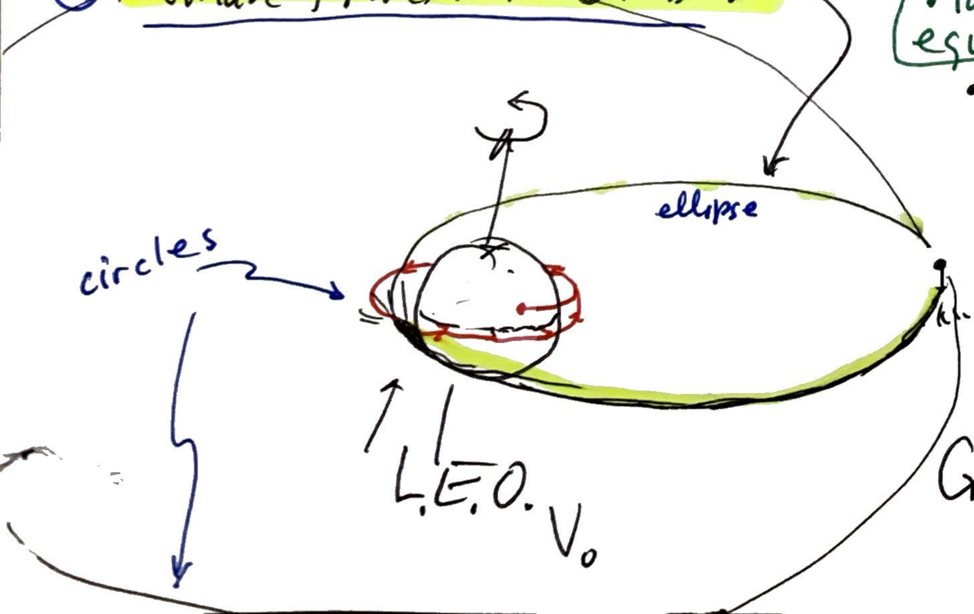
$$V = \sqrt{\left(6.67\times10^{-11}\,\frac{m^2 N}{kg^2}\right)\left(5.98\times10^{24}\,kg\right)\Big/\left(6.38\times10^{6}+2.4\times10^{5}\,m\right)}$$

$$= \boxed{7{,}762\,m/s}\left(\frac{1\,km}{1000\,m}\right)\left(\frac{3600\,s}{1\,hr}\right) \quad N = kg\,m/s^2$$

$$= 27{,}943.9\,km/hr = \boxed{17{,}363.6\,mi/hr}$$

17,500 mph

Hohman Transfer Orbit



- launch to the East near equator to get help from Earth's rot'n
- astrodynamics

play "Kerbel Space" game

- "Delta Vee" used to

$$\boxed{\Delta V = V_f - V_o}$$

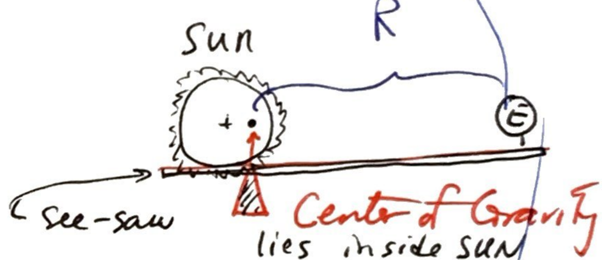
describe the added velocity to get to larger orbits

Kepler's Law

symbol used for the SUN "$\odot$"

Ellipe greatly, greatly exagerated

- Orbits are not perfect circles, but ellipses.
- $R_{orbit}^3 \propto T_{period}^2$
- The sun sits at one focus "points" of the ellipse



- What is Keplers Law?

$$F_G = m\frac{V^2}{R}$$

$$G\frac{M_\odot \cancel{m}}{R^2} = \frac{V^2}{R}\cancel{m}, \quad \overset{\text{but}}{V} = \frac{2\pi R}{T}$$

Circumference

$$G\frac{M}{R^2} = \frac{4\pi^2 R^2}{R \cdot T^2} \rightarrow \left(\frac{GM}{4\pi^2}\right)T^2 = R^3$$

$$\boxed{T^2 \propto R^3}$$

- Newton's 4th Law: analytically showed what Kepler did numerically.
- Compare orbits and periods...

$$\boxed{\frac{T_1^2}{R_1^3} = \text{constant} = \frac{T_2^2}{R_2^3}}$$

planet 1 planet 2

Ex what is the distance from the Center of the Earth to GeoSynchronous Orbit {GSO}?

Kepler's Law:

period of orbit of GSO satellite

$$R = \sqrt[3]{G\frac{M_e T^2}{4\pi^2}}$$

$\sqrt[3]{a} = a^{1/3}$

$$R = \left[\frac{\left(6.67\times10^{-11}\,\frac{Nm^2}{kg^2}\right)\left(5.98\times10^{24}\,kg\right)\Big[\overbrace{24\,hrs\cdot\frac{3600\,s}{hr}}^{T}\Big]^2}{4\pi^2}\right]^{1/3}$$

$R = 42{,}250{,}474.31\ m$

$= 42.23\times10^{6}\ m$

$= \boxed{4.23\times10^{4}\ km}$

42,300 km from center of Earth.

* Gravity is one of the 4 fundamental forces of Nature. The other 3 are

- Electromagnetic force {Physics 07}
- weak-force between subatomic particles
- Strong-Force - force between protons & Neutrons