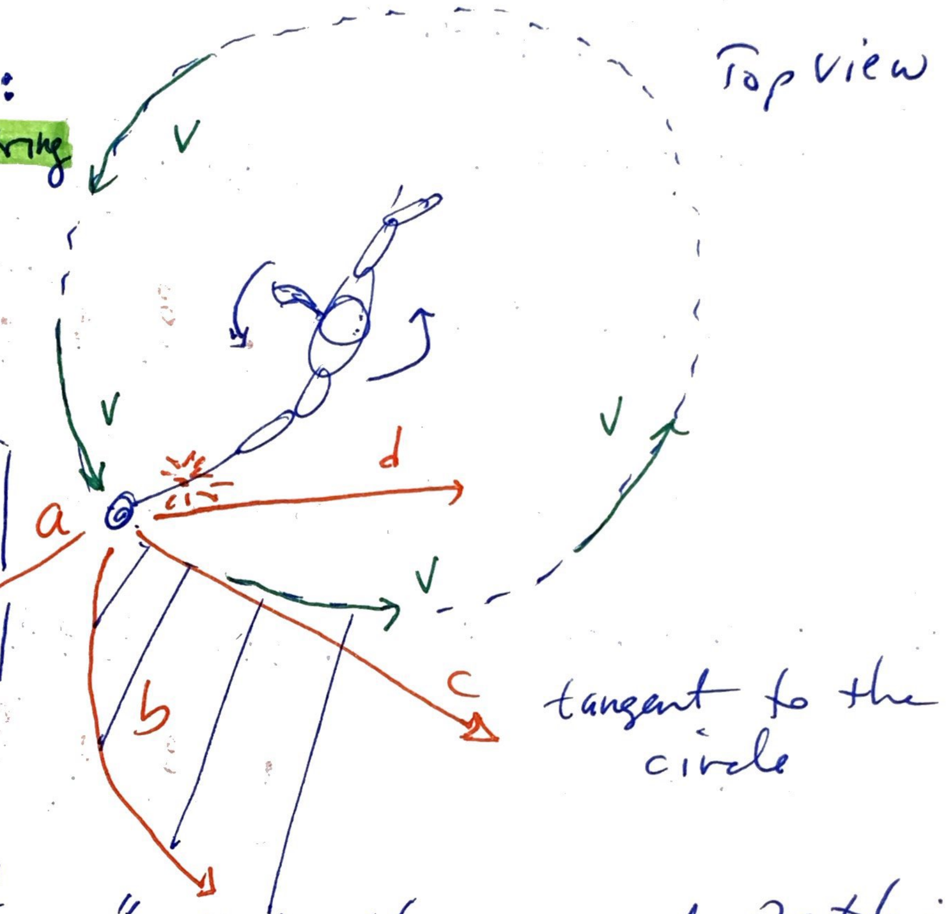
Chapter 5 — Circular Motion

{A : Circ. Motion
B : Gravity

5A

Concept Question:

swing a ball on a string



• The string Breaks !!!

Q: what path does the mass travel?

Q: a, b, c, or "d"?

tangent to the circle

Ans:

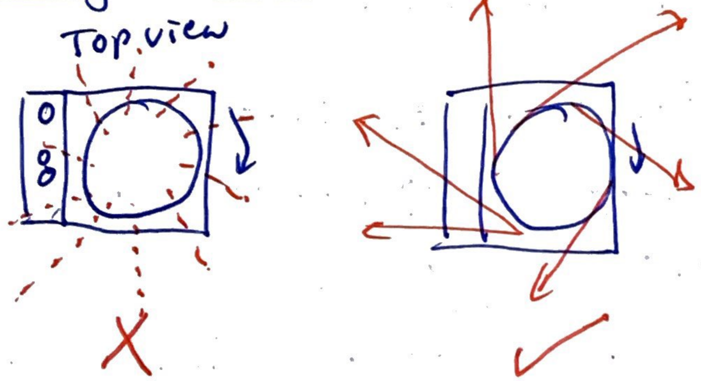
class poll:

	a	b	c ✓	d
poll:	1		1111	1

"c" is the correct path:

• There is no longer a force altering the path of the object & it travels in a straight line.

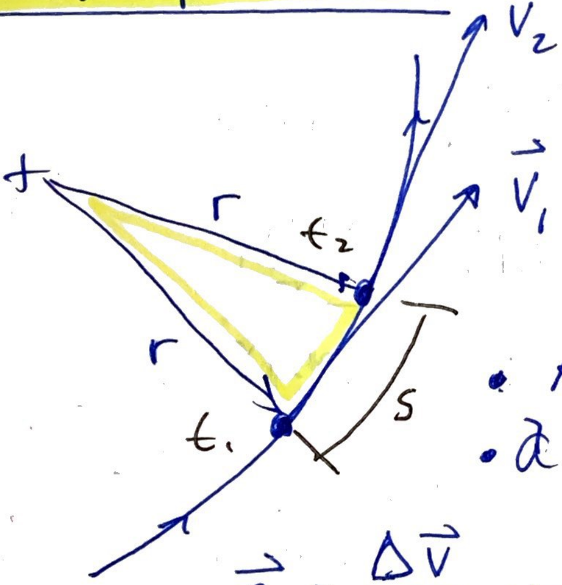
• Washing Machine



2

BTW Newton asked: why is the earth going around the SUN instead of in a straight line past the SUN? He conceived that there must be an invisible force between the two. $\Rightarrow$ Law of Gravity (4th Law)

A. Circular motion



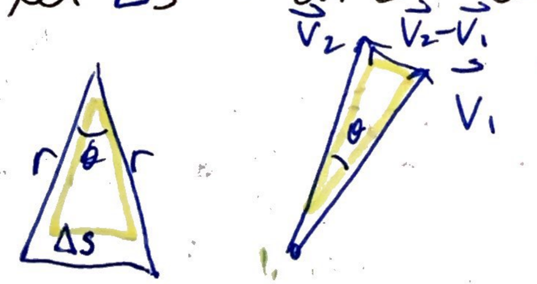
- $\|\vec{V}_1\| = \|\vec{V}_2\|$ but they point in different directions.

- Assume const. speed,
- assume circular motion

- $\vec{a} = \dfrac{\Delta \vec{V}}{\Delta t}$, we need not only change the magnitude, but we can also change only the direction

⊛ we get acc'n due to the change of direction

let Δs = arc length between t_1 & t_2



use similar Δ's:

$\boxed{\|\vec{V}_1\| \text{ is to } \|V_2 - V_1\| \text{ as } r \text{ is to } \Delta s}$

use similar triangles

Cont $\Rightarrow$

- proportional triangles:

Δs is to r AS $\Delta\vec{v}$ is to $\vec{v}$

$$\boxed{\frac{\Delta s}{r} = \frac{\Delta v}{v}}$$

$\div \Delta t$

$$\frac{\Delta s}{r\Delta t} = \frac{\Delta v}{\Delta t}\frac{1}{v}$$

solve for $\frac{\Delta v}{\Delta t}$

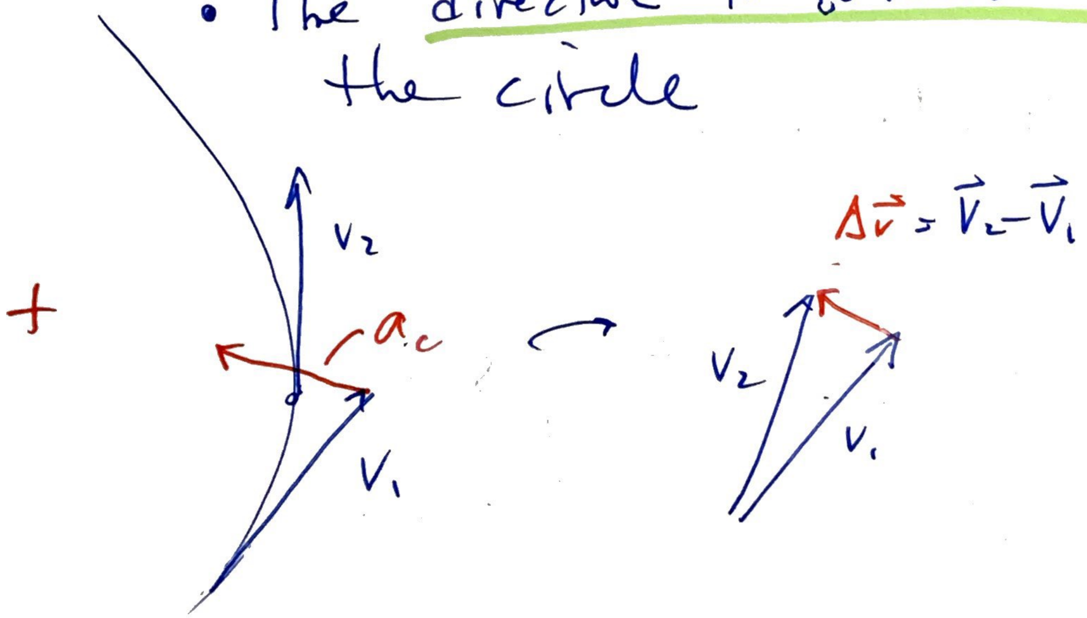
$$\frac{\Delta v}{\Delta t} = v \cdot \left(\frac{\Delta s}{\Delta t}\right) \cdot \frac{1}{r}$$

but $\frac{\Delta \vec{v}}{\Delta t} \equiv \vec{a}$ and $\frac{\Delta s}{\Delta t} = v$

$$\Rightarrow \boxed{a_c = \frac{v^2}{r}}$$

acc'ln due to uniform circular motion
centripetal force

- The direction is towards the center of the circle



⊛ Frequency and Period

T = to be the period = time for one complete revolution about the circle.

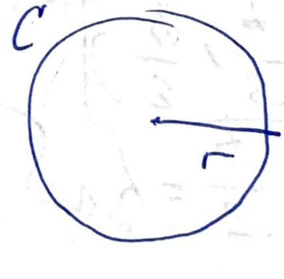
f = the frequency of the motion is the number of complete circuits in a fix time, 1 sec.

• **Dimensions:**

$[T] = \text{sec}/_{\text{cycle}}$, $[f] = \text{cycles}/_{\text{sec}}$

$$\boxed{T = \frac{1}{f} \quad \text{or} \quad f = \frac{1}{T}}$$

• circumerence $C = 2\pi r$



$\Rightarrow$ speed of object $V = \dfrac{\text{Circumference}}{\text{period}}$

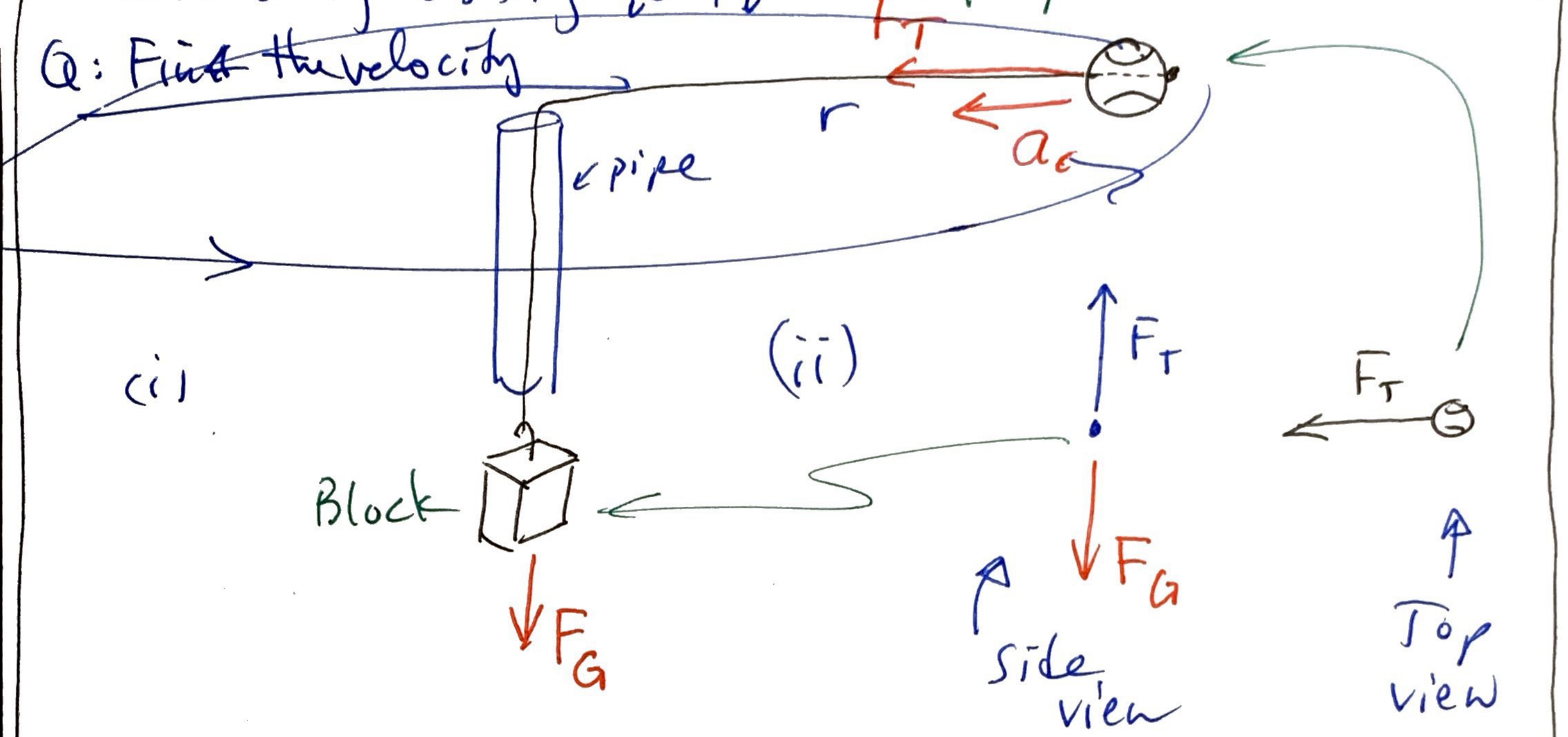
$$\boxed{V = \frac{2\pi r}{T}}$$

{ speed of object with period "T" in radius of circle "r"

OR – $\boxed{V = 2\pi r f}$

EX (a) you swing a Baseball in a circle by attaching a string to it. Keep the block motionless

Q: ~~Find the velocity~~



(iii) eqns for circular motion

radial: $\boxed{\sum F_{radial} = m a_c}$ $\boxed{a_c = \frac{v^2}{r}}$

centripetal acc'ln

• radial: $F_T = m\left(\frac{v^2}{r}\right)$

• vertical: $F_T - F_G = 0$ — no vertical motion of the hanging block

(iv) Solve $m_{Block}\, g = m_{Baseball}\left(\frac{v^2}{r}\right)$

$\Rightarrow \quad r\, m_{Block}\, g = m_{Baseball} \cdot v^2$

$\Rightarrow \quad \boxed{v = \sqrt{\frac{r g\, m_{Block}}{m_{Baseball}}}}$

If hanging mass is not moving then v is given here

Ex (b) the time 10 complete circuits to take 25 s

((10 revolutions in 25 seconds))

Find the centripetal acc'l

(iii) eqns: $a_c = \frac{v^2}{r} = \left(\frac{2\pi r}{T}\right)^2 \cdot \frac{1}{r}$

$v = \frac{2\pi r}{T}$

$\Rightarrow \boxed{a_c = \frac{4\pi^2 r}{T^2}}$ let $r = 20\,cm$

$f = \frac{10 \text{ cycles}}{25 \text{ sec}} = 0.4\ c/s$

$T = \frac{1}{f} = \frac{1}{0.4\,Hz} = \underline{\underline{2.5\,s}}$

BTW

cycle/sec = Hertz

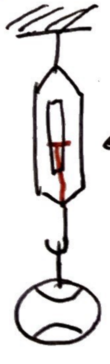
$[f] = Hz$

$\Rightarrow a_c = \frac{4\pi^2 (0.2\,m)}{(2.5\,s)^2}$

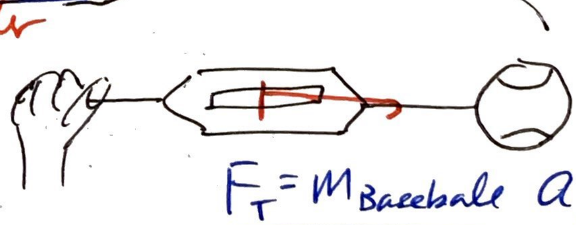
$\boxed{a_c = 1.26\ m/s^2}$

towards the center

Concept:



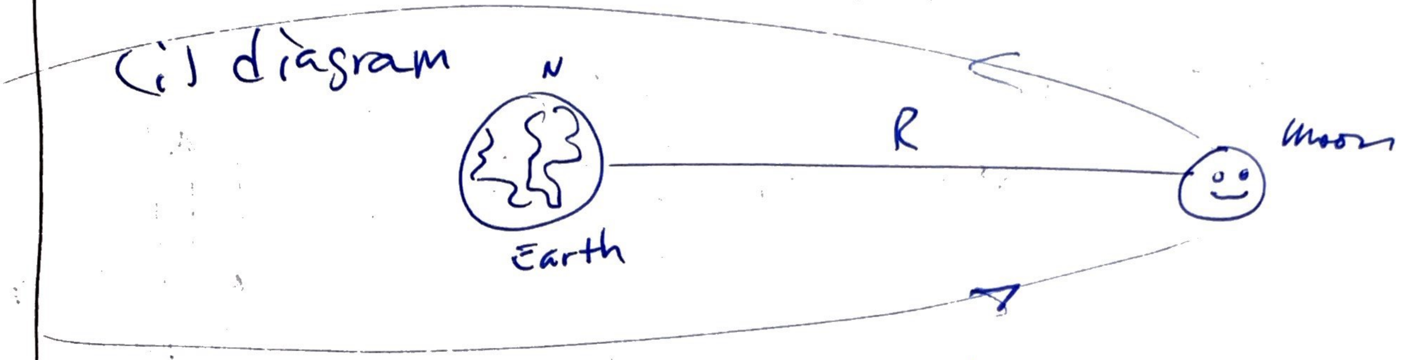
$\leftarrow F_T = m_{Baseball} \cdot g$



$F_T = m_{Baseball}\, a_c$

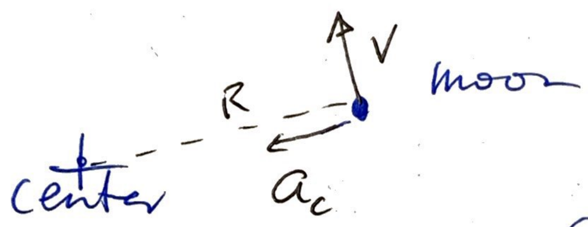
EX Find the moon's centripetal acc'ln

(i) diagram



$$T = 27.3 \text{ days}$$
$$R = 384{,}000 \text{ km}$$

(ii) free body



(iii) eqns: $V = 2\pi r f$, $\boxed{a_c = \frac{V^2}{R}}$, $f = \frac{1}{T}$

(iv) Do the math:

$$V_{moon} = \frac{2\pi(384{,}000{,}000 \text{ m})}{T}, \quad T = 27.3 \text{d}\left(\frac{24 \text{h}}{\text{d}}\right)\left(\frac{3600 \text{s}}{\text{h}}\right)$$

$$= \underline{2{,}358{,}720 \text{ s}}$$

$$V = \frac{2\pi(3.84 \times 10^{+8} \text{ m})}{2358720 \text{ sec}}$$

$$\boxed{V = 1022.9 \text{ m/s}} = \underline{2{,}300 \text{ mi/hr}} = \text{mach } 3$$

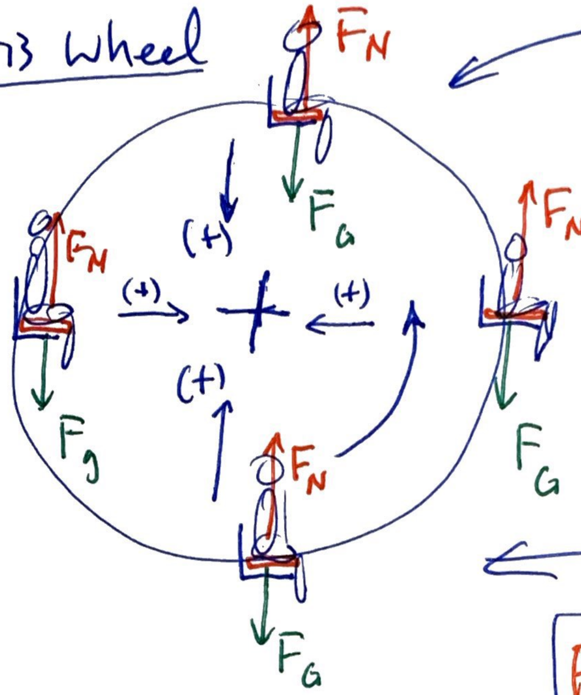
$$\text{So } a_c = \frac{V^2}{R} = \frac{(1022.9 \text{ m/s})^2}{3.84 \times 10^8 \text{ m}} = \boxed{0.0027 \text{ m/s}^2 \text{ towards the earth}}$$

Vertical Circles: Ferris Wheel, pilot in a jet, etc. (8)

* Ferris Wheel

TOP

$$\boxed{F_G - F_N = m\frac{v^2}{r}}$$



$F_N = F_G$

$F_N = F_G$

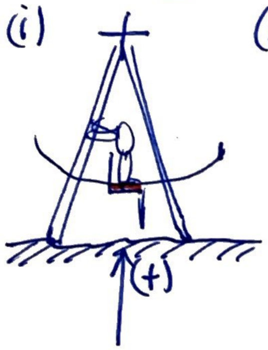
$$\sum F_{radial} = ma_c$$

$$\boxed{F_N - F_G = m\frac{v^2}{r}}$$

BOT

EX How much does a bathroom scale read out when sat upon by a 70kg person at the bottom of the swing of a swing with radius 3m? Assume the speed is 2.2 m/s horizontally

(i)



(ii)

(iii) $F_N - F_G = m\frac{v^2}{r}$

(iv) $F_N = F_G + \frac{mv^2}{r}$

$$F_N = mg + m\frac{v^2}{r} = m\left(g + \frac{v^2}{r}\right)$$

$$F_N = 70\text{kg}\left(9.8\,\text{m/s}^2 + \frac{(2.2\,\text{m/s})^2}{3\text{m}}\right)$$

$$F_N = 70\text{kg}\,(11.4\,\text{m/s}^2) = \boxed{798.9\text{N}}$$

$\div 9.8 = \boxed{1.2\,g}$

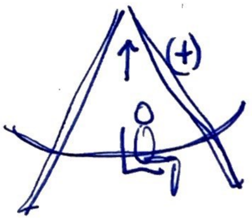
- effective mass (weight)

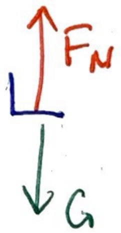
$F_{G''} = mg$

$m = \frac{798.8\text{N}}{9.8} = 81.5\text{kg}$ Like

14% heavier

EX Swing set: How much does a 70 kg person "weigh" at the bottom of a swing if the swing has a radius of 3.0 m and your speed is 2.2 m/s at the bottom





$$\Sigma F = ma$$

$$F_N - F_G = m\frac{v^2}{r}$$

$$\boxed{F_N = F_G + \frac{mv^2}{r}}$$

$$= mg + \frac{mv^2}{r}$$

$$= m\left[g + \frac{v^2}{r}\right]$$

$$= 70\,kg\left[9.8\,\frac{m}{s^2} + \frac{(2.2\,m/s)^2}{3\,m}\right]$$

$$= 70\,k\left[11.41\,m/s^2\right]$$

$$= \underline{\underline{798.9\,N}}$$

"effective weight"

$$m = \frac{F}{g} = \frac{798.9\,N}{9.8}$$

$$= \boxed{81.5\,kg}$$

$$\frac{81.5-70}{70} = 0.16$$

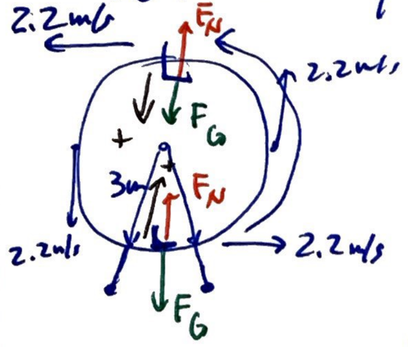
or 16% heavier

$$\frac{11.41\,m/s}{9.8} = \boxed{1.2\,g\text{'s}}$$

you feel 1.2 "g" but weigh one "g"

EX We are on a Ferris wheel that rotates quickly. $R = 3\,m$. If we weigh $70\,kg$ what force is our seat feeling

((Bathroom scale readout))



- Top $\left(-F_N + F_G = \frac{mv^2}{r}\right) * -1 \quad F_N - F_G = -\frac{mv^2}{r}$
- Bot $F_N - F_G = \frac{mv^2}{r}$

feel less

- Top $F_N = F_G - \frac{mv^2}{r} = mg - \frac{mv^2}{r} = m\left(g - \frac{v^2}{r}\right)$
- Bot $F_N = \frac{mv^2}{r} + F_G = mg + \frac{mv^2}{r} = m\left(g + \frac{v^2}{r}\right)$

feel more

- Top $F_N = 70\,kg\left(9.8 - \frac{(2.2\,m/s)^2}{3\,m}\right) = 70 \cdot (8.187\,m/s^2)$
- Bot $F_N = 70\,kg\left(9.8 + \frac{(2.2\,m/s)^2}{3\,m}\right) = 70 \cdot (11.413\,m/s^2)$

- Top $\underline{\underline{F_N = 573.1\,N}}$
- Bot $\underline{\underline{F_N = 798.9\,N}}$

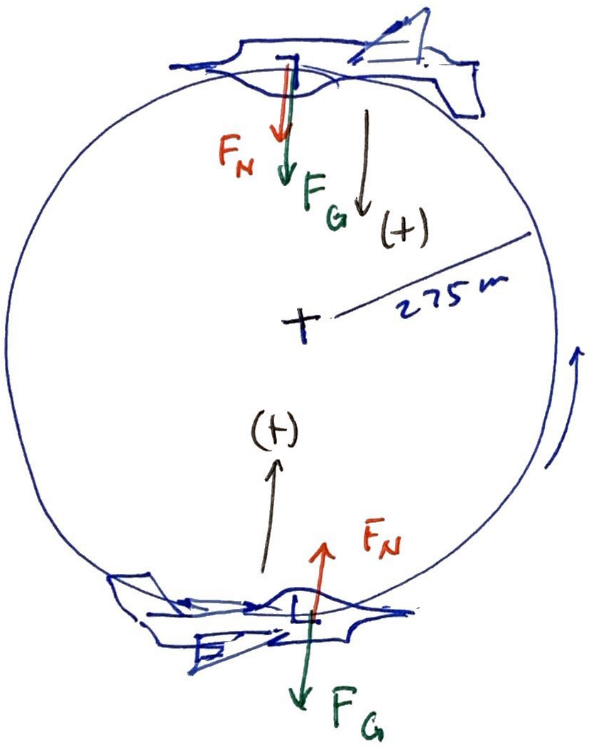
on ground $70(9.8) = \underline{\underline{686\,N}}$

- Top $573.1/9.8 = \boxed{58.5\,kg}$
- Bot $798.9/9.8 = \boxed{81.5\,kg}$

vs $\boxed{70.0\,kg}$ effective "weight"

EX A fighter flys in a vertical circle. What weight does the pilot feel in their at the top and the bottom of the circle. $m_{pilot} = 70.\,kg$
Speed of jet $= 50\,m/s$, $R = 275\,m$

Top:



$$F_N + F_G = m\frac{v^2}{r}$$

Bot:

$$F_N - F_G = \frac{mv^2}{r}$$

Top: $F_N = \frac{mv^2}{r} - mg = m\left(\frac{v^2}{r} - g\right) = 70kg\left(\frac{50\,m/s^2}{275m} - 9.8\,m/s^2\right)$

Bot: $F_N = \frac{mv^2}{r} + mg = m\left(\frac{v^2}{r} + g\right) = 70kg\left(\frac{50\,m/s^2}{275m} + 9.8\,m/s^2\right)$

Top: $F_N = 70kg\left(-0.71\,m/s^2\right) = -6.96N$

Bot: $F_N = 70kg\left(18.89\,m/s^2\right) = 1322.36N$

Top: $-6.96N / 9.8\,m/s^2 = -0.71\,kg$ ← falling off the seat "0kg"

Bot: $1322.36N / 9.8\,m/s^2 = 135.96\,kg$ ← Feel 2times your normal weight "2g's"

EX In the last example, at what speed should the aircraft move so as to feel perfect weightless ness ?

$$F_N = m\left(\frac{v^2}{r} - g\right)$$

$$0 = m\left(\frac{v^2}{r} - g\right)$$

$$\Rightarrow 0 = \frac{v^2}{r} - g \Rightarrow \frac{v^2}{r} = g \Rightarrow \boxed{v = \sqrt{rg}}$$

$$v = \sqrt{(275\,m)\,9.8\,m/s^2}$$

$\boxed{v = 51.9\,m/s}$ so 1.9 m/s faster and the pilot floats at the top of the curve.

Summary: At the top of the circle, pilot upsidedown:

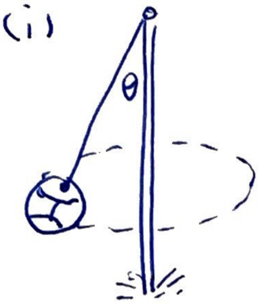
@ 50 m/s the pilot falls but the buckles catch them

@ 51.9 m/s the pilot feels weightless.

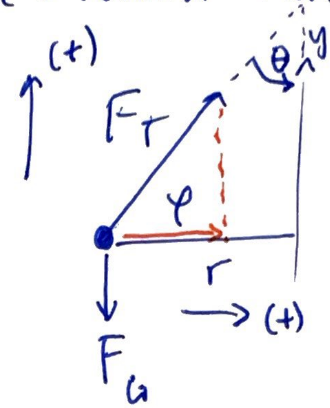
@ 55 m/s the pilot is pushed into the seat and feels a few kg of weight"

Conical Pendulum {a form of Horiz. motion}

(i)



(ii)



(iii)

• radial:

$$\sum F = m\frac{v^2}{r}$$

$$\boxed{F_T \cos\varphi = m\frac{v^2}{r}}$$

• verticle:

$$\sum F = m\cancel{a_y}^{\,0}$$

$$\boxed{F_T \sin\varphi - F_G = 0}$$

• Since $\varphi = 90 - \theta$

$$\sin(\varphi) = \sin(90-\theta) = \cos\theta$$
$$\cos(\varphi) = \cos(90-\theta) = \sin\theta$$

conical pend.

$\Rightarrow$ Newtons Law:

$$\boxed{\begin{array}{ll} r: & F_T \sin\theta = m\frac{v^2}{r} \\ y: & F_T \cos\theta = mg \end{array}}$$

(iv) Some math

Math Trick: divide one eqn by the other

$$\frac{F_T \sin\theta = mv^2/r}{F_T \cos\theta = mg}$$

$$\boxed{\tan\theta = \frac{v^2}{gr}}$$

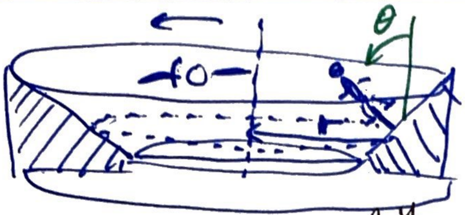
conical pendulum eqn

Ex You play tether ball and keep it moving at $\theta = 30°$. If $r = 1m$ and $m = 2kg$ Find v

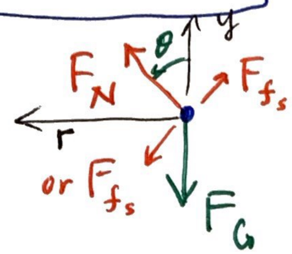
$$v = \sqrt{gr\tan\theta} = \sqrt{(9.8)(1m)(\tan 30°)} = \boxed{2.38\,m/s}$$

⊛ Banked Curve (Velodrome)

(i)



(ii)

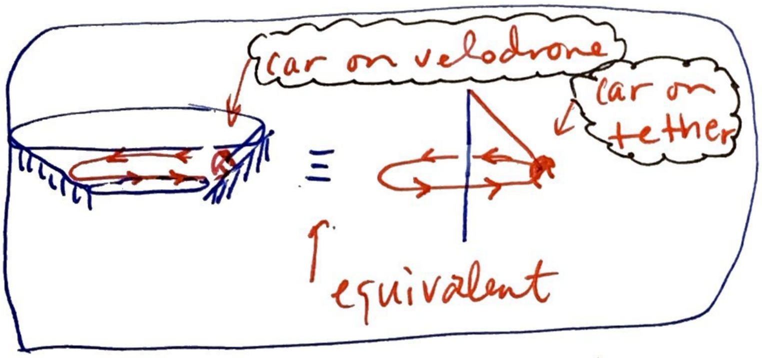


(iii)

- radial $\Sigma F = \frac{mv^2}{r}$
- vertical $\Sigma F = 0$

r: $F_N \sin\theta = \frac{mv^2}{r}$

y: $F_N \cos\theta - F_G = m\cancel{a_y}^{\,0}$



r: $F_N \sin\theta = mv^2/r$

y: $F_N \cos\theta = mg$

divide

$$\Rightarrow \boxed{\tan\theta = \frac{v^2}{rg}} \;*$$

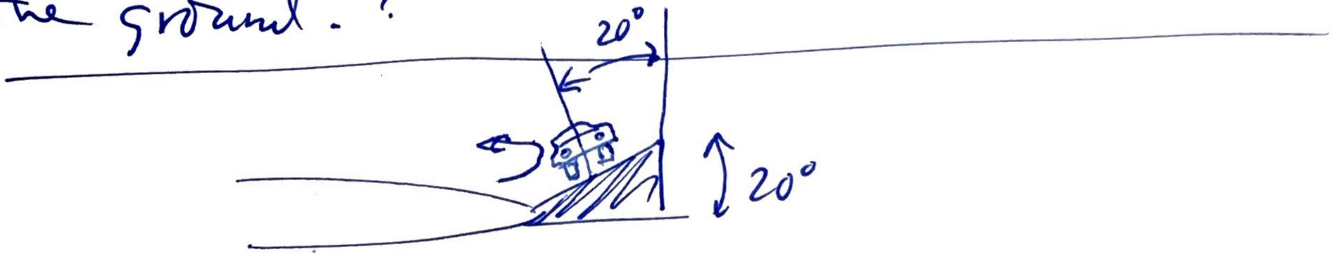
Ideal speed to maintain a radius of r if surface is at an angle θ w.r.t. vertical.

We do not need friction if $v = \sqrt{rg\tan\theta}$, the "design" angle.

- Faster than this ⇒ slip up the ramp and over the top, unless friction is present.
- Slower than this then slip down the ramp.

⊛ This is the same formula for the conical pendulum. motion at this speed is the same wether supported by the ramp of a velodrome or tethered by a rope above the cyclist.

Ex A 1000kg car rounds a banked curve on a road having a radius of 40m. What is the design speed if the bank is 20° w.r.t. the ground.? (14)



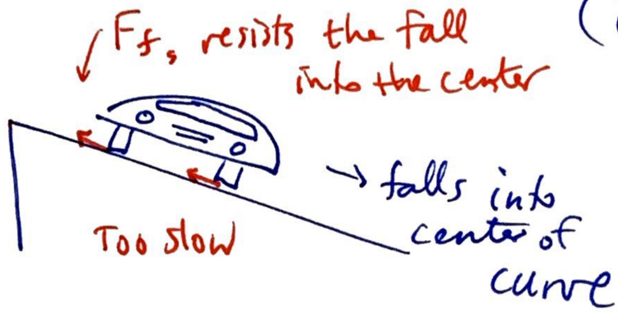
$$v = \sqrt{rg\tan\theta}$$

$$= \sqrt{(40m)(9.8m/s^2)\tan 20°}$$

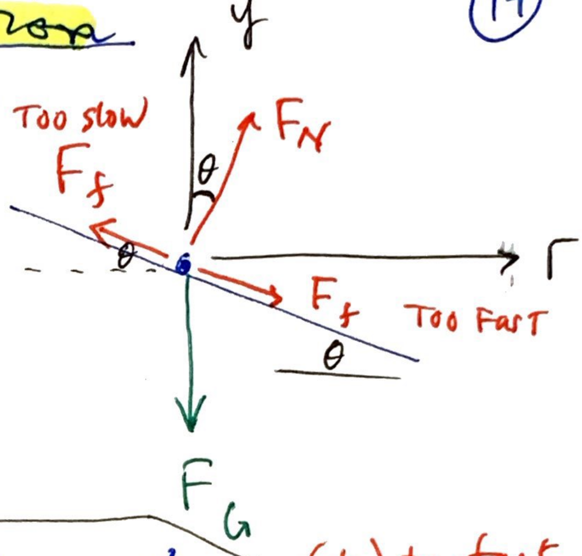
$$= \boxed{11.94 m/s} \quad \sim 25 mph$$

Banked curve with friction

(i)



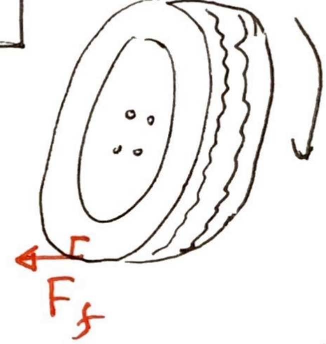
(ii)



(iii) eqns

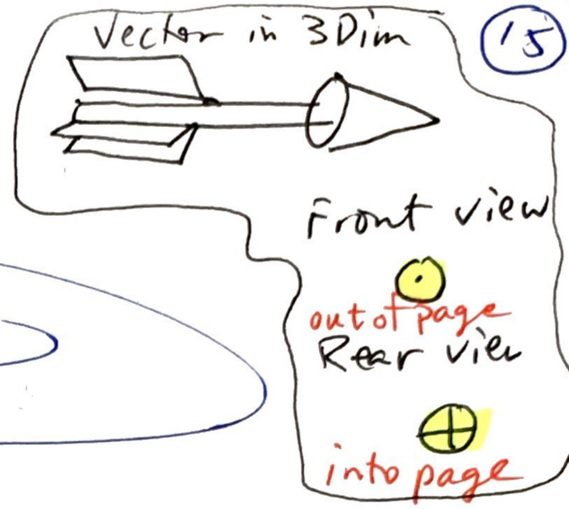
$$\boxed{\begin{aligned} &\text{radial: } F_N \sin\theta \pm F_f \cos\theta = m\frac{v^2}{r} \\ &\text{vertical: } -F_G + F_N \cos\theta \mp F_f \sin\theta = 0 \end{aligned}}$$

(+) too fast
(−) too slow

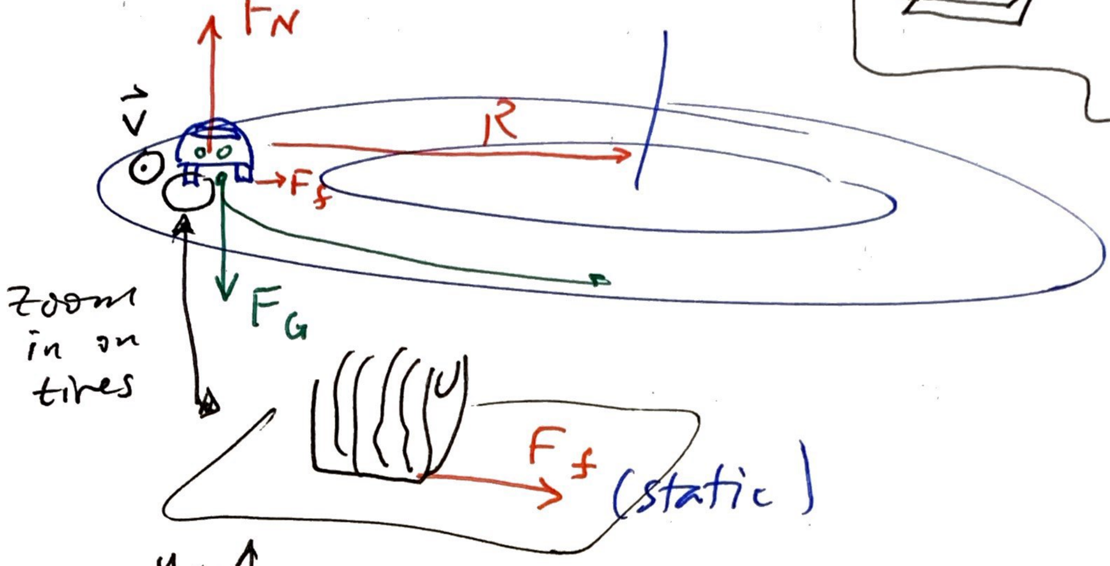


If we are not "skidding sideways"
$F_f = F_{static}$
but friction is applied si

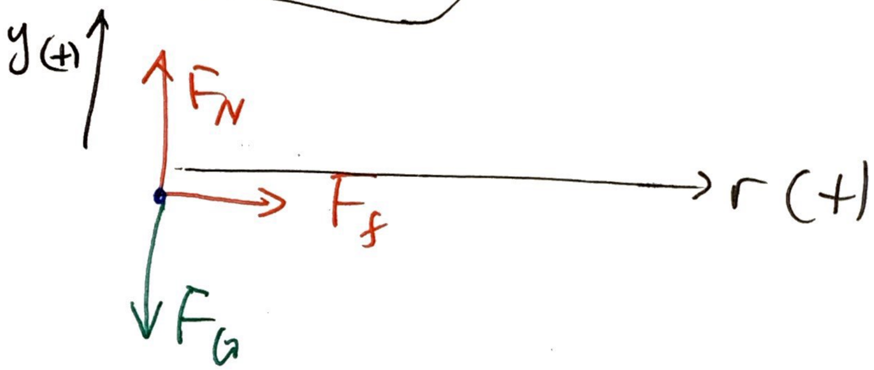
* Flat Curve



(i)



(ii)



(iii)

$$r: \quad F_f = m a_c$$

$$y: \quad F_N - F_G = 0$$

$$f: \quad F_f = \mu_s F_N$$

↖ static

$$a_c = \frac{v^2}{R}$$

$$F_G = mg$$

$$\mu_s F_G = m\left(\frac{v^2}{R}\right)$$

$$\boxed{v = \sqrt{R g \mu_s}}$$

$$\mu_s \cancel{m} g = \frac{\cancel{m} v^2}{R} \quad \rightarrow \quad \boxed{R = \frac{v^2}{\mu_s g}}$$

* Design radius for desired speed & friction

Ex A 1000 kg Car rounds a curve on a road having a radius of 50 m. The car is moving at a speed of 54 km/hr

Q: If the road is dry will the car slip off the curve if $\mu_s = 0.6$ {rubber on cement}

(a) Lets compare the speed to the design speed:

$$V_{des} = \sqrt{R g \mu_s}$$

$$= \sqrt{(50\,m)(9.8\,m/s^2)(0.6)}$$

$$= 17.15\,m/s \left(\frac{1\,km}{1000\,m}\right)\left(\frac{3600\,s}{hr}\right)$$

$$= \boxed{61.7\,km/hr} \text{ design} \ldots$$

• Since $54 < 61.7$ we do NOT slide off the road.

(b) If there is an icy (black ice) region on the curve with $\mu_s = 0.25$ {rubber on ice} will the car stay on the curve?

$$V_{ice} = \sqrt{(50\,m)(9.8\,m/s^2)(0.25)}$$

$$= \boxed{39.8\,km/hr} \text{ max before sliding.}$$

• Car will slide off road!!!