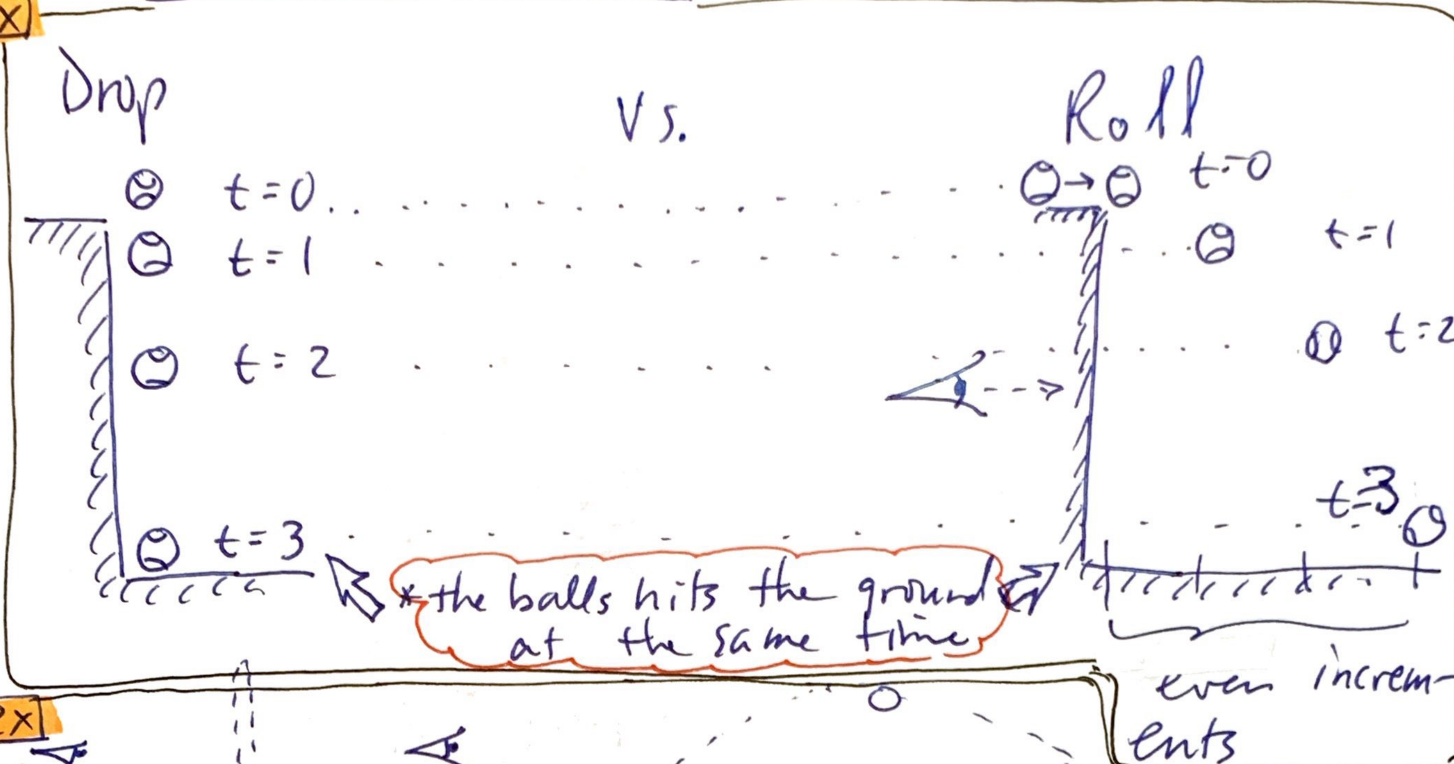
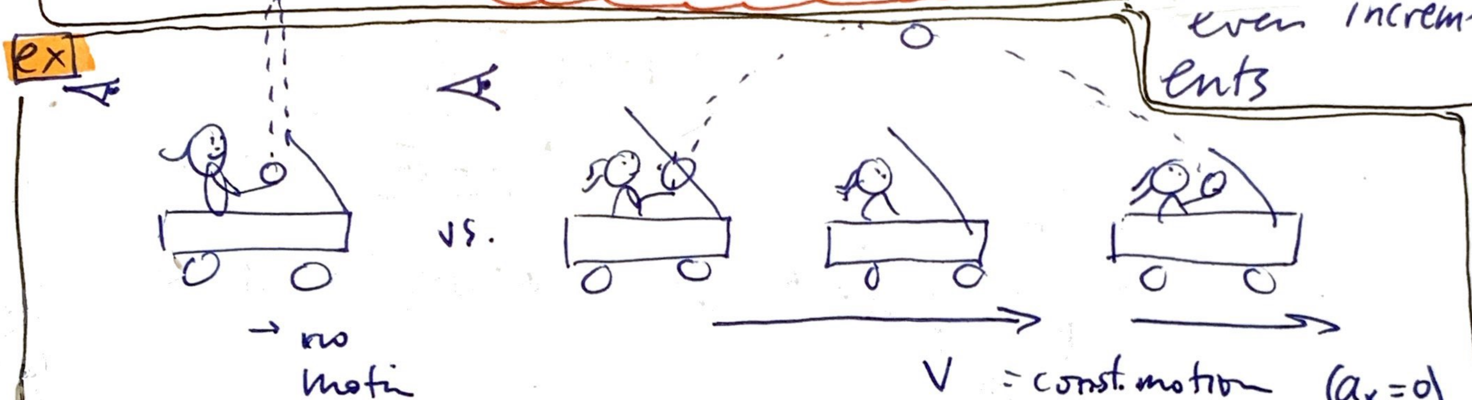
[3b] Pt B : 2-Dim Kinematics

ex



ex



"Inertial Frame of refrence" is in constant uniform motion

If the person in the wagon can't see through side walls they would not know if they were in motion.

⊛ General 2-Dim Motion Eqns:

• Let accln = const (uniform motion) $= a_x\hat{i} + a_y\hat{j}$

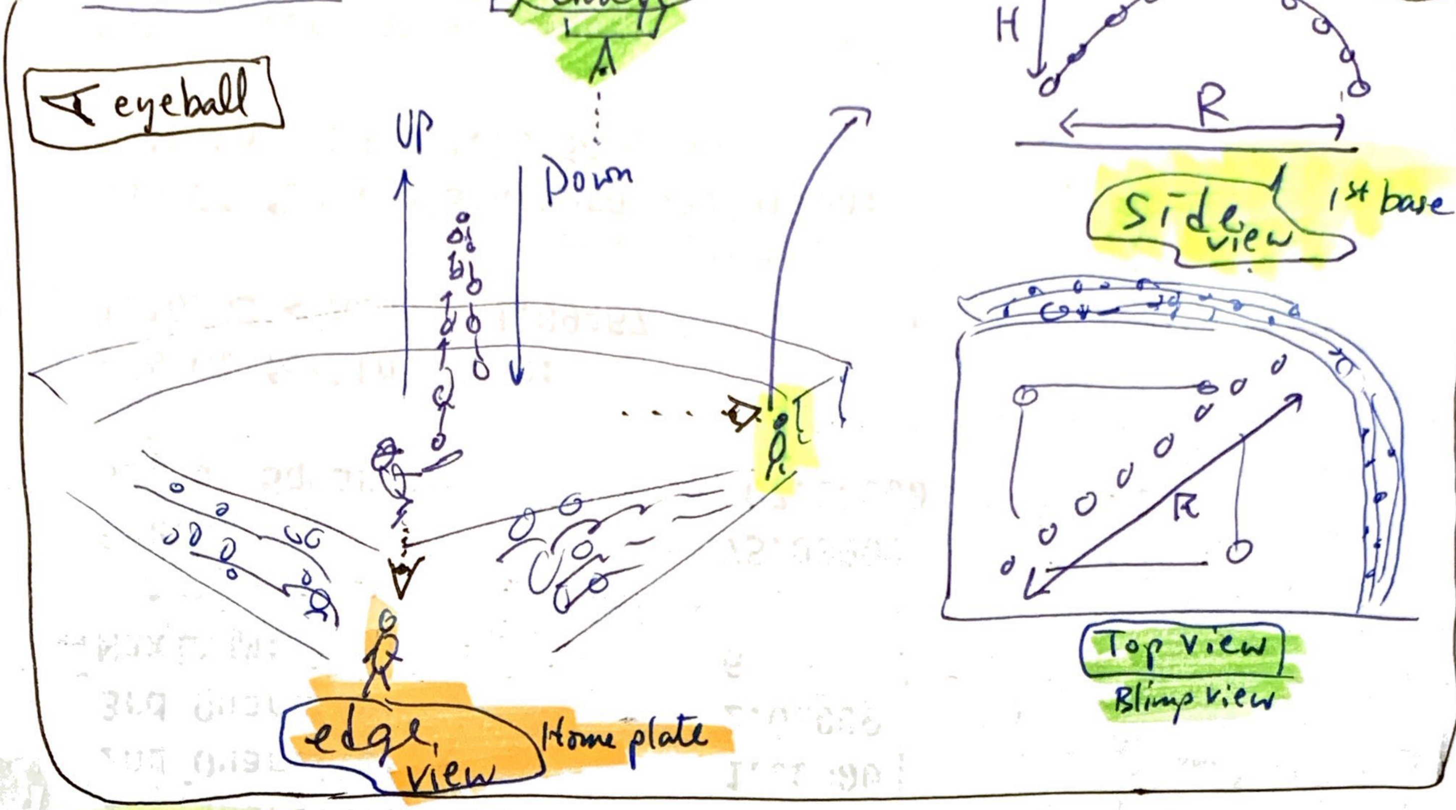
$$\begin{cases} V_x = V_{x_0} + a_x t \\ x = x_0 + V_{x_0} t + \frac{1}{2} a_x t^2 \\ V_x^2 = V_{x_0}^2 + 2a_x (x - x_0) \end{cases}$$

Horizontal Component

$$\begin{cases} V_y = V_{y_0} + a_y t \\ y = y_0 + V_{y_0} t + \frac{1}{2} a_y t^2 \\ V_y^2 = V_{y_0}^2 + 2a_y (y - y_0) \end{cases}$$

Vertical Component

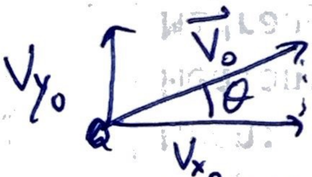
Ex Baseball



Alternative Formula's

- Range Formula $R = V_{x_0}\sqrt{\frac{2H}{g}}$

add in $V_x = \vec{V}\cos\theta$



$V_0 = \|\vec{V}_0\| = \sqrt{V_{x_0}^2 + V_{y_0}^2}$

$$\boxed{R = \frac{V_0^2 \sin(2\theta)}{g}}$$

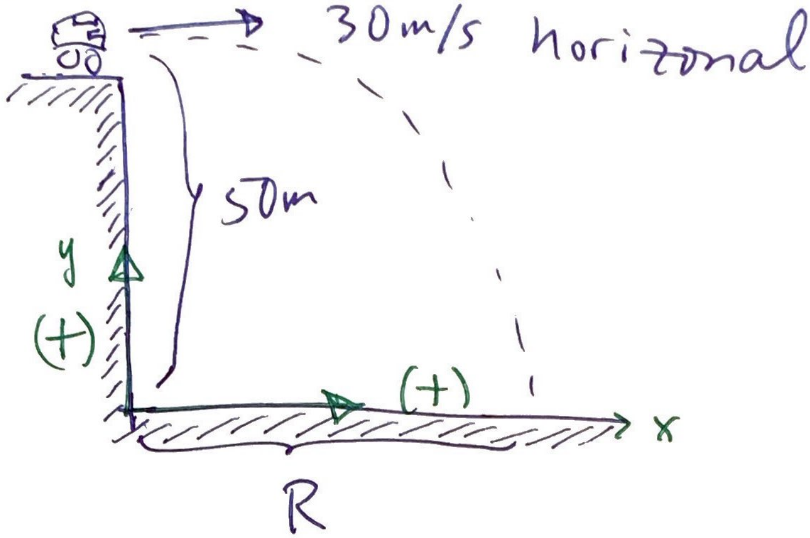
- Height Formula (max): $\boxed{H = \frac{V_{y_0}^2}{2g}}$

- Time of Flight IF landing is the same vertical dist. off ground $\boxed{t = \frac{2v_0\sin\theta}{g}}$

Ex A stunt driver drives a car off of a cliff at 30m/s. The cliff is 50m high

(a) where does the car land?

(i)



(ii) Data

$\vec{V}_0 = 30\,m/s\,\hat{i} + 0\,\hat{j}$

$V_{x_0} = 30\,m/s$, $V_{y_0} = 0\,m/s$

$V_{x_f} = ?$ $\quad V_{y_f} = 0$

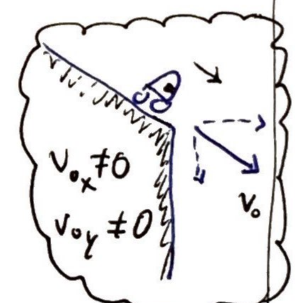
$a_x = 0$, $a_y = g$

$Y_0 = 50\,m$, $X_0 = 0\,m$

$Y_f = 0\,m$, $X_f = ?$

$t_0 = 0\,s$

seek $R = X_f$



(iii) eqns

we need "t" first so...

use $Y_f = Y_0 + V_{y_0} t + \frac{1}{2} a_y t^2$

$0\,m = 50\,m + 0 \cdot t - \frac{1}{2} g t^2$

(iv) Solve for "t"

$\frac{1}{2} g t^2 = H$

$t = \sqrt{\frac{2 \cdot H}{g}}$

$t = \sqrt{\frac{2(50\cancel{m})}{9.8\cancel{m}/s^2}}$

$= \sqrt{\frac{100}{9.8}\,s^2}$

$\boxed{t = 3.19\,s}$ time of flight

Cont. →

With time, lets compute x_f

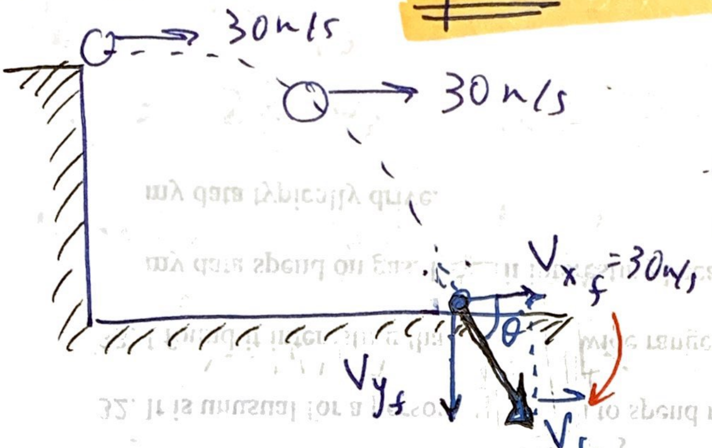
(iii) eqn

$$\underset{\sim}{x_f} = x_0^{\nearrow 0} + \cancel{v_{x_0}}^{30\,m/s} t + \frac{1}{2}\cancel{a_x}^{0} t^2$$

$$R = 30\,m/s\,(3.19\,s)$$

$$R = \boxed{95.7\,m} \text{ Range}$$

(b) what is the <u>speed</u> of impact of the car?



$= \|v_f\|$ speed

$$= \sqrt{v_{x_f}^2 + v_{y_f}^2}$$

need

$$= \sqrt{30\,m/s^2 + v_{y_f}^2}$$

(iii) eqn $v_{y_f} = \cancel{v_{y_0}}^{0} - \cancel{g}^{9.8\,m/s^2} t$

$$v_{y_f} = -9.8\,\tfrac{m}{s^2}\,(3.19\,s)$$

$$\boxed{v_{y_f} = -31.3\,m/s}$$

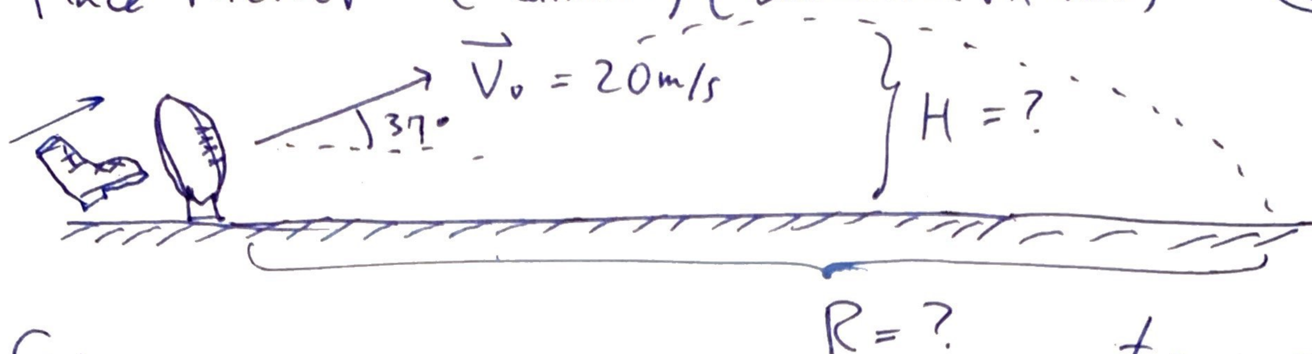
So magnitude: $\sqrt{(30\,m/s)^2 + (-31.26\,m/s)^2}$

$$= \sqrt{900 + 977.188}$$

$$\boxed{v_f = 43.3\,m/s}$$

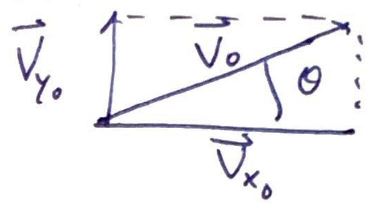
$$\theta = \tan^{-1}\left(\frac{31.3\,m/s}{30\,m/s}\right) = 46.2° \text{ below horizontal}$$

Ex Place Kicker (Cannon) (Baseball off bat)



$t_{flight} = ?$

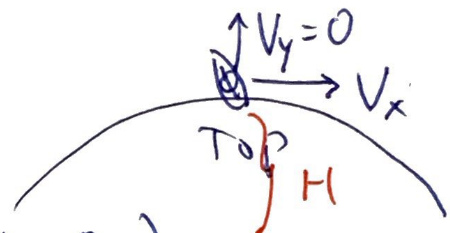
$$\begin{cases} V_{x_0} = V_0 \cos\theta \\ V_{y_0} = V_0 \sin\theta \end{cases}$$



$V_{x_0} = 20\ m/s\ \cos 37° = 16.0\ m/s$ hort. speed

$V_{y_0} = 20\ m/s\ \sin 37° = 12.0\ m/s$ vert. speed

(a) <u>what is max height</u>



$$\boxed{V_y^2 = V_{y_0}^2 + 2a_y(y - y_0)}$$

$$0\ m/s = (12.0\ m/s)^2 - 2(9.8\ m/s^2)(H - 0\ m)$$

$$2(9.8)H = (12.0)^2$$

$$H = \frac{144\ m^2/s^2}{2(9.8\ m/s^2)} = \underline{\underline{7.35\ m}}$$

(b) <u>Range</u>? Since landing is level with the launch point

$$\boxed{R = \frac{V_0^2 \sin(2\theta)}{g}}$$

$$= \frac{(20\ m/s)^2 \sin(2(37°))}{9.8\ m/s^2}$$

$$\boxed{R = 39.2\ m}$$

v = at

(c) <u>time of flight</u>

$$\boxed{t = 2 * \left[\frac{V_{0x}}{g}\right]}$$

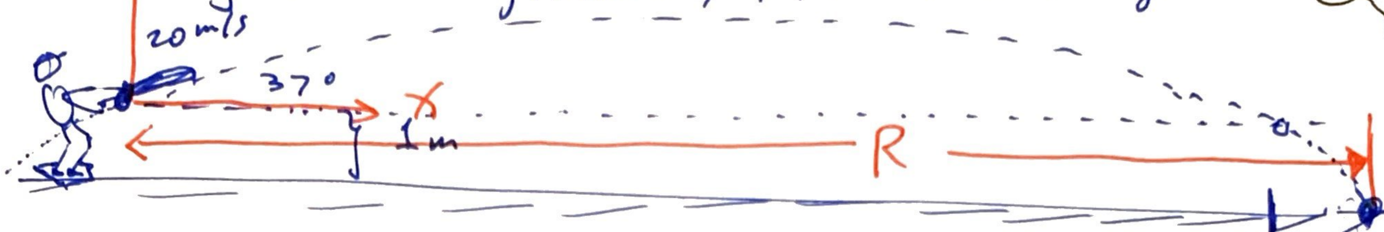
12.0 m/s

$$t = \frac{2(20\ m/s)\sin 37°}{9.8\ m/s}$$

$$\underline{\underline{t = 2.45\ s}}$$

Ex Starting above ground, find the range R

(i)



(ii) $x_0 = 0$ $V_0 = 20\,m/s$ $V_{0x} = 20\cos 37 = 16\ m/s$

$y_0 = 0$ $\theta_0 = 37°$ $V_{0y} = 20\sin 37 = 12\ m/s$

$x_f = R$, $y_f = -1\,m$

Range if we start at $y_0 = 0m$

(iii) eqns: we need time-of-flight 1st, then use $\boxed{x_f = x_0 + V_{x_0} t}$

we need vertical eqns:

$$\boxed{y = y_0 + V_{0y} t + \frac{1}{2} a_y t^2}$$

(iv) populate & solve

$-1m$, $0m$, $12\frac{m}{s}$, $-9.8 m/s^2$

clean up eqn

$$\left(-1 = 0 + 12t - \frac{1}{2} 9.8 t^2\right) * 2$$

$$\Rightarrow -2 = 24t - 9.8t^2$$

$$\Rightarrow \boxed{9.8t^2 - 24t - 2 = 0}$$

$\quad a \qquad b \qquad c$

$ax^2 + bx + c = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$t = \frac{-(-24) \pm \sqrt{(-24)^2 - 4(9.8)(-2)}}{2 \cdot 9.8}$$

$$t = \frac{24 \pm \sqrt{654.4}}{19.6}$$

$$= \frac{24 \pm 25.58}{19.6}$$

$$= \frac{-1.581}{19.6}, + \frac{49.58}{19.6}$$

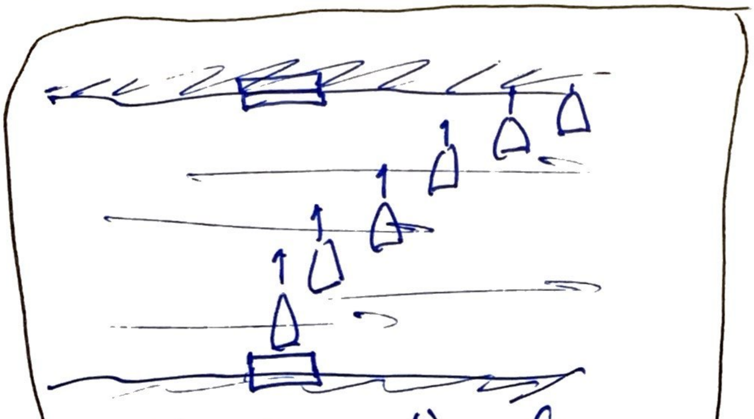
$$\boxed{t = -0.081\,s \quad \boxed{2.53\ s}}$$

So... $R = V \cdot t = (16\,m/s)(2.53\,s) = \boxed{40.48\,m}$

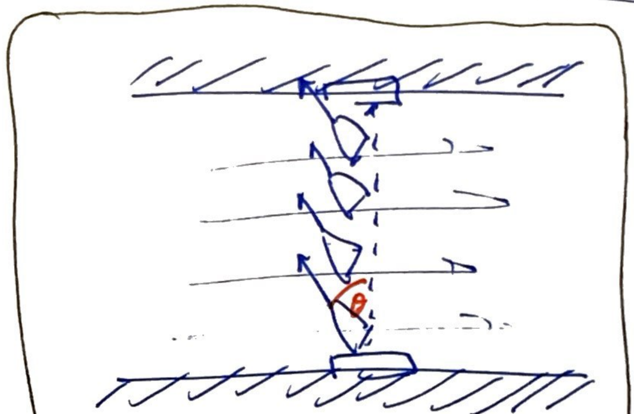
* Relative Velocity

Scenario: A ferry crosses a river. The river flowes to the right at $\vec{V}_{Water}$. The ferry in still water travels at $\vec{V}_{BW}$.

Q: What angle does the captain of the ferry point the ferry at so that it will reach the other side directly across from the launch point?

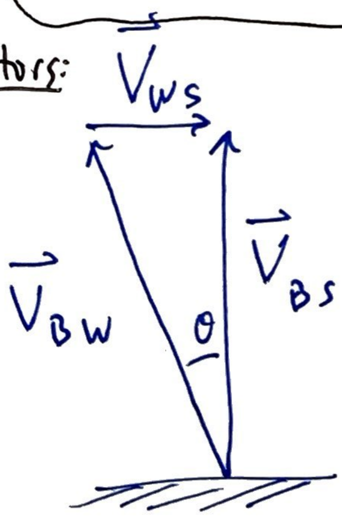


No correction for flow of river



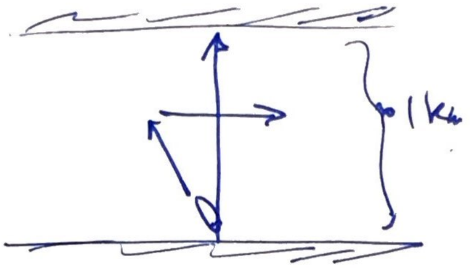
Correction for flow of river

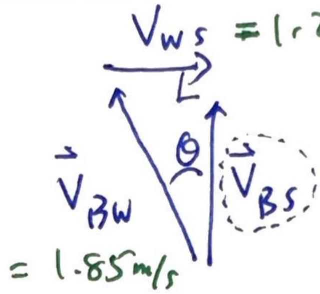
- Vectors:



- $\vec{V}_{BW}$ = velocity of ferry w.r.t. water
- $\vec{V}_{BS}$ = velocity of ferry w.r.t. shore
- $\vec{V}_{WS}$ = velocity of water w.r.t. shore

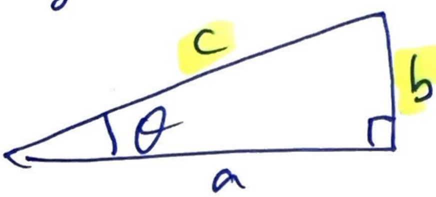
Ex Consider a ferry moving w.r.t. (still) water at 1.85 m/s. If the river flow is 1.20 m/s what angle up stream does the captain point his ferry so he moves $\perp$ to the shore?





We know the opposite and hypt. so use "sin" function

Trig Bubble



$$\begin{cases} b = c\sin\theta \\ a = c\cos\theta \end{cases}$$

$$a^2 + b^2 = c^2$$

$$\sin\theta = \frac{opp}{hyp}$$

$$\sin\theta = \frac{1.20}{1.85}$$

$$\theta = \sin^{-1}\left(\frac{1.20}{1.85}\right)$$

$$\boxed{\theta = 40.44^\circ}$$

Q_2: What is the speed of the ferry w.r.t. the shore? $V_{BS} = ?$

$$V_{BS}^2 + V_{WS}^2 = V_{BW}^2$$

$$V_{BS} = \sqrt{V_{BW}^2 - V_{WS}^2}$$

$$= \sqrt{1.85^2 - 1.20^2}$$

$$= \sqrt{1.9825}$$

$$\boxed{V_{BS} = 1.41 \text{ m/s}}$$

Q_3: At this rate then, how long will it take to reach the opposing dock if the width of the river is 1 km?

$$D = vt$$

$$t = \frac{D}{v} = \frac{1000\text{ m}}{1.41 \text{ m/s}} = 710 \text{ s}$$

$$= \underline{12 \text{ min}}$$