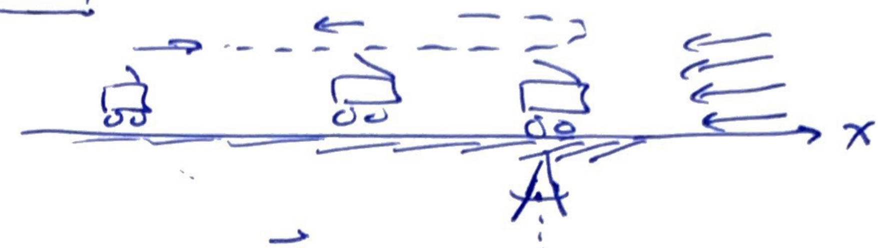
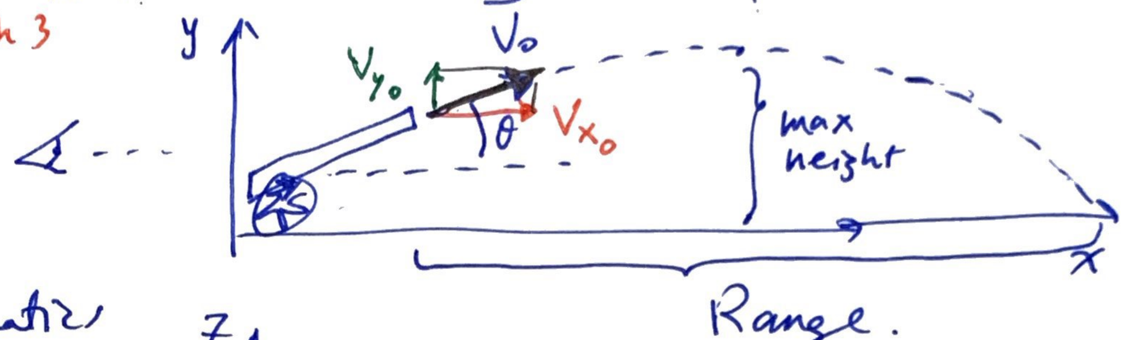
Chapter 3 a: Vectors and 2-Dim Kinematics

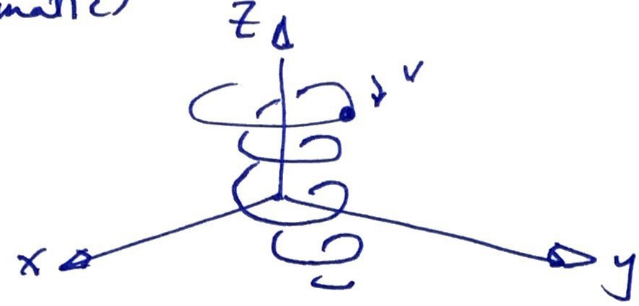
- 1-Dim Kinematics
  ch 2



- 2-Dim Kinematics
  ch 3



- 3-Dim Kinematics



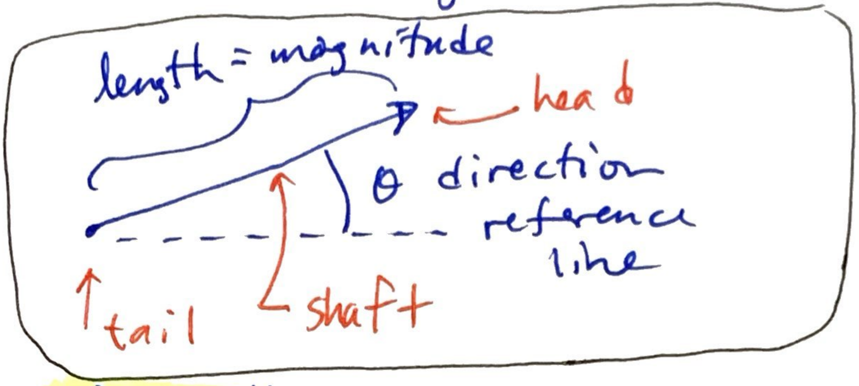
} Phys 101

The equation we saw in 1-Dim we double-up in 2-Dim

x-direction	y-direction
$a_x = \text{const.}\ (=0)$	$a_y = \text{const}\ (=g = 9.8\,\frac{m}{s^2})$
$V_{xf} = V_{x_0} + a_x t$	$V_{y_f} = V_{y_0} + a_y t$
$X = X_0 + V_{x_0} \cdot t$	$y = y_0 + V_{x_0} \cdot t + \frac{1}{2} a_y t^2$
$V_{x_f}^2 = V_{x_0}^2 + 2a(X_f - X_0)$	$V_{y_f}^2 = V_{y_0}^2 + 2a_y(y_f - y_0)$

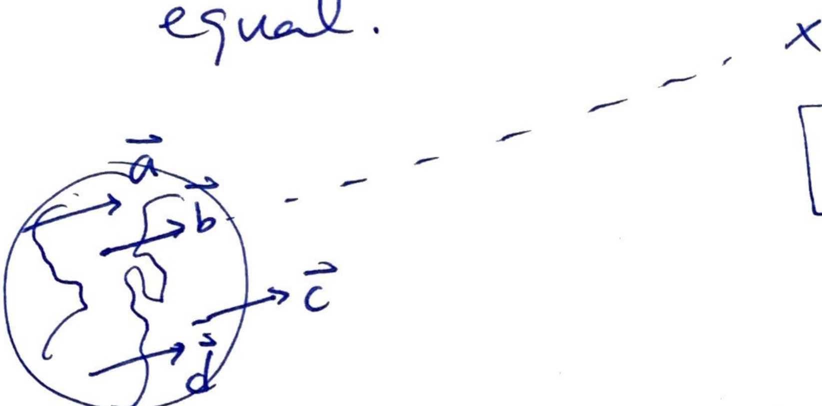
⊛ Vectors

- A vector is a mathematical construct (object) that denotes magnitude and direction.



⊛ Vector Properties

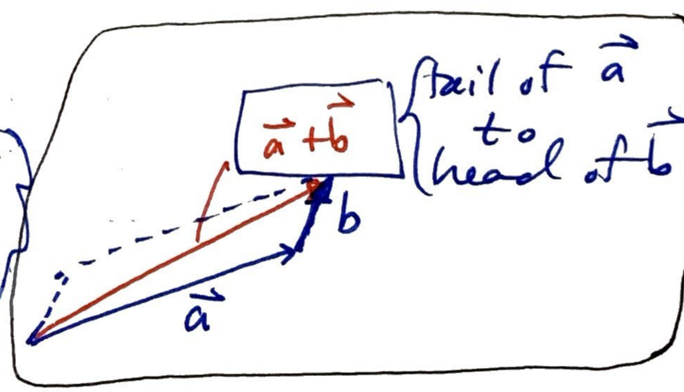
- Equality: any vector pointing at the same far away star, possessing the same length are equal.



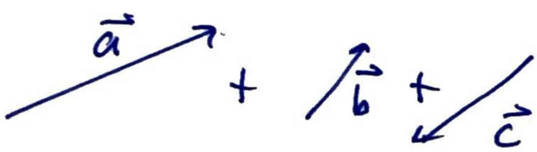
$$\boxed{\vec{a} = \vec{b} = \vec{c} = \vec{d}}$$

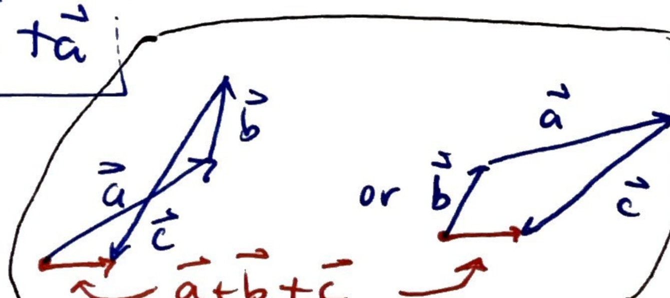
- Addition (graphically)

tail of $\vec{b}$ on head of $\vec{a}$

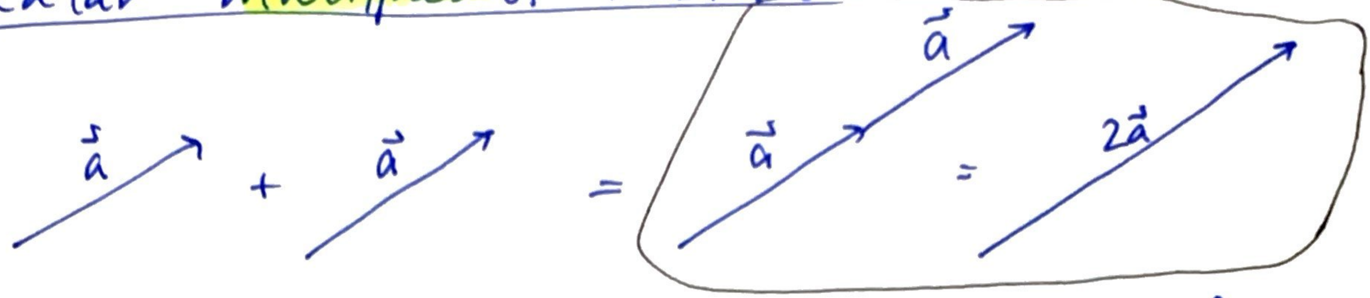


- Commute $\boxed{\vec{a}+\vec{b} = \vec{b}+\vec{a}}$



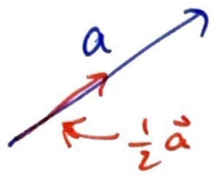


- scalar multiples of vectors

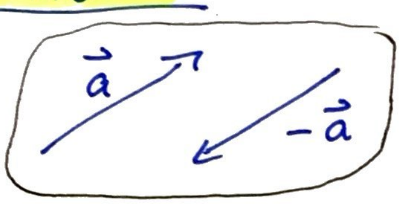


$c\vec{a}$ = a vector in the direction of $\vec{a}$
↖ scalar   but scaled by "c" in length.

ex $\frac{1}{2}\vec{a}$



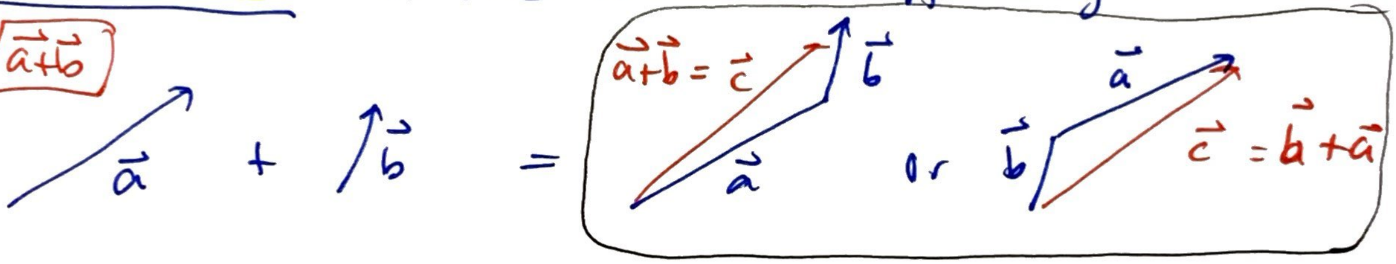
- negation



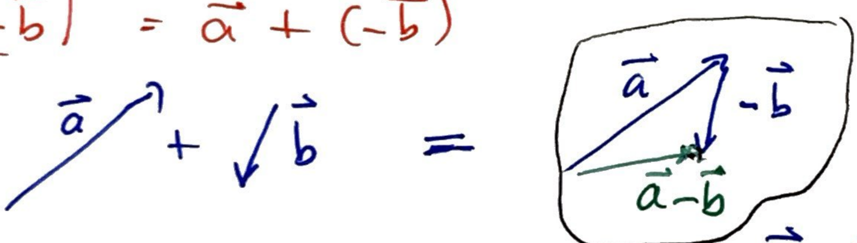
$-\vec{a}$ points in opposite direction but has the length.

- subtraction is the addition of negated vectors.

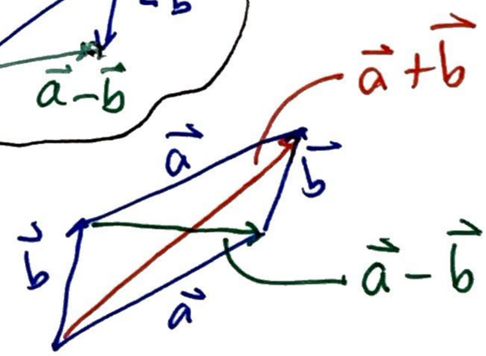
$\boxed{\vec{a}+\vec{b}}$



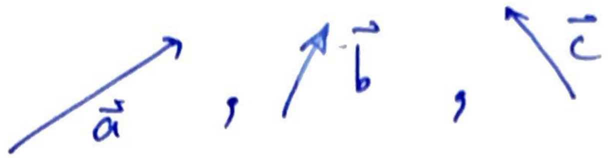
$\boxed{\vec{a}-\vec{b}} = \vec{a}+(-\vec{b})$

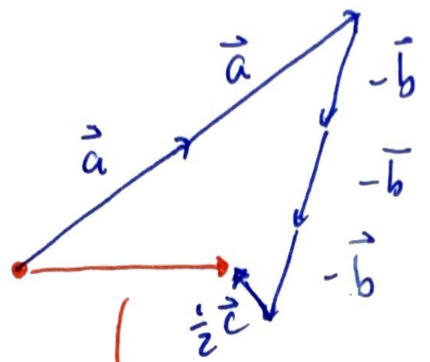


together:



EX perform $2\vec{a} - 3\vec{b} + \frac{1}{2}\vec{c}$ if





$2\vec{a} - 3\vec{b} + \frac{1}{2}\vec{c}$ the resultant vector

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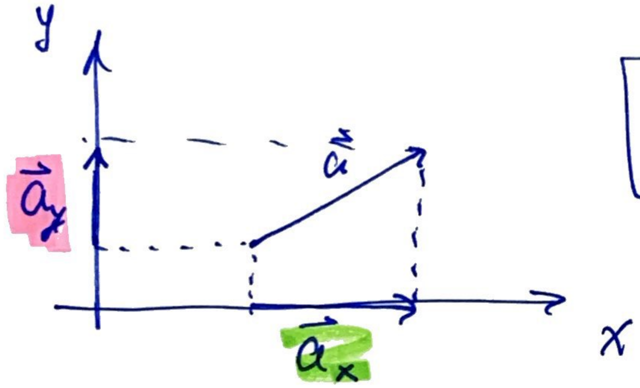
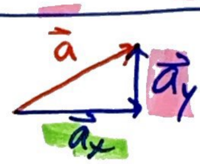
⊗ Components of a vector:

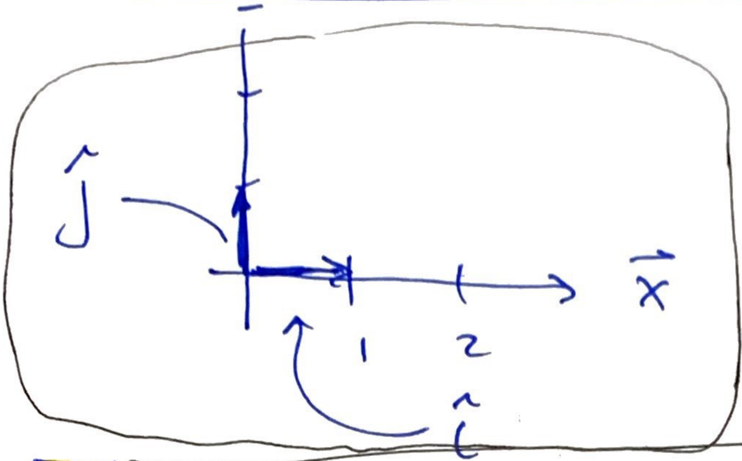


$$\boxed{\vec{a} = \vec{a}_x + \vec{a}_y}$$



we pick (x, y) axis to be ⊥ so we can decompose $\vec{a}$ into x-components (a_x) y-components (a_y)

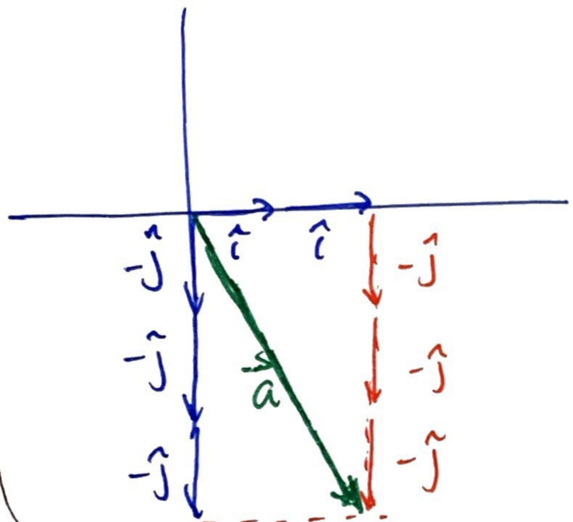
canonical vectors $\hat{i}$ & $\hat{j}$ (aka unit vectors)



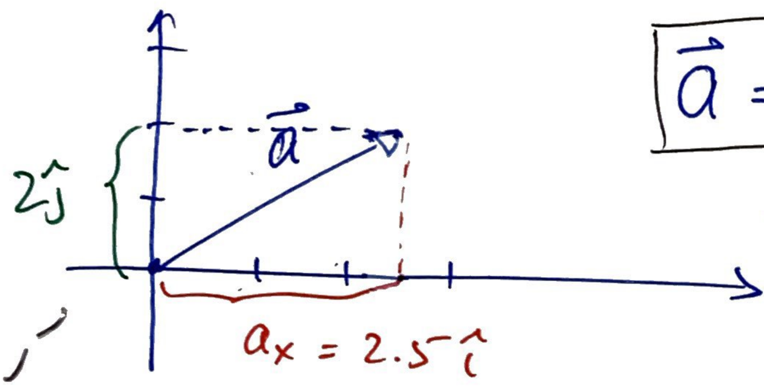
$\|\vec{a}\| = 1$ we use $\hat{a}$

Ex Build an $\vec{a}$ vector if we let

$$\vec{a} = 2\hat{i} - 3\hat{j}$$



- work backwards : decompose $\vec{a}$ into horizontal and vertical components



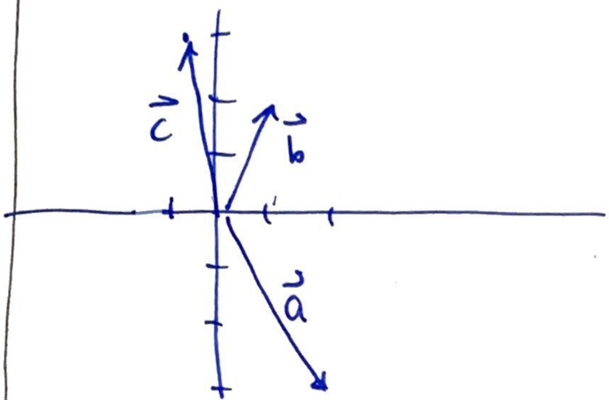
$$\boxed{\vec{a} = 2.5\hat{i} + 2\hat{j}}$$

$= \langle 2.5, 2 \rangle$ bra-ket notation

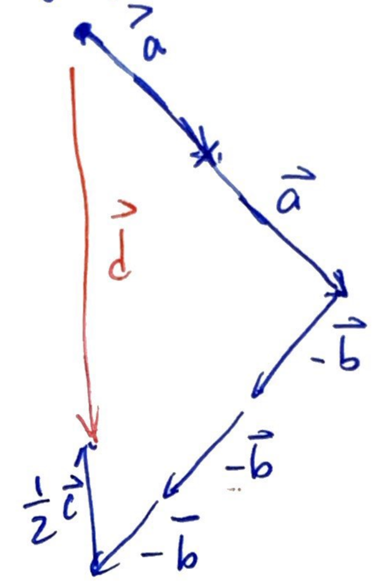
EX Vector addition in bracket notation

let $\vec{a} = \langle 2, -3 \rangle$ & $\vec{b} = \langle 1, 2 \rangle$ & $\vec{c} = \langle -1, 3 \rangle$

Find $2\vec{a} - 3\vec{b} + \frac{1}{2}\vec{c} = \vec{d}$



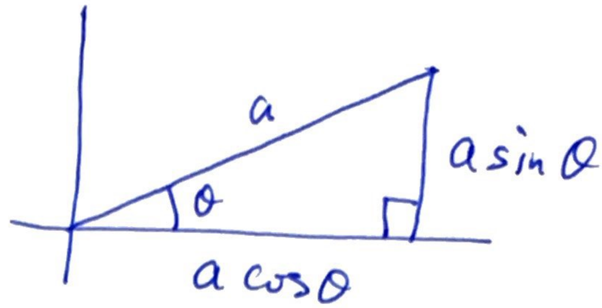
• graphical addition



• bracket addition

$$
\begin{aligned}
2\vec{a} &= 2\langle 2, -3 \rangle &&= \langle 4, -6 \rangle \\
-3\vec{b} &= -3\langle 1, 2 \rangle &&= \langle -3, -6 \rangle \\
+ \quad \frac{1}{2}\vec{c} &= \frac{1}{2}\langle -1, 3 \rangle &&= \langle -\tfrac{1}{2}, \tfrac{3}{2} \rangle \\
\hline
2\vec{a} - 3\vec{b} + \frac{1}{2}\vec{c} & &&= \langle 4 - 3 - \tfrac{1}{2}, -6 - 6 + \tfrac{3}{2} \rangle \\
& && \vec{d} = \boxed{\langle \tfrac{1}{2}, -\tfrac{21}{2} \rangle}
\end{aligned}
$$

- trigonometry



for vectors

$$\boxed{\vec{a} = \langle \|\vec{a}\| \cos\theta, \|\vec{a}\| \sin\theta \rangle}$$

where $\boxed{\|\vec{a}\| = \sqrt{a_x^2 + a_y^2}}$ pythagorean rule (length)

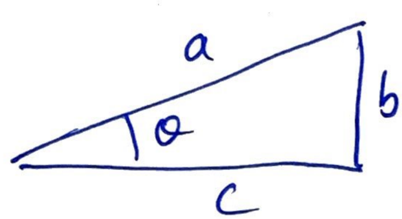
Ex let $\vec{a} = 2\hat{i} - 3\hat{j}$

then $\|\vec{a}\| = \sqrt{2^2 + (-3)^2}$

$= \sqrt{4 + 9}$

$= \underline{\underline{\sqrt{13}}}$

(direction)



then $\tan\theta = \dfrac{opp}{adj}$

$\tan\theta = \dfrac{b}{c}$, $\boxed{\tan^{-1}\left(\dfrac{b}{c}\right) = \theta}$

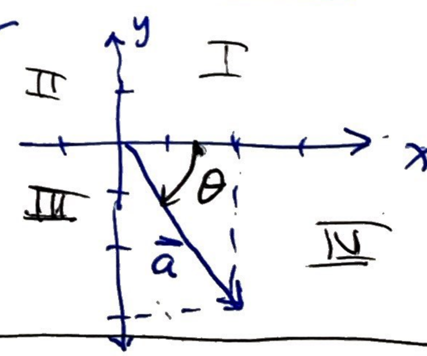
Ex what is the direction of

$\vec{a} = \langle 2, -3 \rangle$ from the example above

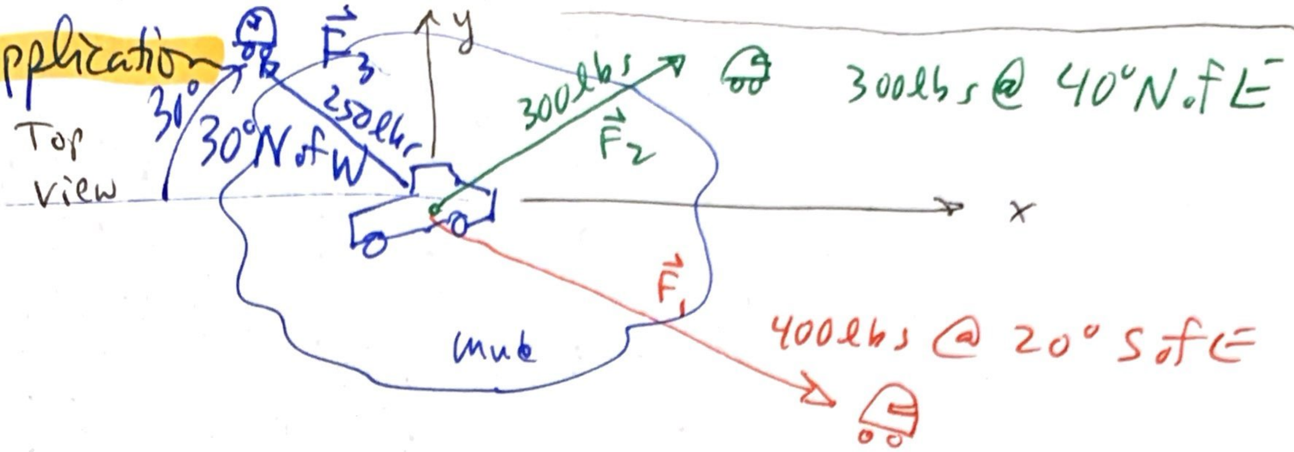
$\tan\theta = \dfrac{opp}{adj} = \dfrac{-3}{2} = -1.5$

$\theta = \tan^{-1}(-1.5)$

$$\boxed{\theta = -56.3^\circ}$$

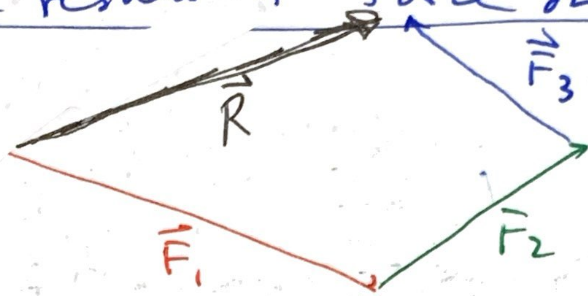


Ex] Application



Q: what is the resultant force on the truck?

· graphically



· components table

	vector	x - component	y component
F_1	400 20° S of E	$400\cos 20° = 375.877$	$-400\sin 20° = -136.808$
F_2	300 40° N of E	$300\cos 40° = 229.813$	$300\sin 40° = 192.836$
F_3	250 30° N of W	$-250\cos 30° = -216.506$	$250\sin 30° = 125.000$
		$\boxed{389.184}$	$\boxed{181.028}$

· component answer: run = adj   rise = opp

$$\vec{R} = 389.184\,\hat{i} + 181.028\,\hat{j}$$

·· mag: $\sqrt{389.184^2 + 181.028^2} = \boxed{429.23}$ lbs

·· dir: $\theta = \tan^{-1}\left(\dfrac{181.028}{389.184}\right) = \boxed{24.9°\text{ N of E}}$

· surveyer's answer:

$$\vec{R} = 429.23 \text{ @ } 24.9° \text{ North of East}$$