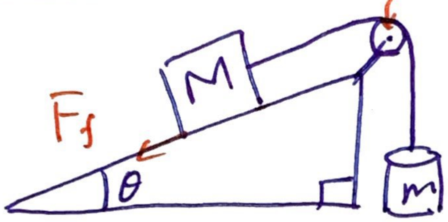
chapter 8 Rotational Motion

This chapter is a repeate of Chpts 3, 4, 5, 6, 7 but we focus on rotation not on linear.

Ex Inclined Plane



This pully is now not taken to be massless,
• It will take energy to spin.

$$\boxed{E_{0T} \pm W_{\substack{\text{added} \\ \text{or lost}}} = E_{fT}}$$

$$PE_0 + KE_0 \pm \mu F_N = PE_f + KE_f$$

$$KE_{\text{translational}} + KE_{\text{rotational}}$$

pulley.

- 8a : Rotational Kinematics
- 8b : Rotational Dynamics

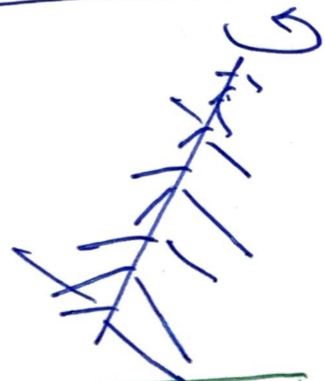
8a : Rotational Dynamics

Def: **Rigid Body** is a group of objects connected together such that their spatial orientation does not change.

Ex artificial christmas tree

w/ folding branches

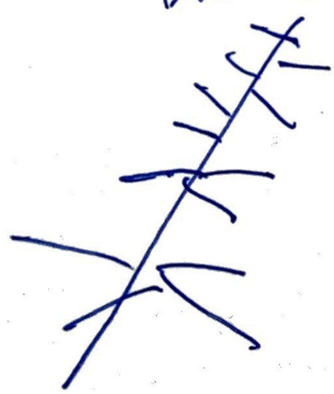
spin



Rigid Body -

- Branches do not fold.
- will not lose orientation when spun

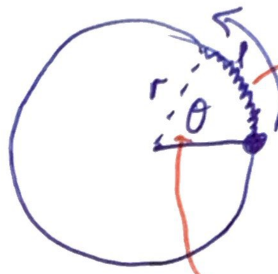
Branches open-up.



Non-Rigid Body

Angular Kinematics

Angular Displacement



length is called arc length

subtended angle

From geometry

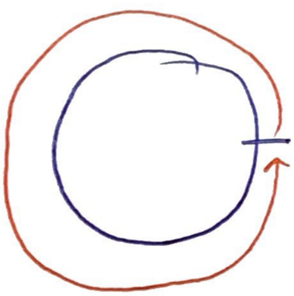
$$\boxed{l = r\theta}$$

radius

angle

arc length

- All formulas presented here require that θ be in radians.



$360° = 2\pi$

degrees radians

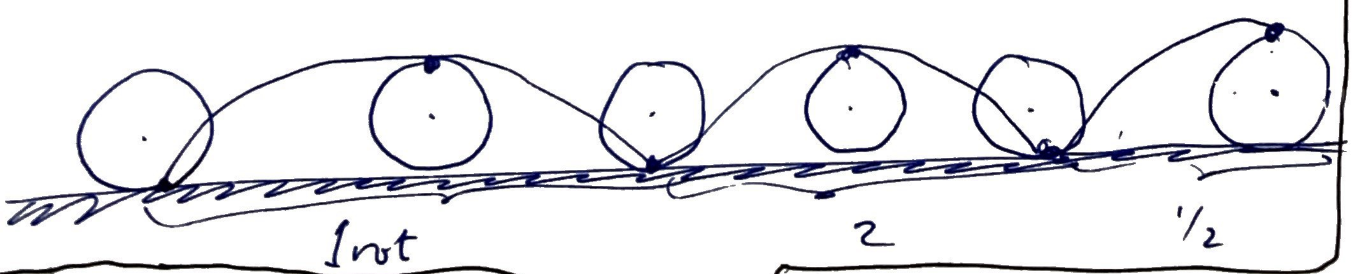
we will assume θ is to be provided in radians unless otherwise noted

equivalents

$$\boxed{360° = 2\pi^{r} = 100^{mil} = 1 \text{ revolution} = 400^{grad}}$$

military usage

Ex A bicycle tire rotates $2\frac{1}{2}$ times. (a) How many radians is this? $2.5\,\cancel{rev}\left(\frac{2\pi^{r}}{1\,\cancel{rev}}\right) = \underline{\underline{5\pi^{r}}} \approx 15^{rads}$



Ex continued

(b) if the tire's radius is 13", how far down the driveway did the "tire" move?

$\boxed{l = r\theta}$ {assumes no slipping}

Dist. traveled = $\underbrace{\text{tire's circumference}}$ * 2.5 rotations

$$l = \left(\frac{2\pi r \text{ in}}{\cancel{\text{rot}}}\right) * 2.5 \cancel{\text{rot}}$$

$$= 5\pi \cdot 13 \text{ in}$$

$$= \underline{\underline{65\pi}} \text{ inches} \quad \approx 195 \text{ in}$$

but the formula...

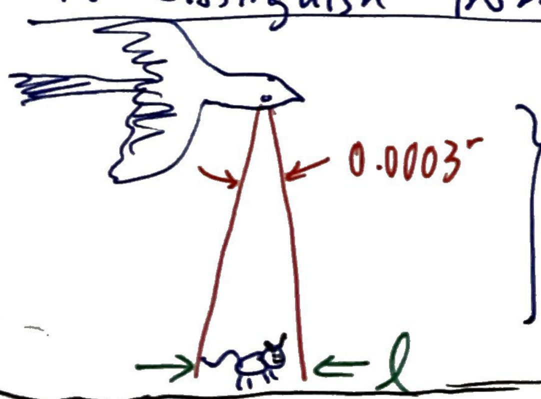
$$l = r\theta$$

$$= \underbrace{(13 \text{ in})}_{r}\overbrace{\left(2.5 \cancel{\text{rot}} \cdot \left(\frac{2\pi^r}{1 \cancel{\text{rot}}}\right)\right)}^{\theta^r}$$

radian

$$= \underline{\underline{65\pi}} \text{ in}$$

Ex A Hawk can distinguish a mouse on the ground as long as the subtended angle is 0.0003^{rad} or larger. If the hawk is at 100 m how small of rodent can it distinguish from a rock? {Resolution Calculation}



$\boxed{l = r\theta}$

$$= (100 \text{ m})(0.0003^r)$$

$$= 0.03 \text{ m}$$

$$= \underline{\underline{3 \text{ cm}}}$$

* Angular Velocity

we introduce small case **omega** "ω" vs Ω
small Big

the average **angular velocity** is given as

$$\boxed{\omega = \frac{\Delta\theta}{\Delta t}}$$

- units $[\![\omega]\!]$ = radians/sec {physics formulas}
- everyday usage : { **rpm** = rotations/minute
  20°/sec

EX A 13 in bicycle tire rotates 2.5 times in 7 sec.

Q: What is its angular speed in rpms?

$$\omega = \frac{\Delta\theta}{\Delta t}$$

$$= \frac{2.5 \text{ rotations}}{7 \text{sec}}$$

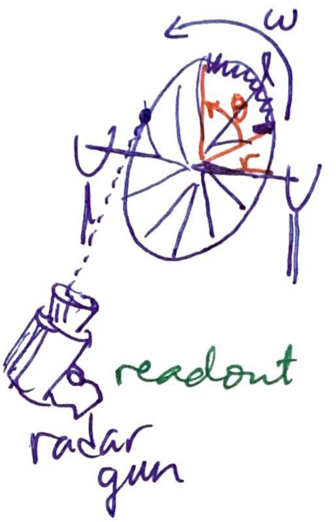
$$= \underline{0.357} \text{ rev}/\cancel{\text{sec}} \left(\frac{60 \cancel{s}}{1 \text{min}}\right)$$

$$= \boxed{21.43 \text{ rpm}}$$

Q: what is its speed in radians/sec

$$0.357 \frac{\text{rev}}{\text{sec}} \left(\frac{2\pi \text{ rad}}{\text{rev}}\right) = \underline{2.24 \text{ rad}/\text{sec}}$$

- angular velocity (vs) linear velocity



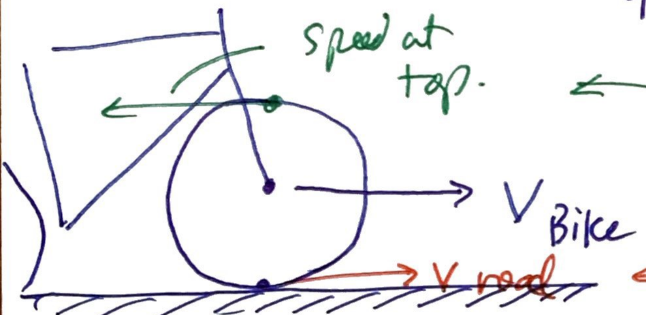
$l = r\theta$ ÷ time

$$\frac{\Delta l}{\Delta t} = r\frac{\Delta \theta}{\Delta t}$$

linear speed, angular speed

$= \boxed{v = r\omega}$ ⊛ connection between v and ω ⊛

speed of a point on the wheel's rim.



$V = r\omega$

$V = 0$

$\frac{r\omega + 0}{2} = \text{ave}$

$\boxed{\frac{1}{2} r\omega}$ axel speed

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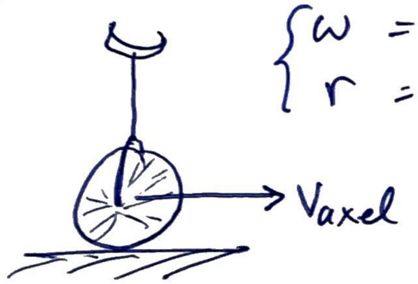
ex In the previous example assume the wheel that did 2.5 rot in 7 sec {0.357 rad/sec} belongs to a unicyclist. How fast is that cyclist moving down the road?



$\left\{\begin{array}{l}\omega = 0.357 \text{ rad/sec} \\ r = 13 \text{ in}\end{array}\right\}$ Top of wheel is

$$\boxed{V_{Top} = r\omega}$$

$= (13 \text{ in})(0.357 \text{ rad/sec})$

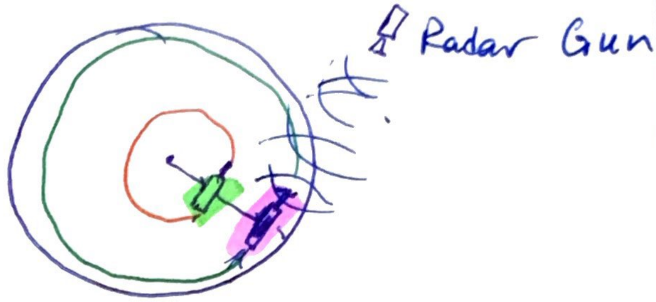
$= \underline{\underline{4.64 \text{ in/sec}}}$

So the axel speed $= \frac{1}{2}$ top of tire speed

$$V_a = 2.32 \text{ in} \; \frac{3600 s}{} \; \frac{1 \text{ ft}}{12 \text{ in}} \; \frac{1 \text{ mi}}{5280 \text{ ft}} = 0.13 \text{ mi/hr}$$

EX A carousel rotates at 1 rotation per 6 sec.

TOP view:



(a) If the outer horse is 2m from the center how fast will the person on the horse move?

$$\boxed{v = r\omega}$$

$$= 2m\left(\frac{1\text{ rot}}{6\text{ sec}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rot}}\right)$$

$$= \boxed{2.09\text{ m/s}}$$

(b) if you move to the inner horse at 1.5m how fast do you move?

$$v = r\omega$$

$$= \underline{\underline{1.5m}}\left(\frac{1\text{ rot}}{6\text{ sec}}\right)\left(\frac{2\pi\text{ rad}}{\text{rot}}\right)$$

$$= \boxed{1.57\text{ m/s}}$$

angular acc'ln

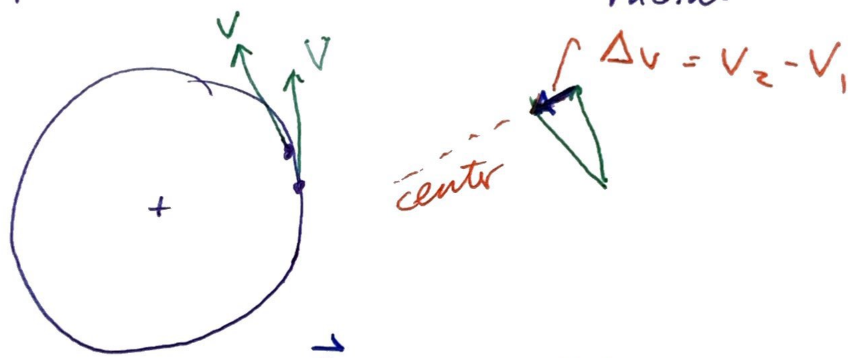
Define $\boxed{\alpha \equiv \frac{\Delta\omega}{\Delta t}}$ rads/sec^2

linear, angular: $\boxed{a_{Tan} = r\alpha}$

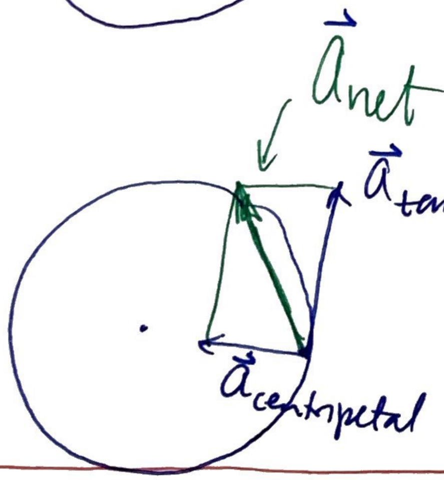
$\omega = \frac{\Delta\theta}{\Delta t}$ $\qquad v = r\omega$

θ $\qquad l = r\theta$

Note: In chpt 5 we use a_{radial} not a_{tan}



Both



The tire can be acc'ltn in speed.

• non-uniform $\boxed{\|\vec{a}_{net}\| = \sqrt{a_{cent}^2 + a_{tan}^2}}$

Non-Uniform motion.... there is angular acc'ln. otherwise $a_{tan} = 0$

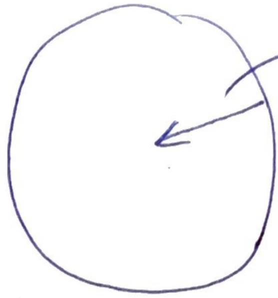
• Uniform: $a_{net} = a_c$, $a_{tan} = 0$

$\Delta\omega = \text{const.}$

$\Delta\alpha = 0$

$a_{tan} = 0$

- **Note:**



$$a_{cent} = \frac{v^2}{r}$$

but $\boxed{v_{tan} = r\omega}$

so $a_c = \frac{[r^2\omega^2]}{r} = \underline{\underline{r\omega^2}}$

$$\boxed{a_{cent} = r\omega^2} = \frac{v^2}{r}$$

Ex A merry-go-round is initially at rest while kids board. A $t=0$ the platform starts to rotate with an angular acc'ln of $\underline{\alpha = 0.06^r/sec^2}$

(a) After 8 sec what is the **angular velocity**?

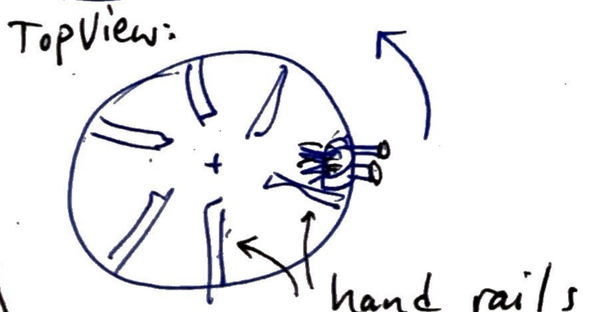
$$\alpha = \frac{\Delta\omega}{\Delta t} \Rightarrow \alpha \cdot \Delta t = \Delta\omega$$

$$\Rightarrow \boxed{\omega_f = \omega_0 + \alpha \cdot t} \quad \alpha = \text{constant}.$$

$$= 0 + (0.06^r/s^2)(8s)$$

$$\omega = \underline{\underline{0.48^r/s}}$$

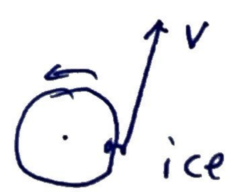
(b) For a child at 1.5 m from the center what is her **linear speed**?



$$v = r\omega$$

$$v = (1.5m)(0.48^r/s)$$

$$= \underline{\underline{0.72\ m/s}}$$



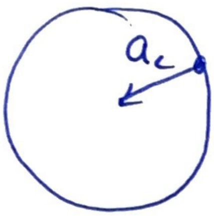
(c) what is the child's tangential acc'n?

$$a_{tan} = r\alpha$$

$$= (1.5\,m)(0.06\,^r/s^2)$$

$$\boxed{a_{tan} = 0.09\,m/s^2}$$

(d) What is the centripetal acc'n @ t=8 s?



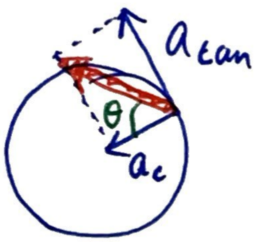
$$a_c = \frac{v^2}{r} = r\omega^2$$

$$a_c = (1.5\,m)(0.48\,^r/s)^2$$

$$\boxed{a_c = 0.35\,m/s^2}$$

(e) Net acc'n both magnitude and direction:

- $$\|\vec{a}_{net}\| = \sqrt{\|a_{centripetal}\|^2 + \|a_{tan}\|^2}$$



$$= \sqrt{(0.35)^2 + (0.09)^2}$$

$$= \underline{0.128\,m/s^2}$$

- $$\theta = \tan^{-1}\left(\frac{0.09}{0.35}\right)$$

$$= \underline{14.4° \text{ off the radial axis}}$$

⊛ angular vs. linear Kinematics

Linear	Connector	Angular
l	$\boxed{l = r\theta}$	θ
$v = v_0 + at$	$\boxed{v = r\omega}$	$\omega = \omega_0 + \alpha t$
$x = x_0 + v_0 t + \frac{1}{2} a t^2$	$\boxed{x = r\theta}$	$\theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$
$v_f^2 = v_0^2 + 2a\Delta x$		$\omega_f^2 = \omega_0^2 + 2\alpha\Delta\theta$
$a = \Delta v / \Delta t$	$\boxed{a = r\alpha}$	$\alpha = \frac{\Delta\omega}{\Delta t}$
$v = \Delta x / \Delta t$		$\omega = \Delta\theta / \Delta t$

nice formulas to have also: $\boxed{\omega = 2\pi f}$ and $\boxed{f = \frac{1}{T}}$ T = period

note: we assume the wheel spins w/ slipping

Ex A centrifuge acc'lts @ 20,000 rpm in 30 sec. Assume the acc'ln is constant.

(a) What is the angular acc'ln?

$$\alpha = \frac{\Delta \omega}{\Delta t} = \frac{\omega_f - \omega_o}{t} =$$

$$= \frac{\boxed{20{,}000 \frac{\cancel{rot}}{\cancel{min}}\left(\frac{1\,\cancel{min}}{60s}\right)\left(\frac{2\pi^r}{\cancel{rev}}\right)} - 0^r/s^2}{30\ sec}$$

$\leftarrow \omega_f = 2094.4^r/s$

$$\boxed{\alpha = 69.81\ ^{rad}/s^2}$$

(b) Through how many rotations did the centrifuge travel to get from rest to 20,000 rpm?

θ

$$\omega_f^2 = \cancel{\omega_o^2}^{\,0} + 2\alpha\Delta\theta$$

want this

$$\frac{\omega_f^2 - 0^2}{2\alpha} = \Delta\theta$$

$$\Delta\theta = \frac{(2094.4^r/s)^2 - 0^2}{2(69.81^r/s^2)}$$

$\theta = 31{,}417.5$ radians to spin up to 20,000 rpm

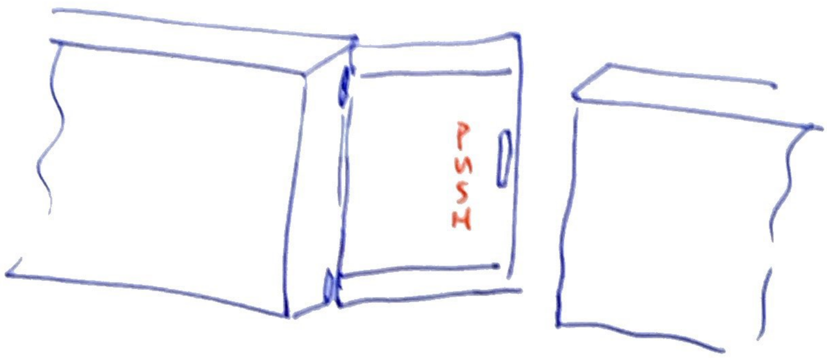
Convert to rotations:

$$31{,}417.5^r\left(\frac{1\,rot}{2\pi^r}\right)$$

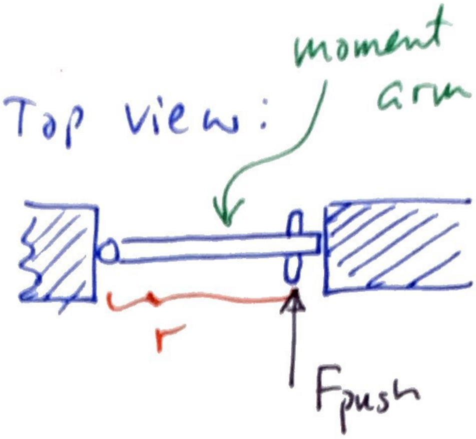
$$= \boxed{5000.3\ \text{rotations}}$$

⊛ Angular Force [Torque]

"Twisting Force"



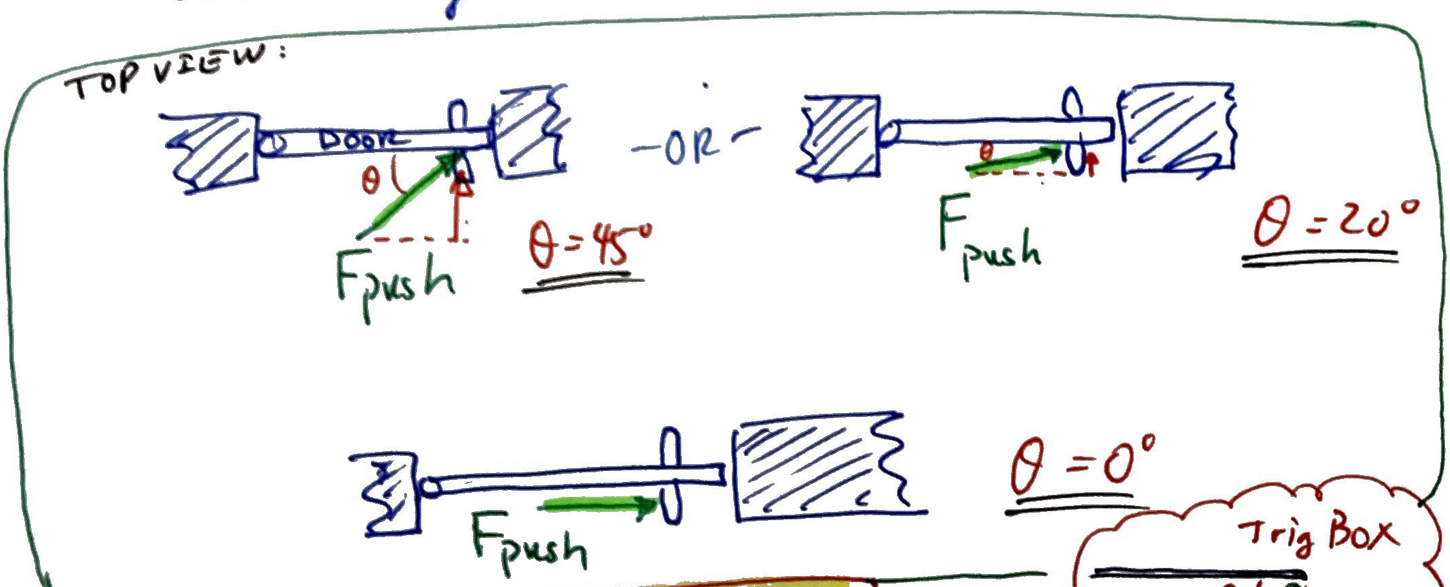
Top view:



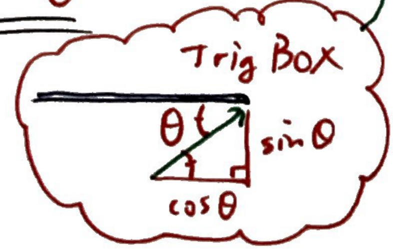
$$\boxed{\overrightarrow{\text{Torque}} \equiv r\,\vec{F}_\perp}$$

$F_\perp$ = Force perpendicular to momentarm

- max torque is acheived when applied force is $\perp$ to the moment arm
- other angle are less efficient

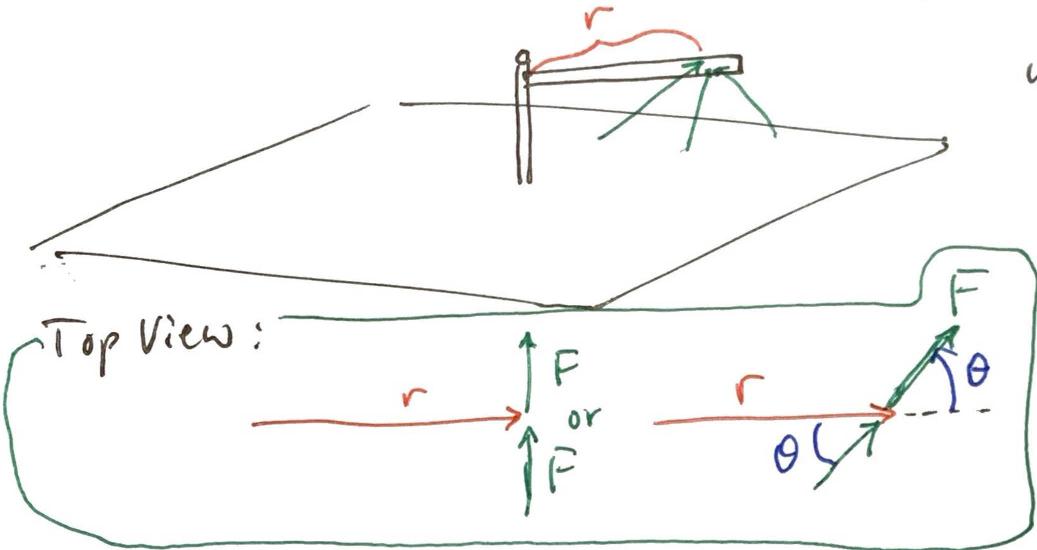


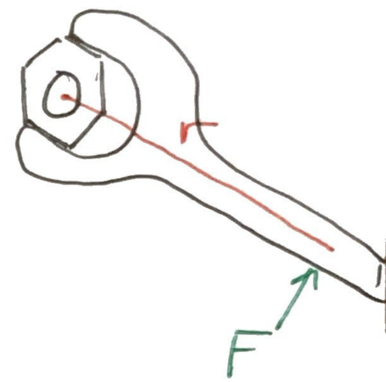
magnitude: $\boxed{\tau = rF\sin\theta}$



• Direction

torque, like force, is a vector quantity

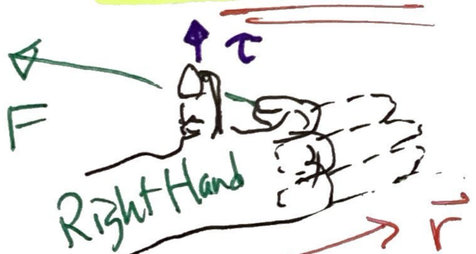


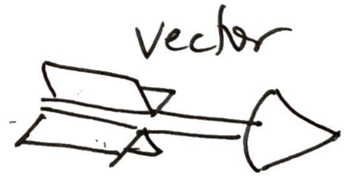


RH rule: "r" cross "F" into applied force

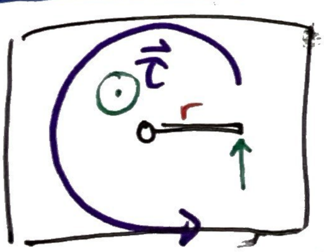
fingers along the moment arm (radial)

⟹ The thumb gives the direction of the torque

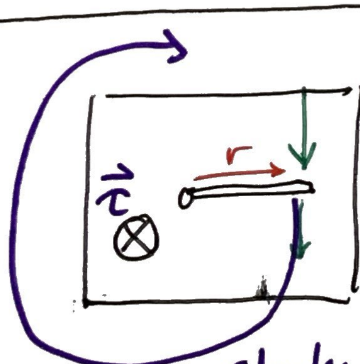




• Conventions



Counter-clockwise "CCW." (+) Torque out of page



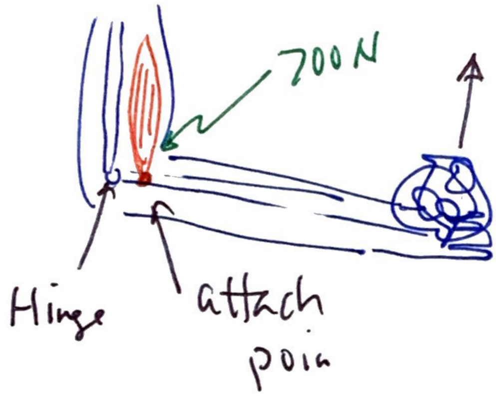
Clockwise (−) torque into the page

$$\boxed{\vec{\tau} = \vec{r} \times \vec{F}}$$

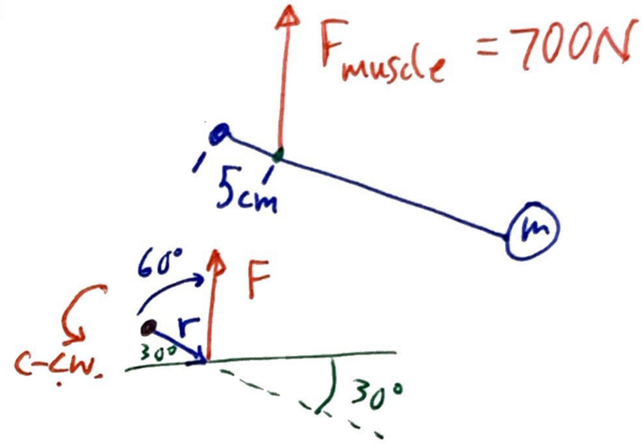
EX

If your forearm is at 30° below your the horizontal and your Bicepts apply 700 N to lift a mass up, what is the torque at the elbow if the Bicepts is attached at 5cm from the hinge point of your elbow?

(i)



(ii)



(iii)

$$\tau = r F \sin\theta$$

(iv)

$$\tau = (0.05m)(700N)\sin 60^\circ$$

$$\boxed{\tau = 30.3\ mN}$$

· U.S. Torque wrench ·

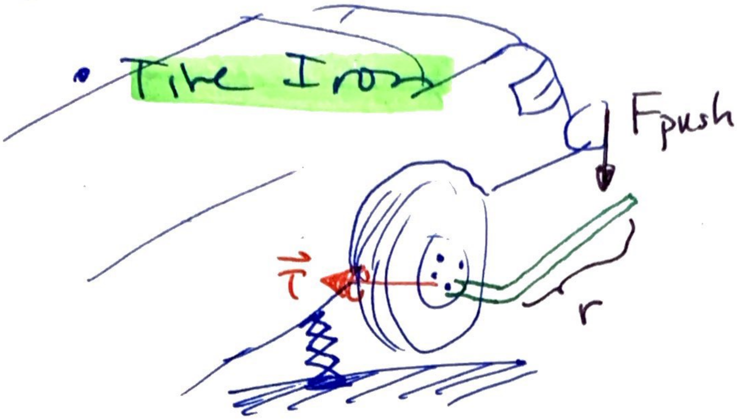
ft-lbs

dyne · cm

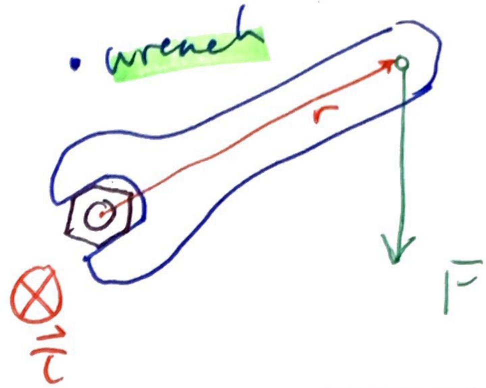
N · cm

Applications

- Tire Iron



- wrench



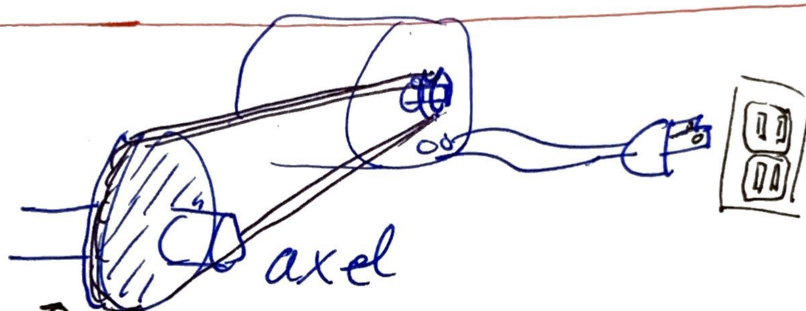
vector bubble

3-D vector 

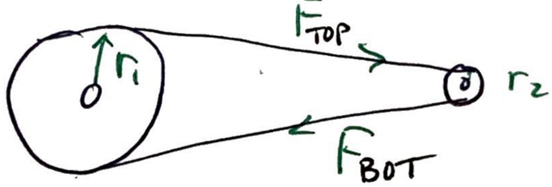
Front view (out of page) 

Rear view (into page) 

- motors



pulley
((b/c it pulls :)))



$F_{TOP} > F_{BOT}$

more torque is being applied to the larger pulley