Except for your name DO NOT WRITE on this sheet. Show ALL work for FULL credit. All problems are 5 pts each unless otherwise noted. Show all work for FULL credit.

Go for the most points… Don't get bogged down on any one problem.

1. (10 pts) Differentiate

(a) $y = \sin(2x + 1)$

(b) $y = (x^3 + 1)^2$

(c) $y = \ln(e^{4x} + 3)$

2. (10 pts) Consider $f(x) = \frac{1}{3}x^3 - x^2 + x + 1.$

(a) find the intervals where the function f is increasing and where it is decreasing, find the location and value of the relative extrema of f.

(b) find the intervals where the graph of f is concave upward and where it is concave downward, and label the inflection points, if any, and t*hen sketch the curve* using this information.

3. (10 pts)

(a) Find the *slope* of the tangent line at the point (1, 2) to the curve $x^2 + 2xy - y^2 = 2$

(b) Evaluate $\int_0^{\pi} cos(x)sin^6(x)\,dx$

4. (10 pts) 900 square centimeters of material is to be used to make an open-topped box which has a square base. What should be the dimensions of the box in order that its volume will be as large as possible?

5. (10 pts) Find the work needed to drain water out of the top of a spherical tank of radius R if only the bottom 'h' feet is full. Sketch the tank and run y from the bottom up. Set up but DO NOT EVALUATE. {Work = Force * Distance= m g y where y = height lifted … so $\Delta W = \Delta m\, g\, y$ and $\Delta m = \rho\, \Delta V$ and $\Delta V = A(y)\Delta y$ where A(y) is the cross-sectional area at station y. Thus $\Delta W = \rho\, g\, A(y)\, y\, \Delta y$. So change this to an integral and insert the proper limits}

6. (10 pts) The biologist G. F. Gause conducted an experiment in the 1930s with the protozoan *Paramecium* and used the population function P(t)

$$P(t) = \frac{64}{1 + 31e^{-0.7944t}}$$

where *t* was measured in days. Use this model to determine when the population was increasing most rapidly.

7. (10 pts) Evaluate the integrals:

(a) $$\int \frac{\cos(\ln x)}{x}\,dx$$

(b) $$\int \tan x \ln(\cos x)\,dx$$

8. (10 pts) Find the volume, via cylindrical shells, of the solid obtained by rotating about the y-axis the region under the curve $y = 1/(1 + x^4)$ from $x = 0$ to $x = 1$

9. (10 pts)

(a) Find the average value of f(x) over [1, 4] for f(x) = 1/x

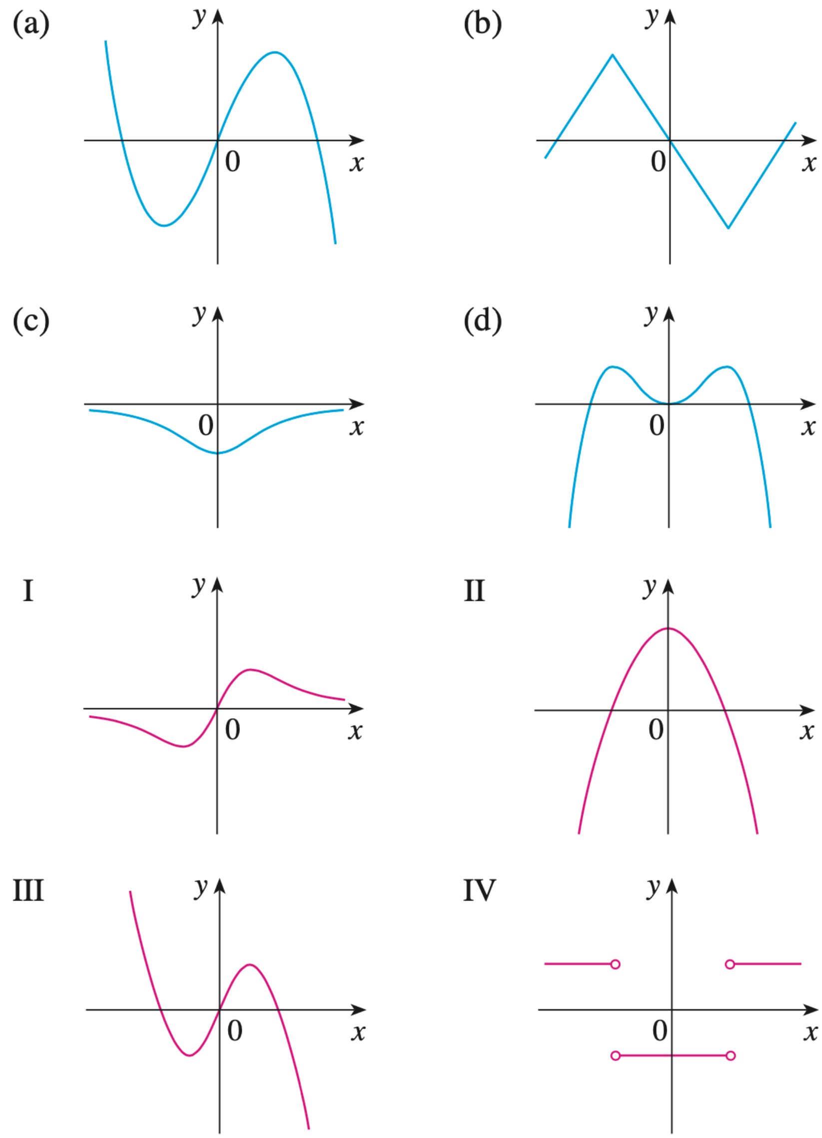
(b) Find the area of the region bounded by the curves $y = e^x$, $y = e^{-x}$, $x = -2$, and $x = 1$.

10. (10 pts) Find an equation of the tangent to the curve at the given point.

(a) $y = (2 + x)e^{-x}, \quad (0, 2)$

(b) $y = x \ln x, \quad (e, e)$

11. (10 pts) Match the graph of each function in (a)–(d) with the graph of its derivative in I–IV. Give reasons for your choices.



(a) Matches with I, II, III or IV (circle). Your reasoning?

(b) Matches with I, II, III or IV (circle). Your reasoning?

(c) Matches with I, II, III or IV (circle). Your reasoning?

(d) Matches with I, II, III or IV (circle). Your reasoning?

EC. (10pts) Any function of the form $f(x) = [g(x)]^{h(x)}$, where $g(x) > 0$, can be analyzed as a power of e by writing $g(x) = e^{\ln g(x)}$ so that $f(x) = e^{h(x) \ln g(x)}$. Using this device, calculate each limit:

(a) $\lim_{x\to\infty} x^{ln(x)}$ is this limit + or - ∞ ?

(b) $\lim_{x\to 0^+} x^{1/x}$ is this limit + or - ∞ or 0?