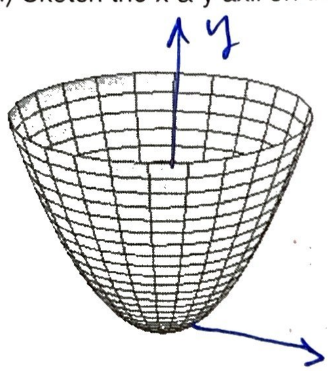
CoC Math 211 Fa25 R. Erickson $\frac{60}{60}$ TEST 5 (5.2-6.2) Name KEY

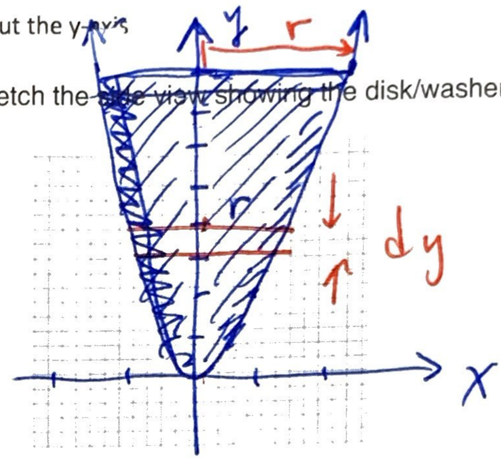
Show work for full credit. Go for the most point. Don't get bogged down on any one problem.

1. [5.2 Volumes] (10 pts). Use disk/washers to determine the volume of the solid obtained by rotating the region bounded by $y = 2x^2$, $y = 8$ and the y-axis about the y-axis

(i) Sketch the x & y axii on this 3-D view:



(ii) Sketch the side view showing the disk/washer:



(iii) What will the thickness be? dx or dy ? (Circle one) Indicate this on the side view sketch above.

(iv) What will the radius and area of the disk/washer be in terms of the variable selected in (iii)?

$y = 2x^2$

radius: r = x → $\boxed{r = \sqrt{y/2}}$ ← $x = \sqrt{\frac{y}{2}}$

area = π r² = πx^2 = $\boxed{\pi\, y/2}$

(v) What will the volume of the disk/washer be? Volume = area * thickness

volume = $\pi\, y/2 \cdot dy$

(vi) What are the variable limits in terms of the varialbe selected in (iii)? circle it below

a: from x or y = 0 to b: x or y = 8

(v) Set the integral up but do not solve it

$$\boxed{V = \int_{a=0}^{b=8} \frac{\pi y}{2}\, dy}$$

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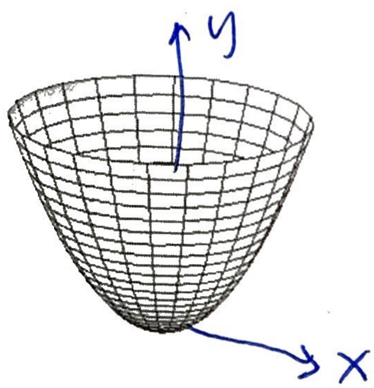
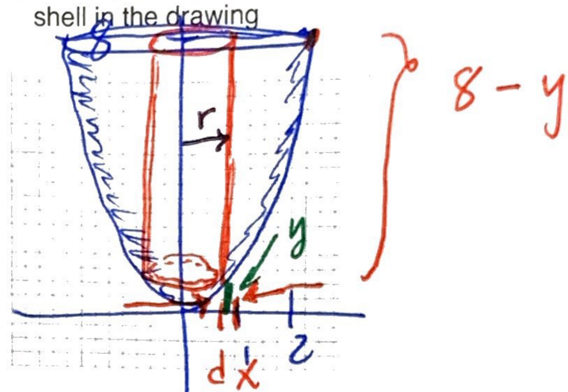
2. [5.3 Volumes by Shells] (10 pts) Use cylindrical shells to determine the volume of the solid obtained by rotating the region bounded by $y = 2x^2$, $y = 8$ and the y-axis about the y-axis {yes, same as prob 1}

(i) Sketch the x & y axii on this 3-D view:



(ii) Sketch the side-view. Place a thin cylindrical shell in the drawing



(iii) What will the thickness of the shell be? dx or dy ? (Circle one) Indicate this on the side-view sketch above.

(iv) What will the radius, height and sidewall area of the shell be in terms of the variable selected in (iii)?

$y = 2x^2$

radius: r = $\boxed{x}$

height: h = $8 - y = \boxed{8 - 2x^2}$

sidewall area = 2π r h = $\boxed{2\pi x(8-2x^2)}$

(v) What will the volume of the shell be? Volume = shell sidewall area * thickness

volume = $\boxed{2\pi x(8-2x^2)\cdot dx}$

(vi) What are the variable limits in terms of the varialbe selected in (iii)? circle it below

a: from x or y = 0 to b: x or y = 2

(v) Set the integral up but do not solve it

$$\boxed{V = \int_{a=0}^{b=2} 2\pi x(8-2x^2)\,dx}$$

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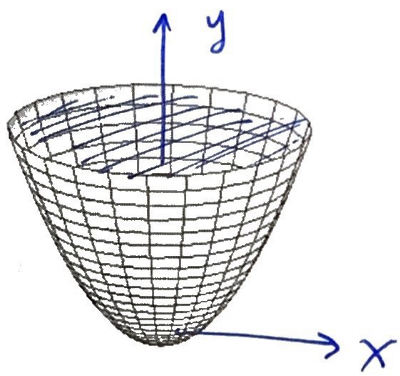
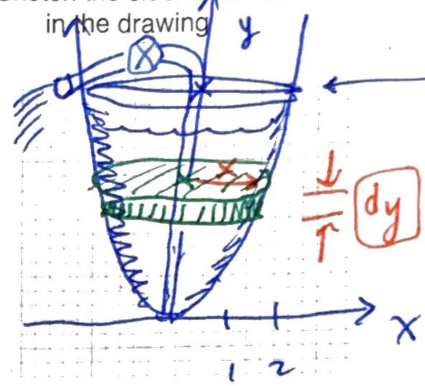
3. [5.4 Work] (10 pts) Determine the work performed by a pump used to drain the liquid contents of a tank that was formed by rotating the region bounded by $y = 2x^2$, $y = 8$ and the y-axis about the y-axis

(i) Sketch the x & y axii on this 3-D view:



(ii) Sketch the side-view. Show a slab of liquid in the drawing



} distance = 8 - y

$y = 2x^2$

$\frac{y}{2} = x^2$

(iii) Let the thickness be 'dy' Indicate this on the side view sketch above.

(iv) What will the radius and area of the slab be in terms of y?

radius: r = x, now convert 'x' to a function of 'y': $\boxed{r = x = \sqrt{y/2}}$

area = πr^2 = $\pi \left(\sqrt{\frac{y}{2}}\right)^2 = \boxed{\frac{\pi y}{2}}$

(v) What will the volume and weight of the slab be?

-> volume = area * thickness, but thickness is 'dy', so just re-write the area in the blank:

volume = $\boxed{\pi y/2 \text{ * dy}}$

-> weight = $\rho_o V g$ {use g = 10 and ρ_o = 1000}

weight = $(1000)\frac{\pi y}{2}(10)$ * dy $= \boxed{5000\pi y}$

(vi) How high will the slab be lifted if the exit is just above the 8 meter rim?

distance from slab height-to-exit: $\boxed{H = 8 - y}$

(vii) What are the integral limits if we drain just over the top of the tank?

a: from y = 0 to b: y = 8

(v) Set the integral up but do not solve it : W = weight · distance

$$\boxed{W = \int_{a=0}^{b=8} 5000\pi y(8-y)\,dy}$$

0

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4. [5.5 Average of a Function] (10 pts) What is the average daily rainfall in Jackson over the course of a year if the rainfall is given by the funtion R(t) = 5 - 5sin(π t / 365) which gives the daily inches of rain?

$$\bar{f} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

$$\boxed{T_{ave} = \frac{1}{365-0}\int_0^{365}\left[5 - 5\sin\left(\frac{\pi t}{365}\right)\right]dt}$$

$$\begin{cases} u = \pi t/365 \\ du = \frac{\pi}{365}dt \end{cases} \qquad \begin{aligned} &\to u_{lower} = 0 \\ &u_{upper} = \pi\, 365/365 = \pi \end{aligned}$$

$$\frac{365\,du}{\pi} = dt$$

$$T_{ave} = \frac{1}{365}\int_{u=0}^{\pi}\left[5 - 5\sin(u)\right]\left(\frac{365}{\pi}du\right)$$

$$T_{ave} = \frac{1}{\cancel{365}}\cdot\frac{\cancel{365}}{\pi}\,5\int_{u=0}^{\pi}\left[1-\sin(u)\right]du$$

integral in "u"

$$= \frac{5}{\pi}\Big[u + \cos(u)\Big]_0^{\pi}$$

$$= \frac{5}{\pi}\Big[\{\pi + \cos(\pi)\} - \{0 + \cos(0)\}\Big]$$

$$= \frac{5}{\pi}\left[\pi - 1 - 1\right] = \frac{5(\pi - 2)}{\pi}$$

$$= \boxed{5\left(1 - 2/\pi\right)} \approx 5/3 \doteq 1.7'' \text{ in/day}$$

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5. [6.2 Exponential Functions] (10 pts) Differentiate the given functions by using the chain, product and or quotient rules:

(a) $h(t) = 4e^{3t}$

$$\frac{dh}{dt} = 4\frac{de^{3t}}{dt}$$

$$= 4\,e^{3t}\cdot\frac{d\,3t}{dt} = \underline{\underline{12e^{3t}}}$$

(b) $f(x) = (1-8x)e^x$

product rule

$$\frac{df}{dx} = (1-8x)'e^x + (1-8x)(e^x)'$$

$$= -8e^x + (1-8x)e^x$$

$$= \boxed{-8xe^x - 7e^x}$$

(c) $g(x) = (x-1)/e^{3x}$

quotient rule

$$\frac{dg}{dx} = \frac{(x-1)'e^{3x} - (x-1)(e^{3x})'}{(e^{3x})^2}$$

$$= \frac{1\cdot e^{3x} - (x-1)(3e^{3x})}{e^{3x}\cdot e^{3x}}$$

$$= \frac{\cancel{e^{3x}}\,[1-3(x-1)]}{\cancel{e^{3x}}\cdot e^{3x}}$$

$$= e^{-3x}\,[1-3x+3]$$

$$= \boxed{-(3x-4)e^{-3x}}$$

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6. [6.2 Exponential Functions] (10 pts) Integrate the given functions:

(a) $\int 3\,(8y-1)\,\mathbf{e}^{4y^2-y}\,dy$ let $u = 4y^2 - y$

$du = (8y-1)\,dy$

$= 3\int e^u\,du$

$= 3e^u + c$

$= \boxed{3e^{4y^2-y} + c}$

(b) $\int_1^2 e^{3x}\,dx$

$u = 3x$
$du = 3\,dx$

$= \int_{x=1}^{2} e^u \left(\frac{du}{3}\right)$

$= \frac{1}{3} e^u \Big|_{x=1}^{2}$

$= \frac{1}{3} e^{3x} \Big|_{x=1}^{2}$

$= \frac{1}{3}\left[e^{3\cdot 2} - e^{3\cdot 1}\right]$

$= \boxed{\frac{1}{3}\left[e^6 - e^3\right]}$

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