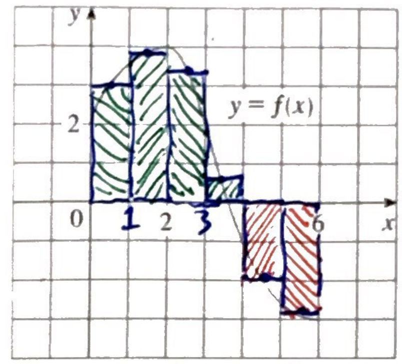
60/60

CoC Math 211 Fa25 R. Erickson TEST 4 (4.1-5.1) Name KEY

Show work for full credit. Go for the most point. Don't get bogged down on any one problem.

1. **[4.1 Riemann Sums] (5 pts)** ea (a) Use the graph of f to find the Riemann sum with six subintervals. Take the sample points to be midpoints of each rectangle. Estimate the heights of each rectangle terminated at the midpoint. Draw the rectangles on the diagram. Write out each the six rectangular area $f(x_{mid})*\Delta x$ then sum.



$\Delta x = 1$

$$R_m = f(\tfrac{1}{2})\cdot\Delta x + f(\tfrac{3}{2})\cdot\Delta x + f(\tfrac{5}{2})\cdot\Delta x$$
$$+ f(\tfrac{7}{2})\cdot\Delta x + f(\tfrac{9}{2})\Delta x + f(\tfrac{11}{2})\cdot\Delta x$$
$$= 3\cdot 1 + 3.75\cdot 1 + 3.33\cdot 1$$
5
$$+ 0.5\cdot 1 + (-2)\cdot 1 + (-2.9)\cdot 1$$
$$= 3 + 3.75 + 3.33 + 0.5 - 2 - 2.9$$
$$\doteq \boxed{5.68}$$

(b) If $f'(x) = x^4 + 3x - 9$ what was $f(x)$?

5 $\boxed{f = \frac{x^5}{5} + 3\frac{x^2}{2} - 9x + C}$ anti-derivative

10

2. [4.2 Define Integrals] (10 pts) (a) Determine the value of $\int_{2}^{10} f(x)\,dx$

Given that: $\int_{2}^{4} f(x)\,dx = -1$, $\int_{4}^{7} f(x)\,dx = 3$ and $\int_{10}^{7} f(x)\,dx = -8$.

$$\int_2^{10} f\,dx = \int_2^4 f\,dx + \int_4^7 f\,dx + \int_7^{10} f\,dx$$

5

$$= (-1) + (3) + \boxed{\{-(-8)\}}$$

$$= \boxed{10}$$

(b) Differentiate the following integral with respect to x. $\int_{-8}^{x} \mathbf{e}^{\cos(t)}\,dt$

can't integrate

Fund. Thm part 1

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

5

So $\frac{d}{dx}\int_{-8}^{x} e^{\cos(t)}\,dt = \boxed{e^{\cos(x)}}$

10

3. **[4.3 Fund Thm] (10 pts)** Show the steps in evaluating the following:

(a) $\int_{-2}^{4} v^3 - 7v^2 + 3v\,dv$

5

$= \left(\frac{v^4}{4} - \frac{7v^3}{3} + \frac{3v^2}{2}\right)\Big|_{-2}^{4}$

$= \left(\frac{4^4}{4} - \frac{7}{3}4^3 + \frac{3}{2}4^2\right) - \left(\frac{(-2)^4}{4} - \frac{7}{3}(-2)^3 + \frac{3}{2}(-2)^2\right)$

$= 4^2\left(\frac{16^8}{4_2} - \frac{7\cdot4}{3} + \frac{3}{2}\right) - (-2)^2\left(\frac{4^2}{4_2} - \frac{-14}{3} + \frac{3}{2}\right)$

$= 16\left(\frac{24-56+9}{6}\right) - 4\left(\frac{6+28+9}{6}\right) = \frac{8}{3}(-23) - \frac{2}{3}(43) = \frac{-184-86}{3} = \frac{-270}{3}$

$\boxed{-90}$

(b) $\int_{0}^{16} 9\sqrt{x} + 10\sqrt[4]{x}\,dx$

$= \int_0^{16}\left(9x^{1/2} + 10x^{1/4}\right)dx$

$= \left(9\frac{x^{3/2}}{3/2} + 10\frac{x^{5/4}}{5/4}\right)\Big|_0^{16}$

5

$= \left(6x^{3/2} + 8x^{5/4}\right)\Big|_0^{16}$

$= \left(6(\sqrt{16})^3 + 8(\sqrt[4]{16})^5\right) - (0+0)$

$= 6\cdot4^3 + 8\cdot2^5$

$= 6\cdot64 + 8\cdot32$

10

$= (360+24) + (240+16) = 384+256$

$= \boxed{640}$

4. [4.4 Indefinite Integrals] (15 pts) Evaluate

(a) $\int \sqrt[7]{w^2} + 3 - 9\sqrt[3]{w^7}\, dw$

$= \int w^{2/7} dw + \int 3\,dw - 9\int w^{7/3} dw$

5

$= \dfrac{w^{9/7}}{9/7} + 3w - 9\dfrac{w^{10/3}}{10/3} + C$

$= \boxed{\frac{7}{9}w^{9/7} + 3w - \frac{27}{10}w^{10/3} + C}$

(b) $\int \frac{6}{y^3} - \frac{1}{7y^6} + \frac{1}{y^2}\, dy$

$= 6\int y^{-3} dy - \frac{1}{7}\int y^{-6} dy + \int y^{-2} dy$

5

$= 6\left(\dfrac{y^{-2}}{-2}\right) - \frac{1}{7}\left(\dfrac{y^{-5}}{-5}\right) + \left(\dfrac{y^{-1}}{-1}\right) + C$

$= \boxed{-3/y^2 + \frac{1}{35y^5} - \frac{1}{y} + C}$

10

5. [4.5 Substitution] (10 pts) Evaluate the following by using a u-substitution. Convert the integral into one involving only the 'u' variable. Box that integral in, then proceed to give the answer.

(a) $\int 12v(7+6v^2)^9\, dv$

let $\boxed{\begin{array}{l} u = 7+6v^2 \\ du = 12v\,dv \end{array}} \rightarrow dv = \dfrac{du}{12v}$

$= \int \cancel{12v}\,(u)^9 \left(\dfrac{du}{\cancel{12v}}\right)$

5

$= \boxed{\int u^9\, du} = \dfrac{u^{10}}{10} + c = \boxed{\dfrac{(7+6v^2)^{10}}{10} + c}$

(b) $\int \dfrac{3+7y}{y^2+3}\, dy$

$u = y^2+3$
$du = 2y\,dy$ } messy on the whole integral so break up

better

$= 3\int \dfrac{dy}{y^2+3} + 7\int \dfrac{y\,dy}{y^2+3}$

let $\left\{\begin{array}{l} y = \sqrt{3}\,w \\ dy = \sqrt{3}\,dw \end{array}\right.$

$= 3\int \dfrac{\sqrt{3}\,dw}{3w^2+3} + 7\int \dfrac{du/2}{u}$

5

$= \sqrt{3}\int \dfrac{dw}{w^2+1} + \dfrac{7}{2}\ln|u| + c$

$= \boxed{\sqrt{3}\tan^{-1}(w) + \dfrac{7}{2}\ln|u| + c}$

$= \boxed{\sqrt{3}\tan^{-1}\left(\dfrac{y}{\sqrt{3}}\right) + \dfrac{7}{2}\ln|y^2+3| + c}$

10

CoC Math 211 Fa25 R. Erickson TEST 4 (4.1-5.1) Name KEY

6. [4.5 More u-subs] (10 pts)

(a) Solve using u-sub, including substituting the limits also. Box in the version of the integral when it is totally an integral in 'u' then STOP with out evaluating it: $I = \displaystyle\int_{-2}^{3} \frac{4}{(5+2x)^3}\,dx$

let $\boxed{u = 5+2x, \quad du = 2dx}$

limits: $\boxed{\begin{array}{l} x=-2 \rightarrow u = 5+2(-2) = 1 \\ x = 3 \rightarrow u = 5+2(3) = 11 \end{array}}$

5

$I = 4\displaystyle\int_{u=1}^{11} \frac{du/2}{u^3}$

$= \boxed{2\displaystyle\int_{u=1}^{11} \frac{du}{u^3}}$

$= \boxed{2\displaystyle\int_{u=1}^{11} u^{-3}du}$ Stop here

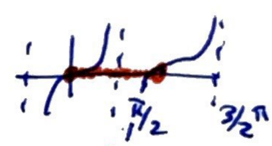
But if your curious..

$= \cancel{2}\, \dfrac{u^{-2}}{\cancel{-2}}\Big|_{u=1}^{11}$

$= \left(-\frac{1}{11^2}\right) - \left(-\frac{1}{1^2}\right) = -\frac{1}{121} + 1 = \boxed{\frac{120}{121}}$

(b) Solve using u-sub, including substituting the limits also. Box in the version of the integral when it is totally an integral in 'u' then STOP with out evaluating it: $I = \displaystyle\int_{0}^{\pi} \sec^2(y)\sqrt{2+\tan(y)}\,dy$

let $\boxed{\begin{array}{l} u = 2+\tan(y) \\ du = \sec^2(y)dy \end{array}}$



problem her!

limits

$\boxed{\begin{array}{l} y=0 : \quad u = 2+\tan(0) = 2+0 = 2 \\ y=\pi : \quad u = 2+\tan(\pi) = 2+0 = 2 \end{array}}$

5

$\boxed{I = \displaystyle\int_{u=2}^{u=2} \sqrt{u}\,du}$ STOP HERE

Note that tan(y) diverges @ $y = \pi/2$ so we need to treat this integral as an improper integral

$\overset{?}{=} 0$ since limits are the same?

-OR-

$= \infty$ since there is a singularity at $y = \pi/2$?

10