Math 211 Fa25 CoC R. Erickson 50/50 TEST 3 Name KEY

Show work for full credit. Go for the most point. Don't get bogged down on any one problem.

1. (5 pts) Long divide to find the oblique asymptote of $y = \dfrac{3x^3 + x^2 + 11x - 2}{x + 7}$

$$\begin{array}{r} 3x^2 - 20x + 151 \\ x+7 \overline{) 3x^3 + x^2 + 11x - 2} \\ -(3x^3 + 21x^2) \\ \hline -20x^2 + 11x \\ (-20x^2 - 140x) \\ \hline 151x - 2 \\ -(151x + 857) \\ \hline -859 \end{array}$$

5

$$\lim_{x \to \pm\infty} \left(\frac{3x^3 + x^2 + 11x - 2}{x+7} \right) = \boxed{3x^2 - 20x + 151}$$

2. (5 pts) Evaluate $\lim\limits_{x\to\infty} \dfrac{4x^3 + 3x + 1}{7x^3 + 2x^2 + x} \cdot \dfrac{1/x^3}{1/x^3}$

$$= \lim_{x \to \infty} \left(\frac{4x^3/x^3 + 3x/x^3 + 1/x^3}{7x^3/x^3 + \frac{2x^2}{x^3} + x/x^3} \right)$$

$$= \lim_{x \to \infty} \left(\frac{4 + \cancel{3/x^2}^{\to 0} + \cancel{1/x^3}^{\to 0}}{7 + \cancel{2/x}^{\to 0} + \cancel{1/x^2}^{\to 0}} \right)$$

5

$$= \boxed{\frac{4}{7}}$$

3. (10 pts) Determine the absolute extrema for the following function. Sketch.

$$y = \frac{1}{6}(x^3 - 6x^2 + 9x + 6)$$

(i) y-intersection:

1

$y(0) = 6/6 = 1$ $\boxed{(0, 1)}$

(ii) critical points (y'=0):

2

$y' = \frac{1}{6}(3x^2 - 12x + 9)$

$0 = 3x^2 - 12x + 9$

$0 = x^2 - 4x + 3$

$0 = (x-1)(x-3)$

$\Rightarrow \boxed{x = 1, 3}$

(iv) concavity (y'' > 0 or < 0):

2

$y'' = \left[\frac{3(x^2 - 4x + 3)}{6}\right]'$

$y'' = \frac{1}{2}(2x - 4)$

$\boxed{y'' = x - 2}$

@ $x = 1$: $y'' = 1 - 2 = -1 < 0$ concave down so $\boxed{x = 1 \text{ is a max}}$

@ $x = 3$: $y'' = 3 - 2 = 1 > 0$ so concave up, $\boxed{x = 3 \text{ is a min}}$

(v) state if critical points are local maximums or minimums

1

x = 1 is a max

x = 3 is a min

$y(1) = \frac{1}{6}(1^3 - 6(1)^2 + 9(1) + 6) = \frac{10}{6} = \frac{5}{3}$

$y(3) = \frac{1}{6}(3^3 - 6 \cdot 3^2 + 9 \cdot 3 + 6) = (27 - 54 + 27 + 6)/6 = 1$

(vi) inflection poin(s)t:

$\boxed{y'' = 0 \text{ @ } x = 2}$

Helper pts :

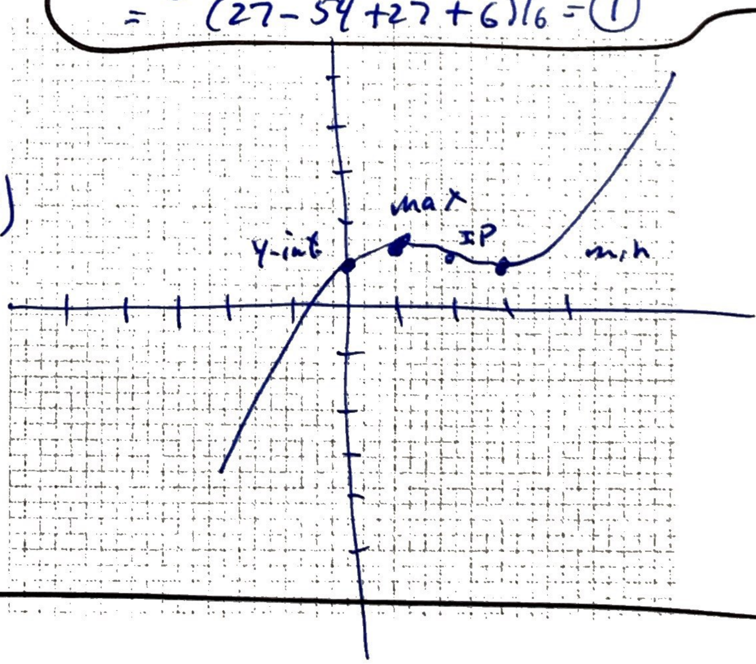
2

$y(2) = \frac{1}{6}(2^3 - 6(2)^2 + 9 \cdot (2) + 6)$

$= \frac{1}{6}(8 - 24 + 18 + 6)$

$= \frac{8}{6}$

$= \frac{4}{3}$



(vii) Sketch

2

10

4. (15 pts) Determine the absolute extrema for the following function and interval. Sketch.

$$y = x + \frac{1}{x} = \frac{x^2}{x} + \frac{1}{x} = \frac{x^2+1}{x}$$

(i) horizontal asymptote (get a common denominator first):

2

HA is a slant (oblique) asymptote

$\boxed{\lim_{x\to\pm\infty} y = x}$ since $\frac{1}{x} \to 0$ as $x \to \pm\infty$

(ii) vertical asymptote: $\boxed{x = 0}$

2

No need to long divide since $y = x + \frac{1}{x}$ would be the results.

(ii) critical points (y'=0):

3

$y' = 1 + (x^{-1})'$

$= 1 - x^{-2}$

$\boxed{y' = 1 - \frac{1}{x^2}}$

@ extrema $0 = 1 - 1/x^2$

$1 = 1/x^2$ $\boxed{x = \pm 1}$

(iv) concavity (y'' > 0 or < 0):

3

$y'' = 0 - (x^{-2})'$

$y'' = 0 - (-2)x^{-3} = \boxed{2/x^3}$

@ $x = -1$: $y'' = 2/(-1)^3 = -2 < 0$ max

@ $x = +1$: $y'' = 2/1^3 = 2 > 0$ min

10

Continued ->

(v) state if critical points are local maximums or minimums

@ $x = -1$ y is a local max

1 @ $x = 1$ y is a local min

(vi) inflection poin(s)t:

1 $y'' = \frac{2}{x^3}$

$0 =$ not possible – No Inflection Point

(vii) Sketch

Helper Pts: $f(-1) = -1 + \frac{1}{-1} = -2$

$f(1) = 1 + \frac{1}{1} = 2$

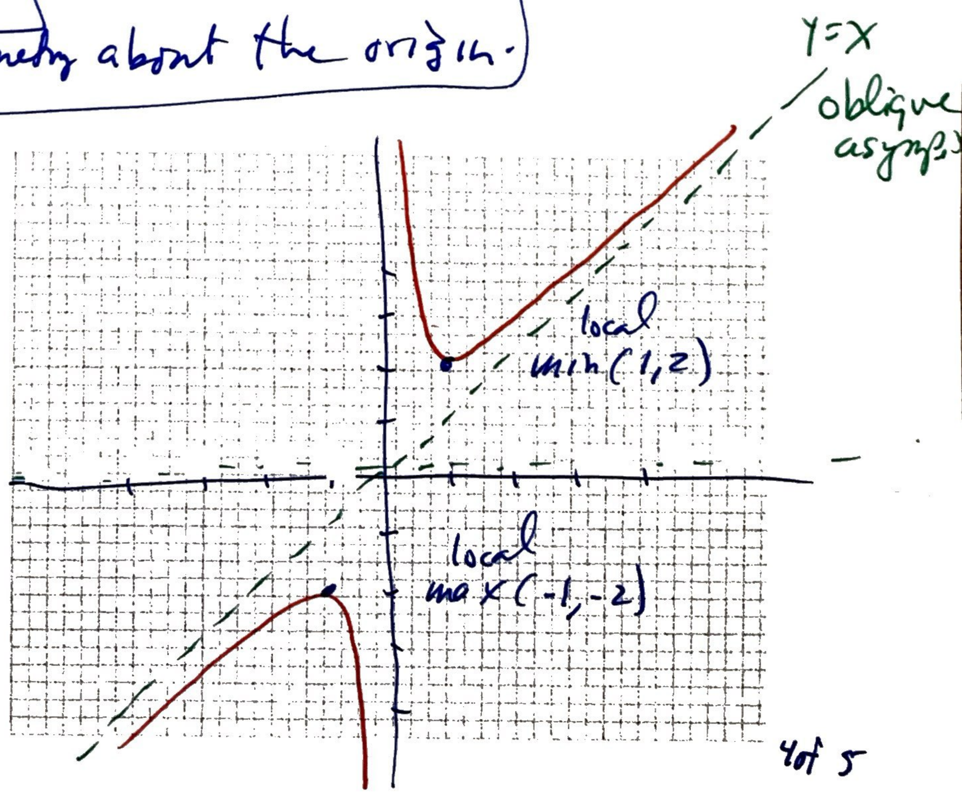
NOTE: $f(x) = x + \frac{1}{x}$

$f(-x) = -x + \frac{1}{-x}$

$\boxed{f(-x) = -f(x)}$

odd function : symmetry about the origin.

3



5)

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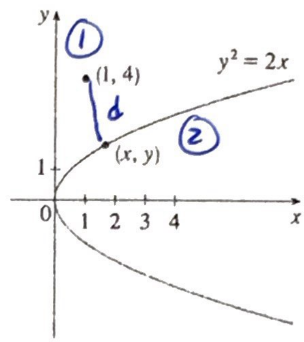
5. (15 pts) Find the point on the parabola $y^2 = 2x$ that is closest to the point (1, 4). Use the distance between two points formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



$d(x,y) = \sqrt{(x-1)^2 + (y-4)^2}$

let optimize d^2 : call it D

$D = (x-1)^2 + (y-4)^2$

5 but constrain the curve to be on $y^2 = 2x$

so as to avoid radicals, $\sqrt{\ }$, lets substitute $x = y^2/2$

$\Rightarrow D(y) = (y^2/2 - 1)^2 + (y-4)^2 \leftarrow$ optimize wrt. "y"

6l

$\frac{dD(y)}{dy} = 2(\frac{y^2}{2} - 1)(y^2/2 - 1)' + 2(y-4)\underbrace{(y-4)'}_{1}$ 7

5 $0 = (y^2 - 2)(y) + 2y - 8$

$0 = y^3 - \cancel{2y} + \cancel{2y} - 8$

$8 = y^3 \rightarrow \boxed{y = 2}$

3 $D'' = 3y^2$ always (+) so $y = 2$ is a min

Q: What is "x"? Use the curve $y = 2x \Rightarrow (2)^2 = 2x$ so $\boxed{x = 2}$

2 Ans: the closest point on the curve $y^2 = 2x$ to the point (1,4) is $\boxed{(x, y) = (2, 2)}$

15