Math 211 Fa25 CoC R. Erickson 60/60 TEST 2 Name KEY

Show ALL work for FULL credit. All problems are 5 pts each unless otherwise noted.

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Go for the most points... Don't get bogged down on any one problem.

1. Sec2.1 Def of Derivative (10 pts) Find the derivative of $f(x) = 2x^2 - 16x + 35$ using the definition of the derivative:

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{[2(x+h)^2 - 16(x+h) + 35] - [2x^2 - 16x + 35]}{h}$$ 5

$$= \lim_{h\to 0} \frac{2(x^2+2xh+h^2) - 16x - 16h + 35 - 2x^2 + 16x - 35}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{2x^2} + 4xh + 2h^2 - \cancel{16x} - 16h + \cancel{35} - \cancel{2x^2} + \cancel{16x} - \cancel{35}}{h}$$

10

$$= \lim_{h\to 0} \frac{4xh + 2h^2 - 16h}{h}$$ 3

$$= \lim_{h\to 0} (4x - 16 + 2h)$$

$$= 4x - 16 + 2\lim_{h\to 0}(h)$$

$$= 4x - 16 + 0$$

$$f'(x) = \boxed{4x - 16}$$ 2

10

2. **Sec 2.2 Derivatives (10 pts)** (a) Find the derivative of $f(x) = (x^3 + 3x^2 + 11)^{3/2}$

$$\boxed{f' = \frac{3}{2}\left(x^2+3x^2+11\right)^{\frac{3}{2}-1} \cdot \left(x^3+3x^2+11\right)'}$$

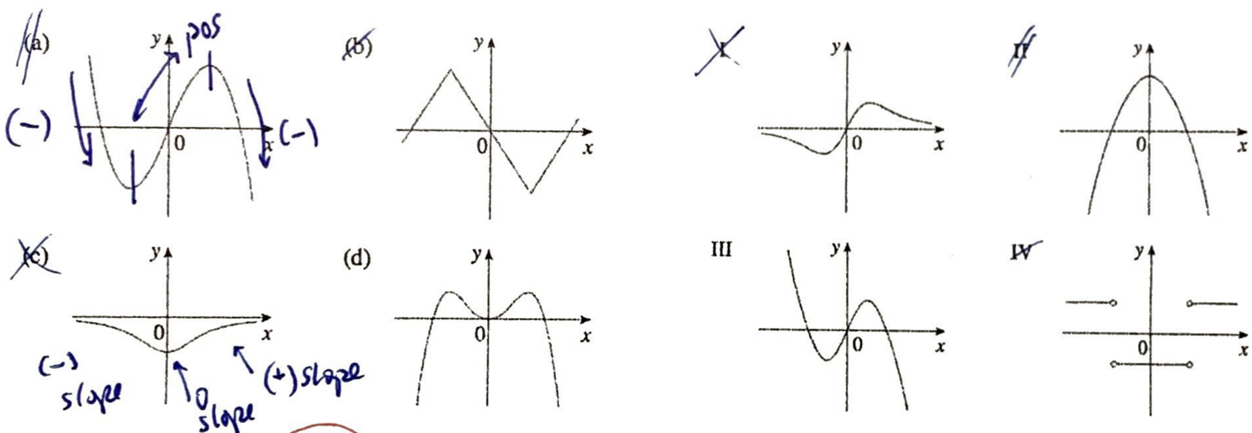
$$= \frac{3}{2}\left(x^2+3x^2+11\right)^{1/2} \cdot \left(3x^2+6x+0\right)$$

5

$$= \frac{3(3x^2+6x)}{2}\sqrt{x^2+3x^2+11}$$

$$= \boxed{\frac{9x^2+18x}{2}\sqrt{x^2+3x^2+11}}$$

(b) Match the graph of each function in (a)–(d) with the graph of its derivative in I–IV. Give reasons for your choices.



5

(a) matches with II because slope goes (+) for a moment the back to (−)

(b) matches with IV because slopes are constants.

(c) matches with I because (−) → 0 → (+) slope

(d) matches with III because quartic has a cubic derivative

10

3. Sec 2.3 Rules (10 pts) Find the derivatives:

(a) $y = \dfrac{x^{1/3}}{1+x^2}$

5

$$y' = \frac{(x^{1/3})'(1+x^2) - (x^{1/3})(1+x^2)'}{(1+x^2)}$$

$$= \boxed{\frac{\frac{1}{3}x^{-2/3}(1+x^2) - x^{1/3}(2x)}{(1+x^2)^2}} \cdot \frac{3}{3}$$

$$= \frac{x^{-2/3}(1+x^2) - 6x\,x^{1/3}}{3(1+x^2)^2} \cdot \frac{x^{2/3}}{x^{2/3}}$$

$$= \frac{(1+x^2) - 6x\,x^{1/3+2/3}}{3x^{2/3}(1+x^2)^2}$$

$$= \frac{1+x^2-6x^2}{3x^{2/3}(1+x^2)^2}$$

$$= \boxed{-\frac{5x^2-1}{3x^{2/3}(1+x^2)^2}}$$

(b) $y = (x - \dfrac{2}{x})(7-2x^3)$

$$y' = (x-\tfrac{2}{x})'(7-2x^3) + (x-\tfrac{2}{x})(7-2x^3)'$$

$$= \left[(x)' - 2(x^{-1})'\right](7-2x^3) + (x-\tfrac{2}{x})(-2\cdot 3x^2)$$

5

$$= \left[1-2(-1)(x^{-2})\right](7-2x^3) + (-6)(x^3-2x)$$

$$\boxed{y' = \left[1+\frac{2}{x^2}\right](7-2x^3) - 6(x^3-2x)}$$

$$y' = 7 + \frac{14}{x^2} - 2x^3 - 4x - 6x^3 + 12x$$

$$= -8x^3 + 8x + 7 + \frac{14}{x^2}$$

10

4. **Sec 2.5 Trig (10pts)** (a) Use the quotient rule to prove $\frac{d}{dx}\frac{\tan(x)}{} = sec^2(x)$

4 $\frac{d}{dx}\left(\frac{\sin(x)}{\cos(x)}\right)$

4 $= \frac{(\sin(x))'\cos(x) - \sin(x)(\cos(x))'}{(\cos(x))^2}$

$= \frac{\cos(x)\cos(x) - \sin(x)(-\cos(x))}{\cos^2 x}$

1 $= \frac{\cos^2(x) + \sin^2(x)}{\cos^2(x)}$ $= 1$

$= \frac{1}{\cos^2(x)}$

1 $= \boxed{\sec^2(x)}$ since $\sec(x) = \frac{1}{\cos(x)}$

(b) Differentiate $H(x) = x^4 - 9\sin(x) + 2\tan(x)$

H'(x) = $4x^3 - 9\cos(x) + 2\sec^2(x)$

10

5. **Sec 2.6 Implicit (10pts)** Find y' for the implicit function: $y = x^4 + 5y^3$

$$\frac{d}{dx}\left(y = x^4 + 5y^3\right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{dx^4}{dx} + 5\frac{dy^3}{dx}$$

7 $$\Rightarrow \boxed{\frac{dy}{dx} = 4x^3 + 15y^2\left(\frac{dy}{dx}\right)}$$

$$y' = 4x^3 + 15y^2 y'$$

$$y' - 15y^2 y' = 4x^3$$

$$y'\left[1 - 15y^2\right] = 4x^3$$

3 $$\boxed{\frac{d}{dx}(y) = \frac{4x^3}{[1 - 15y^2]}}$$

10

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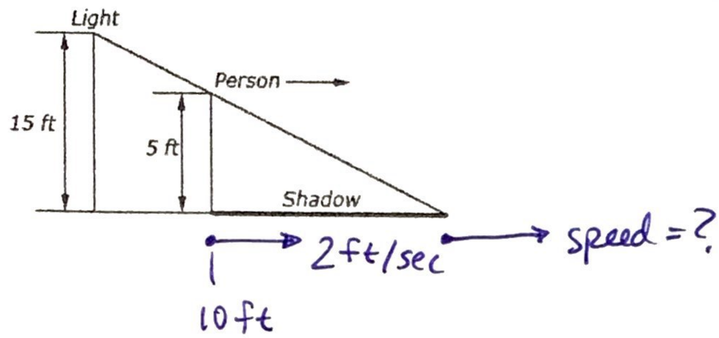
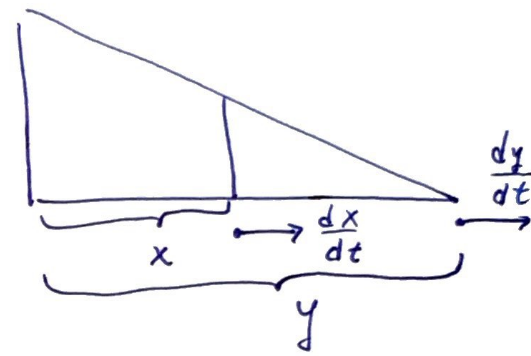
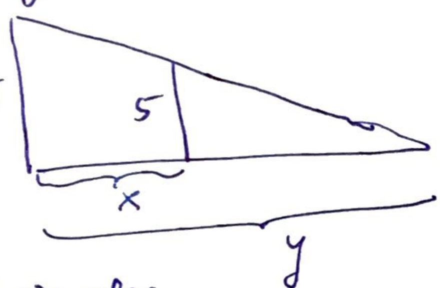
6. Sec 2.8 Related Rates (10pts) A light is on the top of a 15 foot tall pole. A 5 foot tall person starts at the pole and moves away from the pole at a rate of 2 ft/sec. After moving 10 feet from the wall at what rate is the tip of the person's shadow moving away from the ~~wall~~ pole?



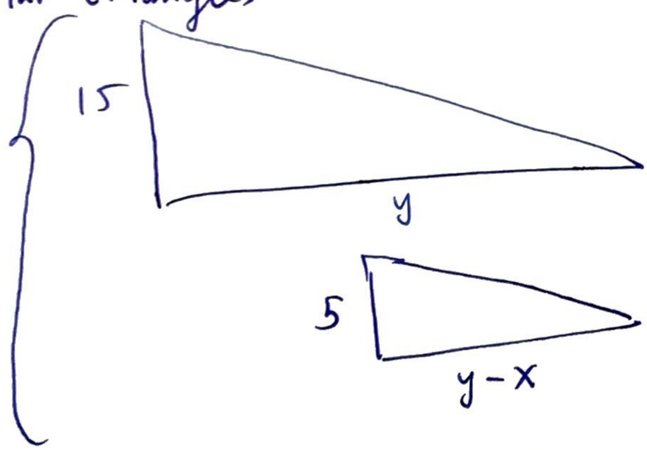


• geometry

1



• similar triangles



• So 15 is to y as 5 is to y−x

5 on $\left(\frac{15}{y}\right) = \left(\frac{5}{y-x}\right)$

$\Rightarrow 15y - 15x = 5y$

$\Rightarrow 10y = 15x$

$\boxed{2y = 3x}$

10

• rates:

$$2\frac{dy}{dt} = 3\frac{dx}{dt}$$

→ 2

$$\boxed{\frac{dy}{dt} = \frac{3}{2}\cdot\frac{dx}{dt}} \quad @ \ \frac{dx}{dt} = 2\text{ft/s}$$

$$= \frac{3}{2}(2\text{ ft/s})$$

→ 2

$$\boxed{\frac{dy}{dt} = 3\text{ ft/s}}$$

{@ x = 10ft is not used only in this case}